# Studies on the Design of Control Systems for High Gain Adaptive Feedback Control of Uncertain Nonlinear Systems

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Ryuji Michino

Graduate School of Science and Technology
KUMAMOTO UNIVERSITY

# Abstract


A nonlinear system is called OFEP(Output Feedback Exponentially Passive) if there exists an output feedback control such that the resulting closed-loop system is EP(Exponentially Passive). The sufficient conditions for a nonlinear system to be OFEP are given as follows:

(1) the system has a relative degree of 1.

(2) the system is globally exponential minimum-phase.

(3) the nonlinearities of the controlled system satisfy the Lipschitz condition.

(4) the coefficients in the control input term are known or bounded.

Under these conditions one can design a robust adaptive control system based on high gain output feedback with a simple controller structure and high robustness with respect to disturbances and unmodelled dynamics. However, since most practical systems do not satisfy the OFEP conditions, the OFEP conditions have imposed very severe restrictions to practical applications of OFEP based adaptive output feedback control.

The objective of this work is to expand the applicable class of the output feedback based robust adaptive control to a wider class of nonlinear systems, including uncertain nonlinear systems with non-Lipschitz nonlinearities, uncertain and unbounded coefficients in the control input terms and uncertain nonlinear systems having a higher order relative degree, etc.

In this thesis, a basic controller design method for OFEP nonlinear systems is reviewed in chapter 2 in order to help the understanding of the following chapters. The definitions of *OFEP* and *relative degree*, and a basic algorithm for robust adaptive output feedback control for OFEP nonlinear systems are presented. In chapter 3, a robust adaptive output feedback controller is designed for uncertain nonlinear systems with non-Lipschitz nonlinearities. This controller can be also applied for the systems with unknown and unbounded functions in the control input term. Chapter 4 presents a controller design scheme for nonlinear systems with nonparametric uncertainties and a higher order relative degree based on a high gain state feedback control. Although the control scheme requires all states of the controlled system, a wider class of controlled systems can be stabilized by this method. In chapter 5, a robust adaptive output feedback control system is designed for non-OFEP nonlinear systems with a higher order relative degree and non-Lipschitz nonlinearities. This output feedback control system is designed by introducing a virtual control input filter instead of a state observer. This control system is developed in chapter 6. Since the control system proposed in chapter 5 has a complex controller structure when a controlled system has a higher order relative degree, a new controller design method, which we call *one-step backstepping*, is proposed. In this method, a compensator is introduced to the virtual control input filter and a robust adaptive control system is designed by backstepping strategy of only one step. In each chapter, the effectiveness of proposed control systems are confirmed through numerical simulations.

# Contents

# Chapter 1

# Introduction

## 1.1 Historical Review

The first step of a control system design is to analyze and construct models for the controlled systems and after that the appropriate controller is designed in order to attain each control objective for the obtained models of controlled systems[1-3]. However, since it is difficult to construct an exact model that reflects the details of the controlled system, most constructed models have some unmodelled uncertainties and/or varying characteristics that disturb the performance of a control system. Therefore, in designing control systems, we have to consider the uncertainties and the unmodelled dynamics adequately. Robust control and adaptive control methods are known as effective control methods for systems with uncertainties.

In the traditional robust control schemes, a controller is designed so that the resulting closed-loop system will be stable for the controlled system with uncertainties whose variations and magnitudes are estimated in advance[4-8]. Although the stability of the control system is assured by applying a controller designed through robust control schemes, control performances of the control system depend on variations and magnitudes of the uncertainties. Further if they become too large then the control system may become unstable.

On the other hand, a controller designed through adaptive control schemes estimates variations of the characteristics in controlled system and adjusts the controller parameters online according to the estimated data. Thus, we can expect adaptive control systems to perform optimal control even if the characteristics varies more than anticipated, because the controller parameters are determined in order to preserve the stability and the optimal conditions in the control system.

The study of adaptive control was started in the middle of 1950's and the theory of adaptive control for time-invariant linear systems was almost completely defined by the late 1970's[9-11]. However, a controller designed through the traditional adaptive control schemes, including MRAC(Model Reference Adaptive Control), has a complex controller structure because the controller is designed under the assumption that the order of controlled system is known and the structure of the control system depends on that order. Additionally, it has poor robustness for unexpected unmodelled uncertainties and disturbances. Most obtained models for controlled systems have some unmodelled uncertainties that occur from linearization or dimension reduction, and the assumption that the order of controlled system is known is impractical. This means that it is difficult to apply the controllers for practical systems.

From the practical aspect, the study of adaptive control based on high gain feedback was started about the beginning of 1980's parallel to the studies of above mentioned adaptive control schemes. High gain feedback strategy with static feedback gain is a traditional controller design scheme, which improves the speed of response of controlled systems and represses the effects from disturbances and/or unmodelled uncertainties by increasing the feedback gain. Further, with the information about the relative degree and stability of zeros of a controlled system, this control scheme is able to give a robust stability of the closed-loop system without information about the order of controlled system by applying a sufficiently large feedback gain[12,13]. Although the parameters in high gain feedback control, which ensure the stability of the closed-loop system, depend on the unknown parameters in controlled systems, we can design stabilizing controllers by means of online adjustment of the parameters in adaptation. Such adaptive controllers are called ***High gain adaptive controllers*** or ***Universal adaptive stabilizers***[13–17]. High gain adaptive control has the following features compared with MRAC:

- The controllers are designed without the information about the order of controlled system.
- It requires a few estimators.
- The controllers have high robustness for disturbances and unmodelled uncertainties.

From the above features, we can design adaptive control systems with simple structure through high gain feedback with less information about the controlled systems than MRAC. Therefore, researches concerning high gain adaptive control have been carried out for several control problems: for example the regulator control[13,14,17,18], the tracking control[16,19,20], the decentralized control[21–23] and the control for the infinite dimensional systems[15,24].

At the same time Sobel *et al.* have proposed a kind of MRAC with the simple controller structure and with a high gain feedback[25]. This controller can attain the output tracking by introducing the CGT(Command Generator Tracker) theory[26], which allows absolute tracking of a reference model output using a feedforward input. This control method is called DMRAC(Direct Model Reference Adaptive Control) or SAC(Simple Adaptive Control). Kaufman[27,28], Bar-Kana[29–31] and Iwai[32–35] have expanded on the theory and applications of this control method. In order to design an adaptive control system through SAC method, the following three assumptions that are required.

- The controlled system has a relative degree of 1.
- The controlled system is minimum-phase.
- The high frequency gain is positive.

The above three conditions are known as ASPR(Almost Strictly Positive Real) conditions[25,27–34]. Under these conditions there exists a static output feedback gain which renders the closed-loop system SPR(Strictly Positive Real)[36]. However, unfortunately ASPR conditions are severe restrictions for practical systems. Therefore it is critically important to alleviate the restrictions for applying the ASPR based adaptive controllers with simple structure and robustness to practical systems.

A popular alleviation methods is the introduction of a PFC(Parallel Feedforward Compensator) in parallel with the non-ASPR systems. In this method, we design a PFC to make the resulting augmented system with the PFC be ASPR. Control systems based on a high gain output feedback are designed for this augmented system. This method alleviates relative degree and minimum-phase restrictions in ASPR conditions. This idea was devised initially by Bar-Kana[29] and after that Iwai *et al.*[32–34], Mizumoto *et al.*[35, 37–39] and Ozcelik *et al.*[40, 41] has proposed some design methods of PFC for the controlled systems with several uncertainties. Further, Shibata *et al.*[42, 43] and Ohtsuka *et al.*[44] have expanded the control method for the discrete time systems. Introducing a PFC is a simple alleviation method, however it has been pointed out that the bias error from the PFC output may remain since the controller is designed for the augmented controlled system with the PFC.

On the other hand Morse has proposed a high gain adaptive control method for systems with a relative degree of 1 or 2[14, 19] and Khalil and Saberi has proposed a high gain adaptive control method for systems with a higher order relative degree[13]. In these methods the adaptive controllers are designed by introducing dynamic compensators, whose order is one less than the relative degree of the controlled system, into the control system. However the parameter adjusting laws of feedback gain become rather complicated.

In 1990's, an alleviation method for the relative degree restriction has been proposed that utilizes backstepping strategy. Originally the backstepping strategy was devised by Kaellakopoulos *et al.* to make a control system be positive real for the controlled system with a higher order relative degree[45]. The backstepping strategy is often utilized for controller designs[46–49], however this strategy requires all state variables of the controlled system. Therefore, in the case where all state variables are not available, we have to design a state estimator such as an observer[50–53]. Later, Takahashi *et al.* designed a high gain output feedback based adaptive control system for the non-ASPR linear systems with a higher order relative degree by introducing a virtual filter and applying backstepping strategy to the filter dynamics without a state estimator[54–57].

The study of adaptive control for linear systems mentioned above has recently led to a lot of study on control system design for nonlinear systems[58–63]. At present, the controllers based on linear control theory are applied in most practical job sites[64]. However, since most systems have some nonlinearities, the controllers based on nonlinear control theory can be expected to give us good control performance. Further, for the systems whose linear approximations are uncontrollable and unobservable we can control them by nonlinear control theory in some cases, even though the linear control theory can not control such systems[60, 65].

The study of adaptive control for nonlinear systems started in the late 1980's and many sort of adaptive strategies for nonlinear controlled systems were proposed in the early 1990's. Most of them, unfortunately, had some restrictions such that the controlled system is feedback linearizable and satisfies the Lipschitz or the matching conditions[66–69]. After that Kaellakopoulos *et al.* presented a design method for an adaptive controller for the controlled systems that do not satisfy the matching condition and have strong nonlinearities[70]. Now, such adaptive control system design has been expanded for generalized systems called *Strict-Feedback nonlinear system* or *Pure-Feedback nonlinear system*[49, 63, 71–75]. Furthermore Krstic *et al.* have solved the problem that the controller designed through traditional adaptive backstepping has a number of estimators by using tuning function technique[49, 76]. The backstepping method solves the matching condition problems and is also applicable to the wider class of nonlinear systems. However, the adaptive method was still less restrictive, that is, the method only handled the parametric uncertainties with unknown constant which appears linearly in the system equations. It might be important from the point of practical application to consider a robust adaptive control strategy for nonlinear uncertain systems with nonparametric uncertainties in nonlinear functions. For this point of view, a lot of adaptive controller design methods have been proposed for uncertain nonlinear systems with nonparametric uncertain nonlinearities and/or exogenous disturbances[77–86]. However, in these methods most controllers are designed by a state feedback. Therefore, these controllers may have rather complicated controller structure with an adaptive observer in the case where all the states are not available.

Around the same time high gain adaptive feedback control for nonlinear systems has been studied. This method has attracted a great deal of attention since this has a strong robustness for system uncertainties in spite of its simple structure. This method is one of the passivity based adaptive control strategy. The ASPR conditions, under which the resulting closed-loop system by an output feedback can be rendered SPR, is reconsidered for nonlinear systems. Fradkov and Hill defined the OFEP(Output Feedback Exponentially Passive) property for nonlinear systems corresponding to the ASPR-ness[87]. For OFEP nonlinear systems as well as ASPR linear systems, we can design a robust adaptive controller with a simple structure. Fradkov *et al.*[87–89], Allgower *et al.*[90] and Mizumoto *et al.*[91] have proposed design methods for OFEP nonlinear systems. However, the OFEP conditions, which are given as:

- The controlled system has a relative degree of 1.
- The controlled system is exponential minimum-phase.
- The nonlinear functions are Lipschitz.
- The coefficients in the control input term are known or bounded.

are really severe restrictions for practical systems as well as ASPR conditions for linear systems. Therefore, to alleviate the OFEP conditions or design a robust control system for non-OFEP nonlinear systems are important issues to extend the practical applications of robust adaptive controllers based on output feedback with simple structures.

An introduction of a PFC is an alleviation method for non-OFEP nonlinear systems as well as for non-ASPR linear systems. Fradkov *et al.*[88,89] and Mizumoto *et al.*[91] have proposed practical design methods of a PFC. However, the bias error from the PFC output may remain since the controller is designed for the augmented controlled system with the PFC.

Marino and Tomei[92], Miyasato[93,94], Xudong[95] and Ding[96] have expanded the method introducing a virtual filter and design a controller through backstepping for nonlinear systems. However, they dealt with the controlled systems with known coefficients in the control input term[93–95] or with parametric uncertainties composed by known functions and unknown constants[92,96].

As mentioned above, an adaptive control system based on high gain output feedback is a powerful control scheme for practical systems because the controller has simple controller structure with a high robustness for disturbances and unmodelled dynamics. However, the control method for nonlinear systems has been researched for only a few years and the basic design of control system for OFEP nonlinear systems and a few alleviation methods for OFEP restrictions have just been proposed. Therefore, it is important to much study high gain feedback based adaptive control strategy for expanding the applicable situations of simple and robust adaptive control.

## 1.2 Outline of the Dissertation

The objective of this work is to design robust adaptive controllers for non-OFEP nonlinear systems and to propose alleviation methods for restrictions in OFEP conditions in order to expand the applicable class of high gain feedback based adaptive control systems. Proposed adaptive controls are very useful and powerful control tools for practical systems with several uncertainties. The contents are organized as follows.

In chapter 2, some definitions concerning OFEP for nonlinear systems and the basic design method for adaptive control system based on high gain output feedback for OFEP nonlinear systems are reviewed. Furthermore, a brief introduction about a PFC, which is an alleviation method for OFEP restrictions, is given.

In chapter 3, an adaptive control system based on high gain output feedback for uncertain nonlinear systems is proposed. This chapter deals with non-OFEP nonlinear systems with non-Lipschitz uncertainties and unknown and unbounded coefficients in the control input term. For such a system, a robust adaptive control system with simple structure is designed without information about the order of controlled system.

In chapter 4, a state feedback based high gain adaptive control system is designed. Although the control system designed in chapter 3 can be applied to non-OFEP nonlinear systems with non-Lipschitz nonlinearities, it is only applicable to systems having a relative degree of 1. In this chapter, under the assumption that all state variables are available we design a high gain based adaptive feedback control system through backstepping strategy. This control system can be applied for uncertain nonlinear systems with a higher order relative degree and non-Lipschitz nonlinearities in the control input terms.

In chapter 5, a design method for an output feedback based adaptive control system for time-varying nonlinear systems with a higher order relative degree is designed. In chapter 4, a high gain feedback based state feedback adaptive control is proposed for nonlinear systems with a higher order relative degree however, all the state variables are not always available in practical applications. In this chapter, an output feedback based adaptive control system is designed for non-OFEP nonlinear systems with a higher order relative degree and non-Lipschitz nonlinearities by introducing a virtual filter and applying backstepping strategy.

In chapter 6, we expand the controller design method proposed in chapter 5. We propose a novel adaptive controller design scheme which can be designed by backstepping strategy of only one step. We call the method *one-step backstepping*. Although the control system proposed in chapter 5 can be applied for nonlinear systems with a higher order relative degree and designed based on output feedback, since the structure of the control system depends on the order of the relative degree, the structure becomes complex for systems with a higher order relative degree. In this chapter, we introduce a PFC in parallel with a virtual filter so that the augmented virtual filter has a relative degree of 1 and design an output feedback based adaptive control system through backstepping of only one step.

# Chapter 2

# Basic Design of High Gain Adaptive Output Feedback Control System

## 2.1 Introduction

The linear plant is said to be ASPR if there exists a static output feedback such that the resulting closed-loop system is SPR[36]. It is well known that, for the ASPR plants, one can design a stable control system via adaptive output feedback with the very simple controller structure[27–35]. Unlike other adaptive methods, under the ASPR condition, we are able to design the adaptive controller without a priori information of the controlled plants (e.g. order of the plant and the size of the uncertainties). As for the nonlinear systems the condition of the high gain output feedback stabilization is recognized as output feedback exponential passivity (OFEP)[87,89]. That is, one can design the adaptive output feedback controller for OFEP nonlinear systems as well as ASPR linear systems without a priori information about the order of the controlled system. However since almost all practical systems are not OFEP nonlinear systems, some alleviative strategies to the OFEP restriction are required in order to apply the high gain adaptive output feedback control system to practical systems.

From the next chapter, several robust control system designs for high gain adaptive feedback control of nonlinear systems will be presented.

In this chapter, for easy understanding of a basic concept for control system design in the following chapters, some definitions concerning *OFEP* and a basic design scheme for a high gain adaptive output feedback control system for OFEP nonlinear systems are reviewed. Further, one of the most popular alleviation method for the OFEP restriction, introduction of a PFC, is presented.

## 2.2 Passivity and System Expressions

Consider the following single input and single output nonlinear system:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{2.1}$$

where $x \in R^n$ is the state variable and $u, y \in R$ are the control input and output, respectively. $f(x) : R^n \to R^n$, $g(x) : R^n \to R^n$ and $h(x) : R^n \to R$ are smooth functions. In the sequel we assume that $f(0) = 0,\ h(0) = 0$.

First of all, some definitions regarding to *passivity* and *relative degree* for the controlled system (2.1) are given as follows.

**Definition 2.1 (Passivity[97,98]).** *The system (2.1) is called passive if there exists a nonnegative function $V : R^n \to R$, $V(0) = 0$, $\forall t \geq 0$ such that for all $t \geq 0$, $u \in R$, $x(0) \in R^n$*

$$V(x(t)) - V(x(0)) \leq \int_0^t y(\tau)u(\tau)d\tau. \tag{2.2}$$

**Definition 2.2 (Strict passivity[97,98]).** *The system (2.1) is called strictly passive if there exists a nonnegative function $V : R^n \to R$, $V(0) = 0$, $\forall t \geq 0$ and positive definite function $S(x) : R^n \to R$ such that for all $t \geq 0$, $u \in R$, $x(0) \in R^n$*

$$V(x(t)) - V(x(0)) \leq \int_0^t y(\tau)u(\tau)d\tau - \int_0^t S(x(\tau))d\tau. \tag{2.3}$$

**Definition 2.3 (Exponential passivity(EP)[87,91]).** *A strictly passive system (2.1) is called exponentially passive if there exist positive numbers $\alpha_1, \alpha_2, \alpha_3$ such that the following inequalities hold:*

$$\begin{aligned} \alpha_1\|x\|^2 &\leq V(x) \leq \alpha_2\|x\|^2 \\ \alpha_3\|x\|^2 &\leq S(x) \end{aligned} \tag{2.4}$$

*for any $x$-solution to equation (2.1).*

**Definition 2.4 (Output feedback exponential passivity(OFEP)[87,91]).** *The system (2.1) is called output feedback exponentially passive, if there exists smooth output feedback:*

$$\begin{aligned} u(t) &= r(y) + \Omega(y)v(t) \\ r(0) &= 0,\ \Omega(0) = 0 \end{aligned} \tag{2.5}$$

*such that the resulting closed-loop systems with an input $v(t)$ and an output $y(t)$ is exponentially passive.*

**Definition 2.5 (Relative degree[58,62,99]).** *A system (2.1) is said to have a relative degree $r$ at $x^0 \in R^n$, if*

$$\begin{aligned} &L_g h(x) = L_g L_f(x) = \cdots = L_g L_f^{r-2} h(x) = 0 \\ &L_g L_f^{r-1} h(x) \neq 0,\ \forall x \in R^n \end{aligned} \tag{2.6}$$

*for all $x$ in a neighborhood of $x^0$. Where $L_f h(x), L_g h(x)$ are expressed Lie derivative of $h(x)$ with respect to $f(x)$ and $g(x)$, respectively.*

**Definition 2.6 (Uniform relative degree[99]).** *The system (2.1) is said to have an uniform relative degree $r$ if it has a relative degree $r$ for all $x$.*

Here, we assume that the system (2.1) has a relative degree $r$. Then, it is well known that there exists a smooth nonsingular variable transformation $z = [z_1, \cdots, z_n]^T = \Phi(x)$ such that the system (2.1) can be transformed into a normal form[58,62]:

$$\begin{aligned} \dot{z}_i &= z_{i+1},\ (i = 1, \cdots, r-1) \\ \dot{z}_r &= a(\xi, \eta) + b(\xi, \eta)u \\ \dot{\eta} &= q(\xi, \eta) \end{aligned} \tag{2.7}$$

where $\boldsymbol{\xi} = [z_1, \cdots, z_r]^T$, $\boldsymbol{\eta} = [z_{r+1}, \cdots, z_n]^T$ and

$$a(0, 0) = L_f^r h(0) = 0, \ q(0, 0) = 0$$
$$b(\boldsymbol{\xi}, \boldsymbol{\eta}) = L_g L_f^{r-1} h(x) \neq 0, \ \forall x \in R^n.$$

## 2.3 Adaptive Output Feedback Control for OEFP Nonlinear Systems

The sufficient conditions for the system (2.1), which can be transformed into the normal form (2.7), to be OFEP are clarified by Fradkov and Hill[87].

**[OFEP conditions]**

(1) The system (2.1) has a relative degree of 1.

(2) The system (2.1) is exponential minimum-phase. That is, the zero dynamics of the system (2.1):

$$\dot{\boldsymbol{\eta}} = q(0, \boldsymbol{\eta}) \tag{2.8}$$

that is defined from the normal form (2.7) is exponentially stable.

(3) The functions $a(\boldsymbol{\xi}, \boldsymbol{\eta})$ and $q(\boldsymbol{\xi}, \boldsymbol{\eta})$ in the normal form (2.7) are Lipschitz with respect to $(\boldsymbol{\xi}, \boldsymbol{\eta})$. Where, $\boldsymbol{\xi} = z_1$ from the condition (1).

(4) The function $b(\boldsymbol{\xi}, \boldsymbol{\eta})$ in the normal form (2.7) is factorized by $b(\boldsymbol{\xi}, \boldsymbol{\eta}) = b_0 b_1(y)$, where $b_0$ is a positive constant and $b_1(y)$ is a known strictly positive function.

Under these OFEP conditions, there exists a positive constant $k_0$ such that the resulting closed-loop system with an input:

$$u(t) = -ky(t)/b_1(y) + v(t)/b_1(y), \ \forall k \geq k_0 \tag{2.9}$$

is rendered EP from the new input $v(t)$ and output $y(t)$[87].

Now, we assume that the system (2.1) is satisfied the OFEP conditions. An adaptive output feedback controller can be designed by

$$u(t) = -k(t)y(t)/b_1(y) \tag{2.10}$$
$$\dot{k}(t) = \gamma y^2(t). \tag{2.11}$$

The following theorem concerning the stability of the control system with the controller (2.10) and (2.11) is given[91].

**Theorem 2.1.** *Suppose that the system* (2.1) *is transformable into a normal form* (2.7) *and satisfies the OFEP conditions. Then, all the signals in the control system with the controller* (2.10) *and* (2.11) *are bounded and* $\lim_{t\to\infty} y(t) = 0$.

**Remark 2.1.** *From Theorem* 2.1, *we can see that one can design a relatively simple adaptive control system for OFEP nonlinear systems without the information about the order of the controlled system. Furthermore, since this control system is designed based on high gain output feedback, basically it has high robustness with respect to bounded disturbances and noise, with a slight modification in the adaptive adjusting law* (2.11)[89, 90].

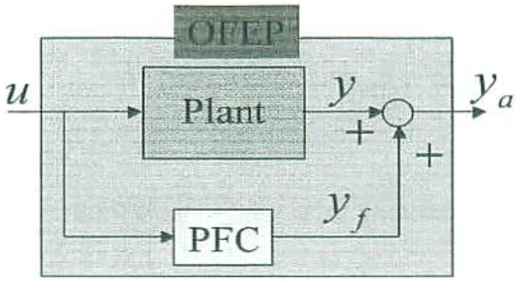


Figure 2.1: Augmented system with a PFC

## 2.4 Introducing a PFC to the non-OFEP Nonlinear Systems

One can design a relatively simple adaptive control system with high robustness with respect to bounded disturbances for OFEP nonlinear systems. Unfortunately however, most real systems do not satisfy the OFEP conditions. In this section, one of the most practical alleviation methods for the OFEP conditions is reviewed. An introduction of a PFC is a simple and easy alleviation method of OFEP conditions. This method can alleviate the relative degree and minimum-phase restrictions.

Introduce a PFC in parallel with the non-OFEP nonlinear system (See Fig.2.1). This PFC is designed so that the augmented system with the PFC is OFEP. Thus we can apply an adaptive controller (2.10) for this augmented system directly. Fradkov *et al.*[89] and Mizumoto *et al.*[91] have proposed design principles of the PFC for alleviating the relative degree restriction and Desaki *et al.*[100] and Kiyama *et al.*[101] have proposed design method for the PFC to alleviate the minimum-phase restriction.

**Remark 2.2.** *An introduction of a PFC expands the applicable class of the high gain output feedback control strategy. However, using this method, it has been pointed out that the bias error, which is occurred from the PFC output, may remain in the tracking control because we design the control system for the augmented system with the PFC.*

## 2.5 Conclusion

In this chapter, some definitions concerning *OFEP* and a basic design of a high gain adaptive output feedback control system for OFEP nonlinear systems are reviewed. Additionally, one of the most practical alleviation methods (introduction of a PFC) for the OFEP conditions is also presented.

# Chapter 3

# Design of High Gain Adaptive Output Feedback Control System for Uncertain Nonlinear Systems

## 3.1 Introduction

In chapter 2, an adaptive control system based on a high gain output feedback for OFEP nonlinear systems, which has a relatively simple controller structure, was presented. However, since most practical systems do not satisfy the OFEP conditions, we have to alleviate the OFEP conditions or have to design a robust controller for non-OFEP nonlinear systems to apply the high gain adaptive output feedback control strategy to practical systems. An introduction of a PFC is an alleviation method for non-OFEP nonlinear systems as well as for non-ASPR systems[88, 89, 91]. Unfortunately, the bias error from the PFC output may remain since the controller is designed for the augmented controlled system with the PFC.

Recently, robust adaptive output feedback control schemes for OFEP nonlinear systems with output dependent non-Lipschitz uncertainties and/or disturbances have been proposed by Fradkov *et al.*[89] and Kohara *et al.*[102]. Considering the nonlinear uncertain function as a kind of output dependent disturbance, the methods are able to deal with robust stabilization problems via high gain adaptive output feedback for nonlinear systems, for which some Lipschitz conditions on nonlinear functions are not satisfied with respect to output signal. In these methods, however, the uncertain nonlinearities in the control input term are restricted to be bounded or known.

In this chapter, we will show that we can remove the restriction that is imposed on the uncertainties in the control input term. That is, we propose a robust high gain adaptive output feedback strategy that can deal with a broader class of uncertain nonlinearities. Unlike previous high gain output feedback strategies, it is shown that we can design an adaptive output feedback controller for non-OFEP nonlinear systems with unbounded uncertainties in the control input term.

## 3.2 Problem Statement

Suppose that the controlled system (2.1) has an uniform relative degree 1. Then, the controlled system (2.1) can be transformed as

$$\begin{aligned} \dot{y} &= a(y,\eta) + b(y,\eta)u + f_1(t,y,\eta) \\ \dot{\eta} &= q(y,\eta) + f_2(t,y,\eta) \end{aligned} \tag{3.1}$$

where $a(y,\eta), q(y,\eta) b(y,\eta)$ and $f_1(t,y,\eta), f_2(t,y,\eta)$ are uncertain nonlinearities.

We impose the following assumptions on the controlled system (3.1).

**Assumption 3.1.** *The nominal part of the system* (3.1) *is exponential minimum-phase. That is, the zero dynamics of the nominal system:*

$$\dot{\eta} = q(0,\eta) \tag{3.2}$$

*is exponential stable.*

**Assumption 3.2.** *The uncertain function $q(y,\eta)$ is globally Lipschitz with respect to $(y,\eta)$, i.e., there exists a positive constant $L_1$ such that for any variables $y_1, y_2, \eta_1, \eta_2$*

$$\|q(y_1,\eta_1) - q(y_2,\eta_2)\| \leq L_1(|y_1 - y_2| + \|\eta_1 - \eta_2\|). \tag{3.3}$$

**Assumption 3.3.** *The uncertain function $a(y,\eta)$ is globally Lipschitz with respect to $(y,\eta)$, i.e., there exists a positive constant $L_2$ such that for any variables $y_1, y_2, \eta_1, \eta_2$*

$$|a(y_1,\eta_1) - a(y_2,\eta_2)| \leq L_2(|y_1 - y_2| + \|\eta_1 - \eta_2\|). \tag{3.4}$$

**Assumption 3.4.** *The uncertain function $f_1(t,y,\eta)$ can be evaluated by*

$$|f_1(t,y,\eta)| \leq \sum_{i=1}^{M_1} d_i|\psi_i(y)| + d_0 \tag{3.5}$$

*with known functions $\psi_i(y)$ and unknown positive constants $d_i$ and $d_0$.*

**Assumption 3.5.** *The uncertain function $f_2(t,y,\eta)$ can be evaluated by*

$$\|f_2(t,y,\eta)\| \leq \sum_{i=1}^{M_2} g_i|\phi_i(y)| + g_0 \tag{3.6}$$

*with unknown positive constants $g_i$ and $g_0$ and known functions $\phi_i(y)$ that have the following property for any variables $y_1$ and $y_2$:*

$$|\phi_i(y_1 + y_2)| \leq |\phi_{1i}(y_1,y_2)||y_1| + |\phi_{2i}(y_2)| \tag{3.7}$$

*with known function $\phi_{1i}(y_1,y_2)$ and unknown function $\phi_{2i}(y_2)$ which is smooth for all $y_2 \in R$.*

**Assumption 3.6.** *The uncertain function $b(y,\eta)$ can be evaluated by*

$$b(y,\eta) \geq b_0 > 0 \tag{3.8}$$

*where $b_0$ is an unknown positive constant.*

The control objective of this work is to design a robust adaptive controller which attains the goal:

$$\lim_{t\to\infty} |y(t) - y^*(t)| \leq \delta \tag{3.9}$$

for a given positive constant $\delta$ and a smooth reference signal $y^*(t)$ such that

$$|y^*(t)| \leq \beta_0, \ |\dot{y}^*(t)| \leq \beta_1 \tag{3.10}$$

where $\beta_0$ and $\beta_1$ are positive constants.

## 3.3 Robust Adaptive Controller Design

Under assumptions 3.1 to 3.6, we design an adaptive controller as follows:

$$u(t) = -\left[k(t)\nu(t) + \sum_{i=1}^{M_1} u_{fi}(t)\right] \tag{3.11}$$

where $\nu(t) = y(t) - y^*(t)$ and k(t) is an adaptive feedback gain which is adjusted by the following adjusting laws:

$$k(t) = k_I(t) + k_p(t) \tag{3.12}$$

$$\dot{k}_I(t) = \gamma_I \nu(t)^2 - \sigma_I k_I(t),\ k_I(0) \geq 0 \tag{3.13}$$

$$k_p(t) = \sum_{i=1}^{M_2} \gamma_{pi}\phi_{1i}(\nu, y^*)^4 \nu(t)^2 \tag{3.14}$$

where $\gamma_I, \gamma_{pi}$ and $\sigma_I$ are any positive constants and the proportional term $k_p(t)$ is the robust control term for the uncertain nonlinearity $f_2(t, y, \eta)$. Further, $u_{fi}(t)$ is also the robust adaptive control term for the uncertain nonlinearity $f_1(t, y, \eta)$ which is given by

$$u_{fi}(t) = \begin{cases} \left[\widehat{d}_i(t)\psi_i(t)\right]^2 \nu(t)/\varepsilon_{fi} & \text{if } \left|\widehat{d}_i(t)\psi_i(y)\nu(t)\right| \leq \varepsilon_{fi} \\ \widehat{d}_i(t)|\psi_i(y)|\mathrm{sign}(\nu(t)) & \text{if } \left|\widehat{d}_i(t)\psi_i(y)\nu(t)\right| > \varepsilon_{fi} \end{cases} \tag{3.15}$$

$$\dot{\widehat{d}}_i(t) = \gamma_{di}|\psi_i(y)||\nu(t)| - \sigma_{di}\widehat{d}_i(t),\ \widehat{d}_i(0) \geq 0 \tag{3.16}$$

where $\gamma_{di}, \sigma_{di}$ and $\varepsilon_{fi}$ are any positive constants.

## 3.4 Boundedness and Convergence Analysis

Applying the controller (3.11) to (3.16) to the controlled system (3.1) which satisfies the assumptions 3.1 to 3.6, the following theorem concerning the boundedness of all the signals in the control system and convergence of the tracking error is obtained.

**Theorem 3.1.** *Under assumptions* 3.1 *to* 3.6*, there exist* $\gamma_I, \gamma_{pi}, \gamma_{di}, \varepsilon_{fi}$ *and the ideal feedback gain* $k^*$ *such that all the signals in the closed-loop system with the controller* (3.11) *to* (3.16) *are bounded and the goal* (3.9) *is attained.*

*Proof.* Setting $\nu = y - y^*$ as the tracking error, the controlled system (3.1) can be rewritten as the following error system:

$$\begin{aligned} \dot{\nu} &= a(\nu + y^*, \eta) - b(\nu + y^*, \eta)\left[k\nu + \sum_{i=1}^{M_1} u_{fi}(\nu + y^*)\right] \\ &\quad + f_1(\nu + y^*, \eta) - \dot{y}^* \\ \dot{\eta} &= q(\nu + y^*, \eta) + f_2(\nu + y^*, \eta). \end{aligned} \tag{3.17}$$

From assumption 3.1 and the converse theorem of Lyapunov on exponential stability[62,66], there exists a positive definite function $W(\eta)$ and positive constants $\tau_1$ to $\tau_4$ such that

$$\begin{aligned} &\frac{\partial W(\eta)}{\partial \eta} q(0, \eta) \leq -\tau_1\|\eta\|^2,\ \left\|\frac{\partial W(\eta)}{\partial \eta}\right\| \leq \tau_2\|\eta\| \\ &\tau_4\|\eta\|^2 \leq \|W(\eta)\| \leq \tau_3\|\eta\|^2. \end{aligned} \tag{3.18}$$

Consider the following positive definite function:

$$V(\nu,\eta,k_I,\widehat{d}_i) = \mu W(\eta) + \frac{1}{2}\nu^2 + \frac{b_0}{2\gamma_I}[k_I - k^*]^2 + \sum_{i=1}^{M_1} \frac{b_0}{2\gamma_{di}}[\widehat{d}_i - d_i/b_0]^2 \tag{3.19}$$

where $\mu$ is any positive constant and $k^*$ is an ideal feedback gain for $k_I$ to be determined later.

The time derivative of $V$ along (3.13),(3.16) and (3.17) yields

$$\begin{aligned}\dot{V} = {} & \mu\frac{\partial W(\eta)}{\partial\eta}[q(\nu+y^*,\eta) + f_2(\nu+y^*,\eta)] + \nu\Big[a(\nu+y^*,\eta) + f_1(\nu+y^*,\eta) \\ & - \dot{y}^* - b(\nu+y^*,\eta)\{k\nu + \sum_{i=1}^{M_1} u_{fi}(\nu+y^*)\}\Big] + \frac{b_0}{\gamma_I}[k_I - k^*][\gamma_I\nu^2 - \sigma_I k_I] \\ & + \sum_{i=1}^{M_1}\frac{b_0}{\gamma_{d_i}}[\widehat{d}_i - \frac{d_i}{b_0}][\gamma_{d_i}|\psi_i(\nu+y^*)||\nu| - \sigma_{di}\widehat{d}_i].\end{aligned} \tag{3.20}$$

It follows from assumptions 3.4 and 3.5 that $\dot{V}$ can be represented by

$$\begin{aligned}\dot{V} \le {} & \mu\frac{\partial W(\eta)}{\partial\eta}q(0,\eta) - b_0k^*\nu^2 + \mu\|\frac{\partial W(\eta)}{\partial\eta}\|\|q(\nu+y^*,\eta) - q(0,\eta)\| \\ & + \mu\|\frac{\partial W(\eta)}{\partial\eta}\|[\sum_{i=1}^{M_2} g_i|\phi_i(\nu+y^*)| + g_0] + |\nu||a(\nu+y^*,\eta)| - b(\nu+y^*,\eta)k\nu^2 \\ & - \nu b(\nu+y^*,\eta)\sum_{i=1}^{M_1} u_{fi}(\nu+y^*) + |\nu|[\sum_{i=1}^{M_1} d_i|\psi_i(\nu+y^*)| + d_0] + |\dot{y}^*||\nu| \\ & + b_0\frac{\sigma_I}{\gamma_I}k^*k_I + b_0k_I\nu^2 - b_0\frac{\sigma_I}{\gamma_I}k_I^2 - b_0\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}\widehat{d}_i^2 - \sum_{i=1}^{M_1} d_i|\psi_i(\nu+y^*)||\nu| \\ & + b_0\sum_{i=1}^{M_1}\widehat{d}_i|\psi_i(\nu+y^*)||\nu| + \sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}d_i\widehat{d}_i.\end{aligned} \tag{3.21}$$

Considering assumptions 3.2 and 3.3 and taking (3.10) and (3.18) into account, $\dot{V}$ can be evaluated as follows:

$$\begin{aligned}\dot{V} \le {} & -\mu\tau_1\|\eta\|^2 + \mu\tau_2\|\eta\|L_1(|\nu| + \beta_0) - b_0k^*\nu^2 + \beta_1|\nu| + d_0|\nu| \\ & + \mu\tau_2\left[\sum_{i=1}^{M_2} g_i(|\phi_{1i}(\nu,y^*)||\nu| + |\phi_{2i}(y^*)|) + g_0\right]\|\eta\| \\ & + L_2(|\nu| + |y^*| + \|\eta\|)|\nu| + b_0\sum_{i=1}^{M_1}\widehat{d}_i|\psi_i(\nu+y^*)||\nu| \\ & - b(\nu+y^*,\eta)[k_I + k_p]\nu^2 + b_0k_I\nu^2 \\ & - \nu b(\nu+y^*,\eta)\sum_{i=1}^{M_1} u_{fi}(t,\nu+y^*) \\ & - \frac{b_0}{\gamma_I}\sigma_I[k_I - k^*]^2 - \frac{b_0}{\gamma_I}\sigma_I[k_I - k^*]k^* \\ & - \sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}[\widehat{d}_i - d_i/b_0]\,d_i - b_0\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}[\widehat{d}_i - d_i/b_0]^2\end{aligned} \tag{3.22}$$

Here, we have from (3.13) and (3.14) that

$$k_I(t) = e^{-\sigma_I t} k_I(0) + \int_0^t e^{-\sigma_I (t-\tau)} \gamma_I \nu(\tau)^2 d\tau \geq 0 \tag{3.23}$$

and

$$k_p(t) \geq 0. \tag{3.24}$$

It follows from (3.23),(3.24) and assumption 3.6 that

$$\begin{aligned} &- b(\nu + y^*, \eta)[k_I + k_p]\nu^2 + b_0 k_I \nu^2 \\ &\leq -b_0[k_I + k_p]\nu^2 + b_0 k_I \nu^2 \\ &= -b_0 k_p \nu^2. \end{aligned} \tag{3.25}$$

From (3.25), $\dot{V}$ can be evaluated by

$$\begin{aligned} \dot{V} \leq &- (b_0 k^* - L_2)\nu^2 - \mu\tau_1 \|\eta\|^2 - b_0 k_p \nu^2 \\ &+ (\mu\tau_2 L_1 + L_2)\|\eta\||\nu| + \mu\tau_2 (L_1\beta_0 + \sum_{i=1}^{M_2} g_i|\phi_{2i}(y^*)| + g_0)\|\eta\| \\ &+ \mu\tau_2 \sum_{i=1}^{M_2} g_i |\phi_{1i}(\nu, y^*)||\nu|\|\eta\| + (L_2\beta_0 + d_0 + \beta_1)|\nu| \\ &- \frac{b_0}{\gamma_I}\sigma_I[k_I - k^*]^2 - \frac{b_0}{\gamma_I}\sigma_I[k_I - k^*]k^* \\ &- b_0 \sum_{i=1}^{M_1} \frac{\sigma_{di}}{\gamma_{di}}[\widehat{d}_i - d_i/b_0]^2 - \sum_{i=1}^{M_1} \frac{\sigma_{di}}{\gamma_{di}}[\widehat{d}_i - d_i/b_0]d_i \\ &- b(\nu + y^*, \eta) \sum_{i=1}^{M_1} u_{fi}(\nu + y^*)\nu + b_0 \sum_{i=1}^{M_1} \widehat{d}_i |\psi_i(\nu + y^*)||\nu| \end{aligned} \tag{3.26}$$

Moreover, taking the following evaluations with any constants $\rho_1$ to $\rho_{6i}$ into consideration

$$\begin{aligned} &(\mu\tau_2 L_1 + L_2)|\nu|\|\eta\| \\ &= \rho_1\|\eta\|^2 - \rho_1(\|\eta\| - \frac{(\mu\tau_2 L_1 + L_2)}{2\rho_1}|\nu|)^2 + \frac{(\mu\tau_2 L_1 + L_2)^2}{4\rho_1}\nu^2 \\ &\leq \rho_1\|\eta\|^2 + \frac{(\mu\tau_2 L_1 + L_2)^2}{4\rho_1}\nu^2 \end{aligned} \tag{3.27}$$

$$\begin{aligned} &\mu\tau_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i|\phi_{2i}(y^*)| + g_0)\|\eta\| \\ &\leq \rho_2\|\eta\|^2 + \frac{[\mu\tau_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i|\phi_{2i}(y^*)| + g_0)]^2}{4\rho_2} \end{aligned} \tag{3.28}$$

$$\mu\tau_2 \sum_{i=1}^{M_2} g_i|\phi_{1i}(\nu, y^*)||\nu||\eta||$$
$$\leq \sum_{i=1}^{M_2} \rho_{3i}||\eta||^2 + \sum_{i=1}^{M_2} \frac{(\mu\tau_2 g_i)^2}{4\rho_{3i}} \phi_{1i}(\nu, y^*)^2 \nu^2 \tag{3.29}$$

$$(L_2\beta_0 + d_0 + \beta_1)|\nu|$$
$$\leq \rho_4 \nu^2 + \frac{(L_2\beta_0 + d_0 + \beta_1)^2}{4\rho_4} \tag{3.30}$$

$$-\frac{b_0}{\gamma_I}\sigma_I[k_I - k^*]^2 - \frac{b_0}{\gamma_I}\sigma_I[k_I - k^*]k^*$$
$$\leq -(1 - \frac{\gamma_I \rho_5}{b_0 \sigma_I})\sigma_I \frac{b_0}{\gamma_I}[k_I - k^*]^2 + \frac{b_0^2 \sigma_I^2}{4\rho_5 \gamma_I^2} k^{*2} \tag{3.31}$$

$$-\sum_{i=1}^{M_1} \frac{b_0}{\gamma_{di}} \sigma_{di}[\widehat{d}_i - d_i/b_0]^2 - \sum_{i=1}^{M_1} \frac{\sigma_{di}}{\gamma_{di}}[\widehat{d}_i - d_i/b_0]d_i$$
$$\leq -\sum_{i=1}^{M_1}(1 - \frac{\gamma_{di}\rho_{6i}}{b_0 \sigma_{di}})\sigma_{di}\frac{b_0}{\gamma_{di}}[\widehat{d}_i - d_i/b_0]^2 + \sum_{i=1}^{M_1} \frac{d_i^2 \sigma_{di}^2}{4\rho_{6i}\gamma_{di}^2} \tag{3.32}$$

the time derivative of $V$ can be evaluated from (3.27) to (3.32) that

$$\begin{aligned}
\dot{V} \leq & -(b_0 k^* - L_2 - \rho_4 - \frac{(\mu\tau_2 L_1 + L_2)^2}{4\rho_1})\nu^2 - (\mu\tau_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2} \rho_{3i})||\eta||^2 \\
& -(1 - \rho_5')b_0 \frac{\sigma_I}{\gamma_I}[k_I - k^*]^2 - b_0 \sum_{i=1}^{M_1}(1 - \rho_{6i}')\frac{\sigma_{di}}{\gamma_{di}}[\widehat{d}_i(t) - d_i/b_0]^2 \\
& + \sum_{i=1}^{M_2} \frac{(\mu\tau_2 g_i)^2}{4\rho_{3i}} \phi_{1i}(\nu, y^*)^2 \nu^2 - b_0 k_p \nu^2 \\
& + b_0 \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i(\nu + y^*)||\nu| - b(\nu + y^*, \eta) \sum_{i=1}^{M_1} u_{fi}(\nu + y^*)\nu \\
& + \frac{[\mu\tau_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i \phi_{2iM} + g_0)]^2}{4\rho_2} + \frac{(L_2\beta_0 + d_0 + \beta_1)^2}{4\rho_4} \\
& + \frac{b_0 \sigma_I k^{*2}}{4\gamma_I \rho_5'} + \sum_{i=1}^{M_1} \frac{\sigma_{di} d_i^2}{4 b_0 \gamma_{di} \rho_{6i}'}
\end{aligned} \tag{3.33}$$

where $\rho_5' = \frac{\gamma_I \rho_5}{b_0 \sigma_I}$, $\rho_{6i}' = \frac{\gamma_{di}\rho_{6i}}{b_0 \sigma_{di}}$ and $\phi_{2iM}$ is a positive constant such taht $|\phi_{2i}(y^*)| \leq \phi_{2iM}$. Such constant exists from assumption 3.5 that $\phi_{2i}(y_2)$ is smooth for all $y_2 \in R$ and $y^*$ is bounded. Here it follows from (3.14) that

$$\begin{aligned}
&\sum_{i=1}^{M_2} \frac{(\mu\tau_2 g_i)^2}{4\rho_{3i}} \phi_{1i}(\nu, y^*)^2\nu^2 - b_0 k_p \nu^2 \\
&= \sum_{i=1}^{M_2} \frac{(\mu\tau_2 g_i)^2}{4\rho_{3i}} \phi_{1i}(\nu, y^*)^2\nu^2 - b_0 \sum_{i=1}^{M_2} \gamma_{pi}\phi_{1i}(\nu, y^*)^4\nu^4 \\
&\leq \sum_{i=1}^{M_2} \frac{1}{4b_0\gamma_{pi}} [\frac{(\mu\tau_2 g_i)^2}{4\rho_{3i}}]^2.
\end{aligned} \tag{3.34}$$

Furthermore, in the case where $|\widehat{d}_i(t)\psi_i(\nu + y^*)\nu(t)| > \varepsilon_{fi}$ for any $i$ we have from (3.15) that

$$\begin{aligned}
&b_0 \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu| - b(\nu + y^*, \eta) \sum_{i=1}^{M_1} u_{fi}(t, \nu + y^*)\nu \\
&= b_0 \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu| - b(\nu + y^*, \eta) \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu|
\end{aligned} \tag{3.35}$$

and since we obtain from (3.16) that $\widehat{d}_i(t) \geq 0$, it follows from the assumption 3.6 that

$$\begin{aligned}
&b_0 \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu| - b(\nu + y^*, \eta) \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu| \\
&\leq b_0 \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu| - b_0 \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu| \\
&= 0.
\end{aligned} \tag{3.36}$$

On the other hand, if $|\widehat{d}_i(t)\psi_i(\nu + y^*)\nu(t)| \leq \varepsilon_{fi}$, we have

$$\begin{aligned}
&b_0 \sum_{i=1}^{M_1} \widehat{d}_i|\psi_i||\nu| - b(\nu + y^*, \eta) \sum_{i=1}^{M_1} \frac{[\widehat{d}_i\psi_i]^2}{\varepsilon_{fi}}\nu^2 \\
&\leq b_0 \sum_{i=1}^{M_1} |\widehat{d}_i\psi_i\nu| \\
&= b_0 \sum_{i=1}^{M_1} \varepsilon_{fi}.
\end{aligned} \tag{3.37}$$

Therefore, we have from (3.34),(3.36) and (3.37) that

$$\begin{aligned}
\dot V \le & -(\mu\tau_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2}\rho_{3i})\|\eta\|^2 - (b_0 k^* - L_2 - \rho_4 - \frac{(\mu\tau_2 L_1 + L_2)^2}{4\rho_1})\nu^2 \\
& -(1-\rho_5')\sigma_I \frac{b_0}{\gamma_I}[k_I - k^*]^2 - \sum_{i=1}^{M_1}(1-\rho_{6i}')\sigma_{di}\frac{b_0}{\gamma_{di}}[\widehat{d_i} - d_i/b_0]^2 \\
& + \frac{[\mu\tau_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i\phi_{2iM} + g_0)]^2}{4\rho_2} + \sum_{i=1}^{M_2}\frac{1}{4b_0\gamma_{pi}}[\frac{(\mu\tau_2 g_i)}{4\rho_{3i}}]^2 \\
& + \frac{(L_2\beta_0 + d_0 + \beta_1)^2}{4\rho_4} + \frac{b_0\sigma_I k^{*2}}{4\gamma_I\rho_5'} + \sum_{i=1}^{M_1}\frac{\sigma_{di}d_i^2}{4b_0\gamma_{di}\rho_{6i}'} + b_0\sum_{i=1}^{M_1}\varepsilon_{fi}.
\end{aligned} \tag{3.38}$$

Finally, setting the values $\rho_1 = \rho_2 = \frac{\mu\tau_1}{8}, \rho_{3i} = \frac{\mu\tau_1}{4M_2}, \rho_5' = \rho_{6i}' = \frac{1}{2}$ we have

$$\begin{aligned}
\dot V \le & -\frac{\mu\tau_1}{2}\|\eta\|^2 - (b_0 k^* - L_2 - \rho_4 - \frac{2(\mu\tau_2 L_1 + L_2)^2}{\mu\tau_1})\nu^2 \\
& -\sigma_I\frac{b_0}{2\gamma_I}[k_I - k^*]^2 - \sum_{i=1}^{M_1}\sigma_{di}\frac{b_0}{2\gamma_{di}}[\widehat{d_i} - d_i/b_0]^2 \\
& + 2\mu\frac{[\tau_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i\phi_{2iM} + g_0)]^2}{\tau_1} + \sum_{i=1}^{M_2}\frac{1}{4b_0\gamma_{pi}}[\mu\frac{M_2(\tau_2 g_i)^2}{\tau_1}]^2 \\
& + \frac{(L_2\beta_0 + d_0 + \beta_1)^2}{4\rho_4} + \frac{b_0\sigma_I k^{*2}}{2\gamma_I} + \sum_{i=1}^{M_1}\frac{\sigma_{di}d_i^2}{2b_0\gamma_{di}} + b_0\sum_{i=1}^{M_1}\varepsilon_{fi}.
\end{aligned} \tag{3.39}$$

Since it follows from (3.18) that

$$\|\eta\|^2 \ge \frac{1}{\tau_3}W(\eta), \tag{3.40}$$

the time derivative of $V$ can be evaluated as follows:

$$\begin{aligned}
\dot V \le & -\frac{\tau_1}{2\tau_3}[\mu W(\eta) + \frac{1}{2}\nu^2] \\
& -\frac{b_0\sigma_I}{2\gamma_I}[k_I - k^*]^2 - \sum_{i=1}^{M_1}\frac{b_0\sigma_{di}}{2\gamma_{di}}[\widehat{d_i} - d_i/b_0]^2 + R
\end{aligned} \tag{3.41}$$

by choosing the ideal feedback gain $k^*$ as

$$k^* \ge \frac{1}{b_0}\left[\frac{\tau_1}{4\tau_3} + L_2 + \frac{2(\mu\tau_2 L_1 + L_2)^2}{\mu\tau_1} - \rho_4\right] \tag{3.42}$$

where

$$\begin{aligned}
R = & 2\mu\frac{[\tau_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i\phi_{2iM} + g_0)]^2}{\tau_1} + \sum_{i=1}^{M_2}\frac{1}{4b_0\gamma_{pi}}[\mu\frac{M_2(\tau_2 g_i)^2}{\tau_1}]^2 \\
& + \frac{(L_2\beta_0 + d_0 + \beta_1)^2}{4\rho_4} + \frac{b_0\sigma_I k^{*2}}{2\gamma_I} + \sum_{i=1}^{M_1}\frac{\sigma_{di}d_i^2}{2b_0\gamma_{di}} + b_0\sum_{i=1}^{M_1}\varepsilon_{fi}.
\end{aligned} \tag{3.43}$$

Consequently the time derivative of the positive definite function $V$ given in (3.19) can be evaluated by

$$\dot{V} \leq -\alpha_v V + R \tag{3.44}$$
$$\alpha_v = \min[\frac{\tau_1}{2\tau_3}, \sigma_I, \sigma_{di}]. \tag{3.45}$$

It is apparent from (3.44) and (3.45) that all the signals in the closed-loop system with the controller (3.11) to (3.16) are bounded and we also obtain

$$\lim_{t\to\infty} V(t) \leq R/\alpha_v \,. \tag{3.46}$$

From the fact that $\nu^2 \leq 2V$, it follows that

$$\lim_{t\to\infty} \nu^2 \leq 2R/\alpha_v \,. \tag{3.47}$$

Thus, the goal (3.9) is achieved for $\delta^2 \geq 2R/\alpha_v$. It can be also confirmed that the appropriate choices of $\mu$ and $\rho_4$ and design parameters $\gamma_I, \gamma_{pi}, \gamma_{di}$ and $\varepsilon_{fi}$ ensure the goal (3.9) for any $\delta$. □

**Remark 3.1.** *For example, one can set design parameters $\gamma_I, \gamma_{pi}, \gamma_{di}$ and $\varepsilon_{fi}$ as follows in order to attain the goal* (3.9) *for any given $\delta$.*

*Let's set $\mu$ and $\rho_4$ such that*

$$\begin{aligned} \mu &\leq \frac{\tau_1\alpha_v\delta^2}{24\{\tau_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i\phi_{2iM} + g_0)\}^2} \\ \rho_4 &\geq \frac{3(L_2\beta_0 + d_0 + \beta_1)^2}{\alpha_v\delta^2} \end{aligned} \tag{3.48}$$

*and consider an ideal feedback gain $k^*$ satisfying the inequality* (3.42). *Then, it is sufficient to choose design parameters such as*

$$\begin{aligned} \gamma_{pi} &\geq \mu\frac{M_2^2\tau_2^2 g_i^4}{8b_0\tau_1(L_1\beta_0 + \sum_{i=1}^{M_2} g_i\phi_{2iM} + g_0)^2} \\ \gamma_I &\geq \frac{6b_0\sigma_I k^{*2}}{\alpha_v\delta^2}, \ \gamma_{di} \geq \frac{6M_1\sigma_{di}d_i^2}{b_0\alpha_v\delta^2}, \ \varepsilon_{fi} \leq \frac{\alpha_v\delta^2}{12M_1b_0}. \end{aligned} \tag{3.49}$$

**Remark 3.2.** *Note that the design parameters $\gamma_I, \gamma_{pi}, \gamma_{di}$ and $\varepsilon_{fi}$, that are set in order to attain the goal* (3.9) *for a given small $\delta$, depend on uncertain constants. However, as shown* (3.49), *if we set sufficiently large $\gamma_I, \gamma_{pi}, \gamma_{di}$ and sufficiently small $\varepsilon_{fi}$, then the control objective will be attained even if we do not know prior information about uncertain constants. Hence,* (3.49) *provides a design principle for design parameters in the controller.*

## 3.5 Numerical Simulation

Here the effectiveness of the proposed control scheme will be confirmed through numerical simulations.

Consider the following SISO affine nonlinear system:

$$\begin{aligned} \dot{y} &= a(y, \eta) + b(y, \eta)u + f_1 \\ \dot{\eta} &= q(y, \eta) + f_2 \end{aligned} \tag{3.50}$$

where

$$
\begin{aligned}
a(y,\eta) &= y + \sin\eta_2 + \cos\eta_3 \\
b(y,\eta) &= \exp(y+\eta_1) \\
q(y,\eta) &= \begin{bmatrix} -\eta_1 + \eta_3 \sin y \\ -\eta_2 - y \\ -\eta_3 + \eta_2 \sin y \end{bmatrix} \\
f_1 &= 3\exp y + \sin\eta_1 \cos^2\eta_3 \\
f_2 &= [\cos^2 y, \sin\eta_1 \cos y, -y^3 \cos\eta_2]^T.
\end{aligned}
$$

The controlled system given in (3.50) has a relative degree 1 and the nominal part of the system is exponential minimum-phase. In this simulation, it is supposed that we have prior information about the controlled system such that nonlinearities $a(y,\eta)$ and $q(y,\eta)$ are Lipschitz in $(y,\eta)$ and that nonlinear functions $f_1$ and $f_2$ are not Lipschitz, but can be evaluated by

$$
\begin{aligned}
|f_1| &\le d_1|\psi(y)| + d_0 \\
\|f_2\| &\le g_1|\phi_1(y)| + g_0
\end{aligned}
\tag{3.51}
$$

with known functions $\psi(y) = \exp(y), \phi_1(y) = y^3$. It is also assumed that nonlinearity in the control input term $b(y,\eta)$ is unknown. Note that $b(y,\eta) = \exp(y+\eta_1)$ is an unbounded nonlinear function with respect to (y,$\eta$).

We consider the following reference signal $y^*(t)$ for the output signal $y(t)$:

$$
y^*(t) = \begin{cases} \frac{1}{0.2s+1}[r] & \text{for } 0 \le t < 8 \\ r(t) & \text{for } 8 \le t \end{cases}, \quad r(t) = \begin{cases} 0.3 & 0 \le t < 2 \\ 0 & 2 \le t < 4 \\ 0.6 & 4 \le t < 6 \\ 0 & 6 \le t < 8 \\ 0.3\sin(\pi(t-8)) & 8 \le t < 15. \end{cases}
\tag{3.52}
$$

In this simulation, the design parameters of the controller are set as follows:

$$
\begin{aligned}
&\gamma_I = 1000, \ \gamma_p = 1000, \ \gamma_d = 100 \\
&\sigma_I = 0.1, \ \sigma_d = 0.1, \ \varepsilon_f = 0.001.
\end{aligned}
$$

Fig. 3.1 to 3.4 show the simulation results of the proposed method. To illustrate the effectiveness of the proposed method, the simulation results for the controller without robust control input term $k_p$ and $u_{fi}$ against uncertain nonlinearities $f_1$ and $f_2$, *i.e.*, a controller with only high gain adaptive output feedback, are shown in Fig. 3.5 to 3.8. It is clear that performance of the proposed control system is better than that of the conventional high gain adaptive output feedback control system.

## 3.6 Conclusion

In this chapter, the design scheme for a robust adaptive controller for non-OFEP nonlinear systems based on high gain output feedback was proposed. It was shown that using the robust high gain adaptive output feedback control method, one can design a stable adaptive output feedback control system even if the controlled system has unbounded uncertainty in the control input term.

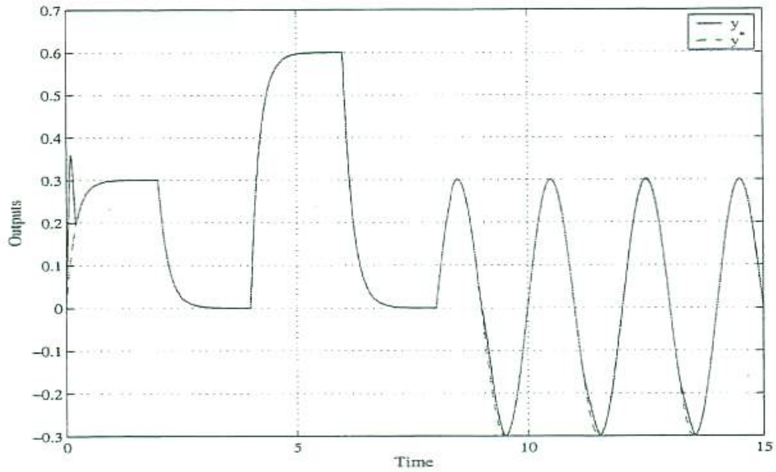


Figure 3.1: Control result by the proposed control system: $y$

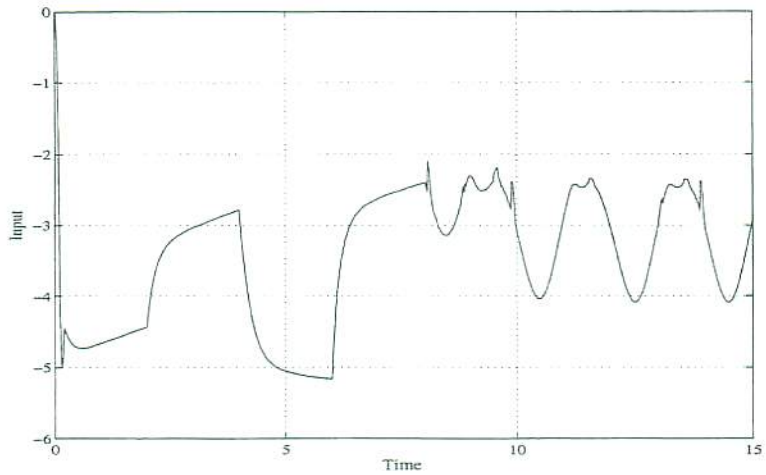


Figure 3.2: Control result by the proposed control system: $u$

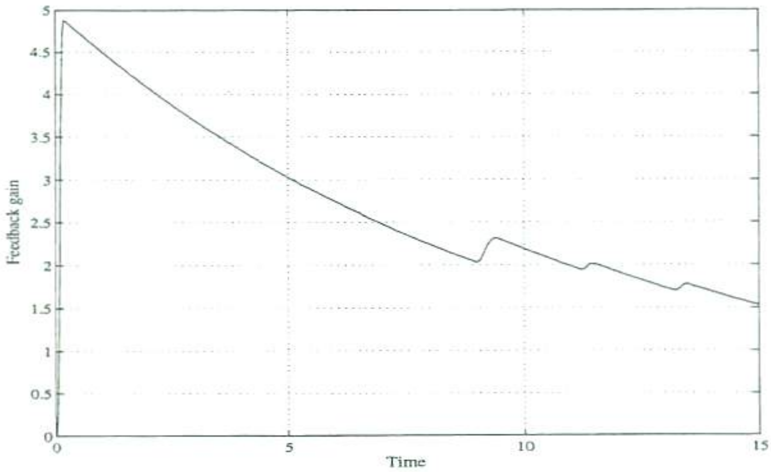


Figure 3.3: Control result by the proposed control system: $k$

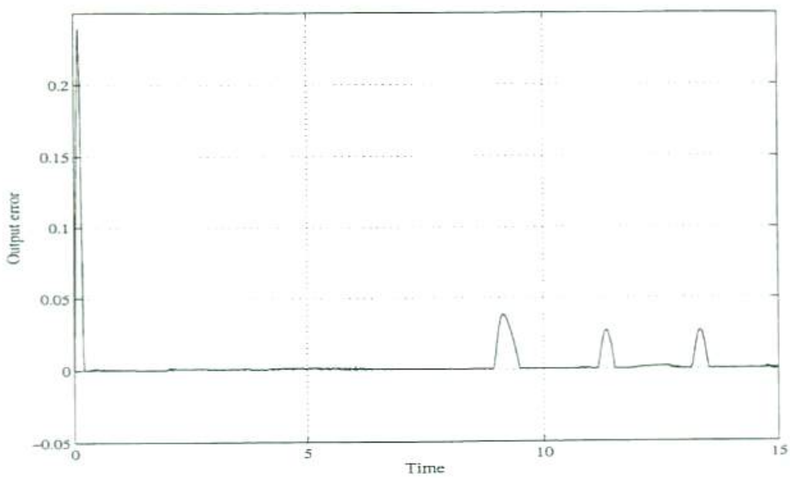


Figure 3.4: Control result by the proposed control system: $\nu$

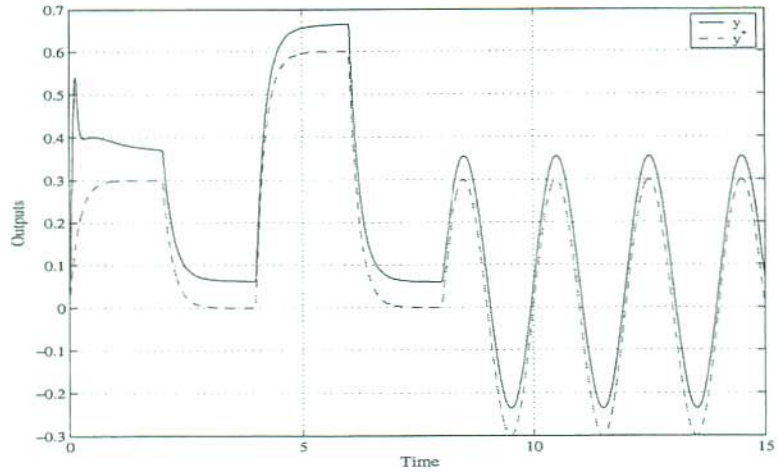


Figure 3.5: Control result by the proposed control system without robust adaptive control term for $f_1, \boldsymbol{f}_2$: $y$

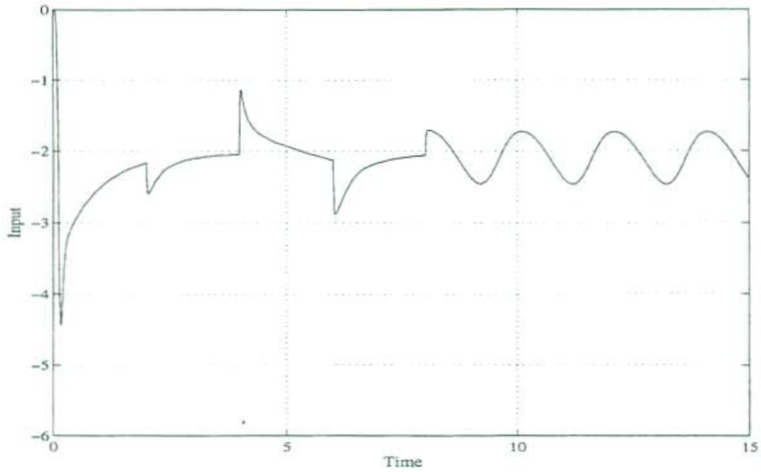


Figure 3.6: Control result by the proposed control system without robust adaptive control term for $f_1, \boldsymbol{f}_2$: $u$

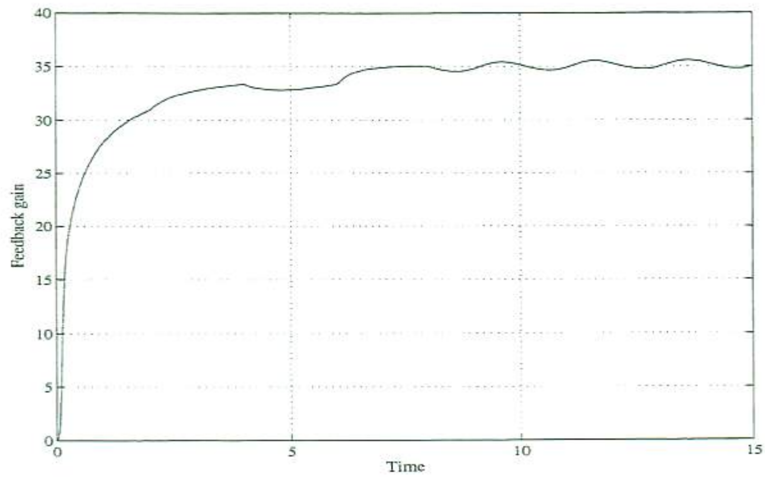


Figure 3.7: Control result by the proposed control system without robust adaptive control term for $f_1, \boldsymbol{f}_2$: $k$

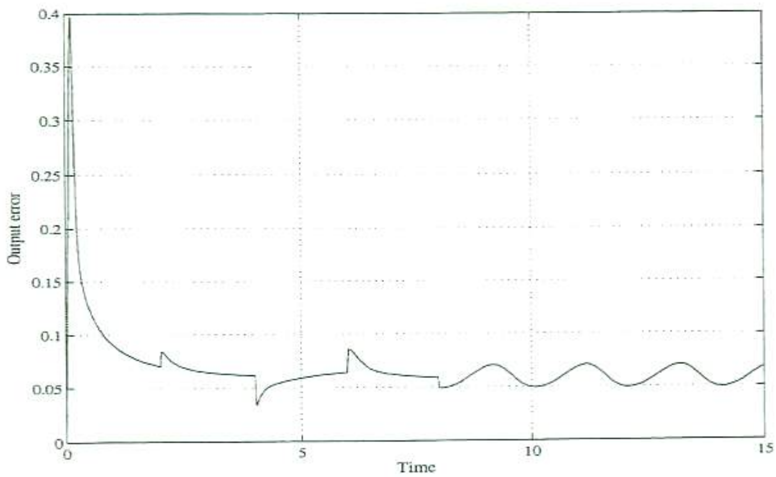


Figure 3.8: Control result by the proposed control system without robust adaptive control term for $f_1, \boldsymbol{f}_2$: $\nu$

# Chapter 4

# Design of State Feedback Control System through High Gain Adaptive Backstepping

## 4.1 Introduction

It is relatively easy to control a system in a wide class of nonlinear systems by state feedback controllers when all the states of the controlled system are available. In particular, backstepping[45] using full state information is one of the most powerful controller design tool. Therefore many researchers have applied backstepping in controller designs[46–53]. *Adaptive backstepping* is a design scheme for an adaptive controller for uncertain systems[49]. However, the traditional adaptive backstepping has been applied to parametric uncertain systems, whose uncertainties are separated by unknown constants and known functions.

In this chapter, an adaptive feedback control system is designed based on high gain state feedback for uncertain nonlinear systems with nonparametric uncertainties. Recently, a number of papers that deal with the robust adaptive control problem for uncertain nonlinearities and/or exogenous disturbances have been presented[77–86]. Despite that the methods provide significant progress in robust adaptive control for uncertain nonlinear systems, they have not dealt with uncertainties appeared in the control input term. Besides some papers dealt with controller design methods for nonlinear systems with uncertain coefficients in the control input terms, but the uncertainties are unknown constants or unknown time-varying bounded functions[103–109].

The proposed method can be applied to uncertain nonlinear systems with nonparametric uncertain nonlinearities and unbounded state depending uncertain coefficients in the control input terms. The main assumption on nonparametric uncertain functions is a so-called *triangular bounds condition*, i.e. it is assumed that the unknown nonlinearities are evaluated by unknown constants and known functions that give some growth conditions. Even though some information about uncertain nonlinearities such as triangular bounds condition is necessary, the proposed method extends the applicable class of nonlinear uncertain systems with uncertain nonlinear function in control input term. To illustrate the effectiveness of the proposed robust adaptive control, the application to a continuous stirred tank reactor (CSTR), which is known as the difficult system to control, is considered and simulation results will be shown.

## 4.2 Problem Statement

Consider the following $n$th order SISO nonlinear system: $(1 \le i \le n-1)$

$$\begin{aligned} \dot{x}_i &= f_i(x,t) + g_i(x,t)x_{i+1} \\ \dot{x}_n &= f_n(x,t) + g_n(x,t)u \\ y &= x_1 \end{aligned} \tag{4.1}$$

where $x = [x_1, \cdots, x_n]^T \in R^n$ is a state variable and $u, y \in R$ are a control input and an output, respectively. $f_i$ and $g_i$ are unknown smooth functions.

We make the following assumptions that are concerned with prior knowledge of the nonlinear controlled system (4.1).

**Assumption 4.1.** *For the unknown nonlinear function $f_i(x,t)$, there exist a unknown positive constant $f_{0i}$ and a known smooth function $f_{Mi}(x_1, \cdots, x_i)$ such that for all $x \in R^n$ and $t \in R^+$*

$$|f_i(x,t)| \le f_{0i}|f_{Mi}(x_1, \cdots, x_i)|. \tag{4.2}$$

**Assumption 4.2.** *For the unknown nonlinear function $g_i(x,t), (1 \le i \le n-1)$, there exist unknown positive constants $g_{mi}$ and $g_{0i}$ and a known smooth function $g_{Mi}(x_1, \cdots, x_{i+1})$ such that for all $x \in R^n$ and $t \in R^+$*

$$0 < g_{mi} \le g_i(x,t) \le g_{0i}|g_{Mi}(x_1, \cdots, x_{i+1})|. \tag{4.3}$$

**Assumption 4.3.** *For the unknown nonlinear function $g_n(x,t)$ which is bounded for all $t$, there exists a positive constant $\beta_m$ such that for all $x \in R^n$ and $t \in R^+$*

$$0 < \beta_m \le g_n(x,t). \tag{4.4}$$

The objective of this work is to find a robust adaptive controller that has the output $y$ track a given reference signal $y_r$ with the goal:

$$\lim_{t\to\infty} |y(t) - y_r(t)| \le \delta \tag{4.5}$$

for any small positive constant $\delta$, where $y_r$ is any signal that satisfies the following conditions:

$$|y_r(t)| \le d_0, \ |\dot{y}_r(t)| \le d_1 \quad \forall t \in [0, \infty) \tag{4.6}$$

with positive constants $d_0$ and $d_1$.

**Remark 4.1.** *Under these assumptions the controlled system (4.1) can be considered as a strict-feedback nonlinear system or a pure-feedback nonlinear system.*

## 4.3 Robust High Gain Adaptive Controller Design via Backstepping

Here, we show the design scheme for a robust high gain adaptive controller for uncertain nonlinear system (4.1) satisfying the assumptions 4.1~4.3.

[Step 1] Let $z_1 = x_1 - y_r = y - y_r$ be a tracking error. The $z_1$-system is given by

$$\dot{z}_1 = f_1(x,t) + g_1(x,t)x_2 - \dot{y}_r. \tag{4.7}$$

Now we design a virtual control input $\alpha_1$ for $x_2$ in the $z_1$-system as follows:

$$\alpha_1 = -\widehat{k}_1 z_1 \tag{4.8}$$

where $\widehat{k}_1$ is a feedback gain that is adaptively adjusted by the following integral and proportional adjusting laws:

$$\begin{aligned} \widehat{k}_1 &= \widehat{k}_{1I} + \widehat{k}_{1P} \\ \dot{\widehat{k}}_{1I} &= \gamma_{1I} D(z) z_1^2, \;\; \widehat{k}_{1I}(0) \ge 0, \;\; \gamma_{1I} > 0 \\ \widehat{k}_{1P} &= \gamma_{1P}\eta_1, \;\; \eta_1 = f_{M1}^2, \;\; \gamma_{1P} > 0. \end{aligned} \tag{4.9}$$

Here, $D(z)$ is defined by

$$D(z) = \begin{cases} 0 & \text{for } z \in \Omega_{z0} \\ 1 & \text{for } z \in \Omega_{z1} \end{cases} \tag{4.10}$$

$$\Omega_{z0} = \{ z \in R^n \mid \|z\|^2 \le \delta_z^2 \}$$
$$\Omega_{z1} = \{ z \in R^n \mid \|z\|^2 > \delta_z^2 \}$$

where $z^T = [z_1, z_2, \cdots, z_n]$ and $z_i, i = 2, \cdots, n$ are variables given by error signals such as $z_i = x_i - \alpha_{i-1}$. The functions $\alpha_i, i = 1, \cdots, n-1$ will be referred to as *virtual control inputs*, which will be designed in each step $i$ by according to backstepping strategy.

Consider the following positive definite function $V_1$ for $z \in \Omega_{z1}$:

$$V_1 = \frac{1}{2} z_1^2 + \frac{g_{m1}}{2\gamma_{1I}} (\widehat{k}_{1I} - k_1^*)^2 \tag{4.11}$$

where $k_1^* > 0$ is an unknown ideal feedback gain for $\widehat{k}_{1I}$. The time derivative of $V_1$ is given by

$$\dot{V}_1 = f_1(x,t) z_1 + g_1(x,t)(\alpha_1 + z_2) z_1 - \dot{y}_r z_1 + g_{m1}(\widehat{k}_{1I} - k_1^*) z_1^2 \tag{4.12}$$

where $z_2 = x_2 - \alpha_1$. Considering (4.8) and (4.9), we have

$$\begin{aligned} \dot{V}_1 =& f_1(x,t) z_1 - g_1(x,t)\widehat{k}_{1I} z_1^2 - g_1(x,t)\widehat{k}_{1P} z_1^2 \\ &+ g_1(x,t) z_1 z_2 - \dot{y}_r z_1 + g_{m1}(\widehat{k}_{1I} - k_1^*) z_1^2. \end{aligned} \tag{4.13}$$

Since $\widehat{k}_{1I} \ge 0$ from (4.9) and assumption 4.2,

$$\begin{aligned} & -g_1(x,t)\widehat{k}_{1I} z_1^2 + g_{m1}(\widehat{k}_{1I} - k_1^*) z_1^2 \\ & \le -g_{m1}\widehat{k}_{1I} z_1^2 + g_{m1}(\widehat{k}_{1I} - k_1^*) z_1^2 \\ & = -g_{m1} k_1^* z_1^2. \end{aligned} \tag{4.14}$$

Further from assumption 4.1 and (4.9), we have

$$\begin{aligned} & f_1(x,t) z_1 - g_1(x,t)\widehat{k}_{1P} z_1^2 \\ & \le f_{01}|f_{M1}||z_1| - g_{m1}\gamma_{1P} f_{M1}^2 z_1^2 \\ & \le -g_{m1}\gamma_{1P}(|f_{M1}||z_1| - \frac{f_{01}}{2 g_{m1}\gamma_{1P}})^2 + \frac{f_{01}^2}{4 g_{m1}\gamma_{1P}} \\ & \le \frac{f_{01}^2}{4 g_{m1}\gamma_{1P}}. \end{aligned} \tag{4.15}$$

The time derivative of $V_1$ is then evaluated by considering $|\dot{y}_r| \leq d_1$ that

$$
\begin{aligned}
\dot{V}_1 \leq & -g_{m1}k_1^* z_1^2 + d_1|z_1| + \frac{f_{01}^2}{4g_{m1}\gamma_{1P}} + g_1(x,t)z_1 z_2 \\
\leq & -g_{m1}k_1^* z_1^2 + \rho_1 z_1^2 - \rho_1 z_1^2 + d_1|z_1| + \frac{f_{01}^2}{4g_{m1}\gamma_{1P}} + g_1(x,t)z_1 z_2 \\
\leq & -(g_{m1}k_1^* - \rho_1)z_1^2 - \rho_1(|z_1| - \frac{d_1}{2\rho_1})^2 + \frac{d_1^2}{4\rho_1} + \frac{f_{01}^2}{4g_{m1}\gamma_{1P}} + g_1(x,t)z_1 z_2 \\
\leq & -(g_{m1}k_1^* - \rho_1)z_1^2 + R_1 + g_1(x,t)z_1 z_2
\end{aligned}
\tag{4.16}
$$

where

$$
R_1 = \frac{d_1^2}{4\rho_1} + \frac{f_{01}^2}{4g_{m1}\gamma_{1P}}
$$

and $\rho_1$ is any positive constant.

**[Step i $(2 \leq i \leq n-1)$]** In step $i$, we design a virtual control input $\alpha_i$ for $x_{i+1}$ in the $\dot{z}_i$-system, where $z_i = x_i - \alpha_{i-1}$, as follows:

$$
\alpha_i = -\widehat{k}_i z_i \tag{4.17}
$$

where $\widehat{k}_i$ is adaptive feedback gain which is adaptively adjusted by following adjusting laws:

$$
\begin{aligned}
\widehat{k}_i &= \widehat{k}_{iI} + \widehat{k}_{iP} \\
\dot{\widehat{k}}_{iI} &= \gamma_{iI} D(z) z_i^2 \ , \ \widehat{k}_{iI}(0) \geq 0 \ , \ \gamma_{iI} > 0 \\
\widehat{k}_{iP} &= \gamma_{iP}\eta_i \\
\eta_i &= f_{Mi}^2 + \sum_{k=1}^{i-1}(f_{Mk}\frac{\partial \alpha_{i-1}}{\partial x_k})^2 + \sum_{k=1}^{i-1}(g_{Mk}\frac{\partial \alpha_{i-1}}{\partial x_k}x_{k+1})^2 \\
&\quad + \sum_{k=1}^{i-1}(\frac{\partial \alpha_{i-1}}{\partial \widehat{k}_{kI}} z_k^2)^2 + (\frac{\partial \alpha_{i-1}}{\partial y_r})^2 + (g_{Mi-1}z_{i-1})^2
\end{aligned}
\tag{4.18}
$$

Since the time derivative of $\alpha_{i-1}$ is given by

$$
\dot{\alpha}_{i-1} = \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_k}(f_k(x,t) + g_k(x,t)x_{k+1}) + \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \widehat{k}_{kI}}\dot{\widehat{k}}_{kI} + \frac{\partial \alpha_{i-1}}{\partial y_r}\dot{y}_r, \tag{4.19}
$$

the $\dot{z}_i$-system is expressed by

$$
\begin{aligned}
\dot{z}_i =& f_i(x,t) + g_i(x,t)x_{i+1} - \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \widehat{k}_{kI}}\dot{\widehat{k}}_{kI} - \frac{\partial \alpha_{i-1}}{\partial y_r}\dot{y}_r \\
&- \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_k}(f_k(x,t) + g_k(x,t)x_{k+1}).
\end{aligned}
\tag{4.20}
$$

For $z_i$-system, consider the following positive definite function $V_i$ for $z \in \Omega_{z1}$ such as

$$
V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{g_{mi}}{2\gamma_{iI}}(\widehat{k}_{iI} - k_i^*)^2 \tag{4.21}
$$

where $k_i^* > 0$ is an unknown ideal feedback gain for $\widehat{k}_{iI}$. The time derivative of $V_i$ is evaluated by

$$\begin{aligned}
\dot{V}_i \leq & -(g_{m1}k_1^* - \rho_1)z_1^2 + \sum_{k=2}^{i-1} g_{mk}k_k^* z_k^2 + g_{i-1}(x,t)z_{i-1}z_i \\
& + f_i(x,t)z_i - g_i(x,t)\widehat{k}_{iI}z_i^2 - g_i(x,t)\widehat{k}_{iP}z_i^2 \\
& + g_i(x,t)z_i z_{i+1} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(f_k(x,t) + g_k(x,t)x_{k+1})z_i \\
& - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{k}_{kI}}\dot{\widehat{k}}_{kI}z_i - \frac{\partial \alpha_{i-1}}{\partial y_r}\dot{y}_r z_i + g_{mi}(\widehat{k}_{iI} - k_i^*)z_i^2.
\end{aligned} \tag{4.22}$$

Since $\widehat{k}_{iI} \geq 0$ from (4.18) and assumption 4.2, we have

$$\begin{aligned}
& -g_i(x,t)\widehat{k}_{iI}z_i^2 + g_{mi}(\widehat{k}_{iI} - k_i^*)z_i^2 \\
& \leq -g_{mi}\widehat{k}_{iI}z_i^2 + g_{mi}(\widehat{k}_{iI} - k_i^*)z_i^2 \\
& \leq -g_{mi}k_i^* z_i^2.
\end{aligned} \tag{4.23}$$

Further from assumptions 4.1,4.2 and (4.18) we have

$$\begin{aligned}
& g_{i-1}(x,t)z_{i-1}z_i + f_i(x,t)z_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(f_k(x,t) + g_k(x,t)x_{k+1})z_i \\
& - \frac{\partial \alpha_{i-1}}{\partial y_r}\dot{y}_r z_i - \sum_{k=1}^{i-1} \gamma_{kI}\frac{\partial \alpha_{i-1}}{\partial \widehat{k}_{kI}}z_k^2 z_i - g_i(x,t)\widehat{k}_{iP}z_i^2 \\
\leq & g_{0i-1}|g_{Mi-1}||z_{i-1}||z_i| + f_{0i}|f_{Mi}||z_i| + \sum_{k=1}^{i-1} |\frac{\partial \alpha_{i-1}}{\partial x_k}|(f_{0k}|f_{Mk}| + g_{0k}|g_{Mk}||x_{k+1}|)|z_i| \\
& + d_1|\frac{\partial \alpha_{i-1}}{\partial y_r}||z_i| + \sum_{k=1}^{i-1} \gamma_{kI}|\frac{\partial \alpha_{i-1}}{\partial \widehat{k}_{kI}}||z_k^2||z_i| - g_{mi}\gamma_{iP}\left[f_{Mi}^2 + \sum_{k=1}^{i-1}(f_{Mk}\frac{\partial \alpha_{i-1}}{\partial x_k})^2\right. \\
& \left. + (\frac{\partial \alpha_{i-1}}{\partial y_r})^2 + \sum_{k=1}^{i-1}(g_{Mk}\frac{\partial \alpha_{i-1}}{\partial x_k}x_{k+1})^2 + \sum_{k=1}^{i-1}(\frac{\partial \alpha_{i-1}}{\partial \widehat{k}_{kI}}z_k^2)^2 + (g_{Mi-1}z_{i-1})^2\right]z_i^2 \\
\leq & \frac{1}{4g_{mi}\gamma_{iP}}(\sum_{k=1}^{i} f_{0k}^2 + \sum_{k=1}^{i-1} g_{0k}^2 + d_1^2 + \sum_{k=1}^{i-1} \gamma_{kI}^2 + g_{0i-1}^2).
\end{aligned} \tag{4.24}$$

The time derivative of $V_i$ can be evaluated by

$$\dot{V}_i \leq -(g_{m1} - \rho_1)z_1^2 - \sum_{k=2}^{i} g_{mk}k_k^* z_k^2 + g_i(x,t)z_i z_{i+1} + R_1 + \sum_{k=2}^{i} R_k \tag{4.25}$$

where

$$R_k = \frac{1}{4g_{mk}\gamma_{kP}}\left(\sum_{l=1}^{k} f_{0l}^2 + \sum_{l=1}^{k-1} g_{0l}^2 + \sum_{l=1}^{k-1} \gamma_{lI}^2 + d_1^2 + g_{0k-1}^2\right), \ (2 \leq k \leq i, 2 \leq i \leq n-1). \tag{4.26}$$

[Step n] The actual control input $u$ is designed in this final step as follows:

$$u = -\widehat{k}_n z_n \tag{4.27}$$

where $\widehat{k}_n$ is an adaptive feedback gain which is adjusted by

$$\begin{aligned}
\widehat{k}_n &= \widehat{k}_{nI} + \widehat{k}_{nP} \qquad (4.28)\\
\dot{\widehat{k}}_{nI} &= \gamma_{nI} D(z) z_n^2 \ , \ \widehat{k}_{nI}(0) \geq 0 \ , \ \gamma_{nI} > 0 \\
\widehat{k}_{nP} &= \gamma_{nP} \eta_n \\
\eta_n &= f_{Mn}^2 + \sum_{k=1}^{n-1} (f_{Mk} \frac{\partial \alpha_{n-1}}{\partial x_k})^2 + \sum_{k=1}^{i-1} (g_{Mk} \frac{\partial \alpha_{n-1}}{\partial x_k} x_{k+1})^2 \\
&+ \sum_{k=1}^{n-1} (\frac{\partial \alpha_{n-1}}{\partial \widehat{k}_{kI}} z_k^2)^2 + (\frac{\partial \alpha_{n-1}}{\partial y_r})^2 + (g_{Mn-1} z_{n-1})^2.
\end{aligned}$$

In this final step, we consider the following overall positive definite function $V_n$ for $z \in \Omega_{z1}$ as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{\beta_m}{2\gamma_{nI}} (\widehat{k}_{nI} - k_n^*)^2 \qquad (4.29)$$

The time derivative of $V_n$ can be evaluated by

$$\dot{V}_n \leq -(g_{m1} k_1^* - \rho_1) z_1^2 - \sum_{k=2}^{n-1} g_{mk} k_k^* z_k^2 - \beta_m k_n^* z_n^2 + \sum_{k=1}^{n} R_k \qquad (4.30)$$

using the same mannars in step $i$. $R_k$ is defined in (4.26) and with $g_{mn} = \beta_m$. Finally, by considering a positive constant $K^*$ such that

$$K^* = \min\{g_{m1} k_1^* - \rho_1, g_{m2} k_2^*, \cdots, g_{mn-1} k_{n-1}^*, \beta_m k_n^*\} \qquad (4.31)$$

the time derivative of $V_n$ can be evaluated as follows:

$$\dot{V}_n \leq -K^* \|z\|^2 + \sum_{k=1}^{n} R_k. \qquad (4.32)$$

## 4.4 Stability and Convergence Analysis

For the designed control system in section 4.3, we have the following theorem concerning the boundedness of all signals in the control system and the convergence of tracking error.

**Theorem 4.1.** *Consider the nonlinear system* (4.1) *which satisfies assumptions* 4.1~4.3. *Then, all the signals in the resulting closed-loop system with control input* (4.27) *are bounded and the tracking error* $z_1$ *converges to any given bound* $\delta$ *such that*

$$\lim_{t \to \infty} z_1^2 \leq \delta_z^2. \qquad (4.33)$$

*Thus, the control objective* (4.5) *can be attained by setting* $\delta_z = \delta$.

*Proof.* For designed $z$-system, we consider the following continuous function $V$.

$$V = \begin{cases} \frac{1}{2}\delta_z^2 + \sum_{i=1}^{n-1} \frac{g_{mi}}{2\gamma_{iI}} \Delta k_{iI}^2 + \frac{\beta_m}{2\gamma_{nI}} \Delta k_{nI}^2, & \text{if } z \in \Omega_{z0} \\ V_n & \text{if } z \in \Omega_{z1} \end{cases} \qquad (4.34)$$

where $\delta_z$ is any positive constant which is set in (4.10) and $\Delta k_{iI} = \widehat{k}_{iI} - k_i^*$. Additionally the ideal feedback gains $k_i^*$, which are not necessarily to be known, are given so as to satisfy the following inequality:

$$K^* > \frac{1}{\delta_z^2}\sum_{k=1}^{n} R_k. \tag{4.35}$$

The continuous function given by (4.34) can be evaluated by

$$V \geq \frac{1}{2}\delta_z^2 + \sum_{i=1}^{n-1}\frac{g_{mi}}{2\gamma_{iI}}\Delta k_{iI}^2 + \frac{\beta_m}{2\gamma_{nI}}\Delta k_{nI}^2 \geq \frac{1}{2}\delta_z^2 > 0. \tag{4.36}$$

Furthermore, since $k_i^*$ are given as (4.35), we have $\dot{V} < 0$ for $z \in \Omega_{z1}$ and $\dot{V} = 0$ for $z \in \Omega_{z0}$. That is, $\dot{V} \leq 0$ for all $t \geq 0$. This means that $V$ is bounded and $z, \Delta k_{iI}$ are also bounded. It is apparent from the facts that all the signals in the resulting closed-loop system are bounded.

Next we analyze the convergence of the tracking error. From (4.32) and (4.34), the time derivative of $V$ can be evaluated for $z \in \Omega_{z1}$ by

$$\dot{V} \leq -K^*\delta_z^2 + \sum_{k=1}^{n} R_k = -\gamma_z \tag{4.37}$$

where $\gamma_z = K^*\delta_z^2 - \sum_{k=1}^{n} R_k > 0$ from (4.35). Assume that there exists a time $t_0$ such that $\|z\|^2 \geq \delta_{z1}^2 > \delta_z^2$ for all $t \geq t_0$. This assumption implies that $V \geq \frac{1}{2}\delta_{z1}^2$ for $\forall t \geq t_0$. On the other hand we have from (4.37) that

$$V(t) = V(t_0) + \int_{t_0}^{t} \dot{V}(\tau)d\tau \leq V(t_0) - \gamma_z(t - t_0). \tag{4.38}$$

Since the right hand side of (4.38) will eventually become negative as $t \to \infty$, the inequality (4.38) contradicts the assumption that $V \geq \frac{1}{2}\delta_{z1}^2$. Therefore, there exists a finite interval $(t_0, t_1)$ such that $z \in \Omega_{z1}$ and there exists a finite time $t_2$ at which lies on the boundary $\Omega_{z0}$, *i.e.* $z \in \Omega_{z0}$. Then $\|z(t_0)\|^2 = \|z(t_2)\|^2 = \delta_z^2$ and from the fact that $\dot{V} \leq -\gamma_z < 0$ for $z \in \Omega_{z1}$, we have

$$\sum_{i=1}^{n-1}\frac{g_{mi}}{2\gamma_{iI}}\Delta k_{iI}^2(t_2) + \frac{\beta_m}{2\gamma_{nI}}\Delta k_{nI}^2(t_2) < \sum_{i=1}^{n-1}\frac{g_{mi}}{2\gamma_{iI}}\Delta k_{iI}^2(t_0) + \frac{\beta_m}{2\gamma_{nI}}\Delta k_{nI}^2(t_0) \tag{4.39}$$

and hence the parameter error decreases a finite amount every time $z$ leaves $\Omega_{z0}$ and re-enter $\Omega_{z0}$. Finally, we can conclude that the parameter error converges to a constant. This implies that

$$\lim_{t\to\infty} \|z\|^2 \leq \delta_z^2 \tag{4.40}$$

and we obtain the final evaluation

$$\lim_{t\to\infty} z_1^2 \leq \delta_z^2. \tag{4.41}$$

□

**Remark 4.2.** *It is not necessary to know the exact value of the ideal feedback gains $k_i^*$ which are set in* (4.35), *because they are adaptively adjusted by parameter adjusting laws* (4.9) *and* (4.18).

## 4.5 Application to A CSTR Model

In this section, we apply the proposed method to a certain model of a continuous stirred tank reactor (CSTR)[110,111]. The CSTR model is known as the difficult system to control, since it has strong nonlinearities and unstable zero dynamics.

### 4.5.1 CSTR Model and Problem Formulation

In many chemical processes, the main reaction which yields the desired product is accompanied by consecutive and parallel reactions which produce undesired by-products. This time we consider the following reaction mechanism:

$$
\begin{aligned}
A &\xrightarrow{k_1} B \xrightarrow{k_2} C \\
2A &\xrightarrow{k_3} D
\end{aligned}
$$

where $A$ is the reactant, $B$ the desired product, $C$ and $D$ are the unwanted by-products and $k_i$ is the reaction rate.

The dynamics of the reactor can be described by following set of differential equations[110,111].

$$
\begin{aligned}
\dot{c}_A &= \frac{\dot{V}}{V_R}(c_{A0} - c_A) - k_1(\nu)c_A - k_3(\nu)c_A^2 \\
\dot{c}_B &= -\frac{\dot{V}}{V_R}c_B + k_1(\nu)c_A - k_2(\nu)c_B \\
\dot{\nu} &= \frac{\dot{V}}{V_R}(\nu_0 - \nu) + C_1(\nu_K - \nu) - \frac{1}{\rho C_p}(k_1(\nu)c_A d_1 + k_2(\nu)c_B d_2 + k_3(\nu)c_A^2 d_3) \\
\dot{\nu}_K &= \frac{1}{m}Q_K + \frac{C_2}{m}(\nu - \nu_K)
\end{aligned}
\tag{4.42}
$$

The concentrations of $A$ and $B$ are $c_A$ and $c_B$ respectively ($c_A \geq 0, c_B \geq 0$). The temperature in the reactor is denoted by $\nu$ while the temperature in the cooling jacket is given by $\nu_K$, both temperatures are expressed in absolute degrees $K$. The reaction velocities are assumed to depend on the temperature via the Arrhenius law:

$$
k_i(\nu) = k_{i0} \exp \frac{E_i}{\nu}, \; i = 1, 2, 3 \tag{4.43}
$$

The inflow of the reactor is composed only of substance $A$ with inflow concentration and temperature given by $c_{A0}$ and $\nu_0$, respectively. Values for the physical and chemical parameters are given in Table 4.1.

For the model (4.42), we set $x = [\ c_A\ ,\ c_B\ ,\ \nu\ ,\ \nu_K\ ]^T$ and $u_1 = \frac{\dot{V}}{V_R}$ , $u_2 = Q_K$, the CSTR model is represented as follows:

$$
\begin{aligned}
\dot{x}_1 &= u_1(c_{A0} - x_1) - k_1(x_3)x_1 - k_3(x_3)x_1^2 \\
\dot{x}_2 &= -u_1 x_2 + k_1(x_3)x_1 - k_2(x_3)x_2 \\
\dot{x}_3 &= u_1(\nu_0 - x_3) + C_1(x_4 - x_3) - \frac{1}{\rho C_p}(k_1(x_3)x_1 d_1 + k_2(x_3)x_2 d_2 + k_3(x_3)x_1^2 d_3) \\
\dot{x}_4 &= \frac{1}{m}u_2 + \frac{C_2}{m}(x_3 - x_4)
\end{aligned}
\tag{4.44}
$$

Table 4.1: Process parameters

| Symbol | Values | unites |
|---|---|---|
| $k_{10}$ | $3.575 \times 10^8$ | $\sec^{-1}$ |
| $k_{20}$ | $3.575 \times 10^8$ | $\sec^{-1}$ |
| $k_{30}$ | $2.512 \times 10^6$ | $mol\ A.\sec^{-1}$ |
| $E_1$ | $-9758.3$ | $\deg K$ |
| $E_2$ | $-9758.3$ | $\deg K$ |
| $E_3$ | $-8560$ | $\deg K$ |
| $d_1$ | 4.20 | $kJ/mol\ A$ |
| $d_2$ | $-11.0$ | $kJ/mol\ B$ |
| $d_3$ | $-41.85$ | $kJ/mol\ A$ |
| $\rho$ | 0.9342 | $kg/l$ |
| $C_p$ | 3.01 | $kJ/kg.K$ |
| $C_1$ | $85.6347 \times 10^{-4}$ | $\sec^{-1}$ |
| $C_2$ | 0.2408 | $kJ/\sec.K$ |
| $m$ | 10.0 | $kJ/K$ |
| $V_R$ | 0.01 | $m^3$ |

where $u_1$ and $u_2$ are control inputs given by the flow rate normalized by the reactor volume $\dot{V}/V_R$ and the heat removal $\dot{Q}_k$, respectively. It is assumed that the state variables $x_1, x_2, x_3$ satisfy the following conditions:

$$c_{A0} > x_1 > x_2 > 0\ ;\ x_3 > \delta_\nu \tag{4.45}$$

with a small positive constant $\delta_\nu$. Further, it is assumed that $E_1 = E_2$ , $k_{10} = k_{20}$ and that the upper bounds $E_{01}$ of $E_1$ and the value of $c_{A0}$ are known.

Under these assumptions, define the following new variable with arbitrarily chosen $E(> E_{01})$ and a scaling factor $k_n$ ($E$ and $k_n$ can be considered as design parameters):

$$\bar{x}_3 = k_n \exp\frac{E}{x_3} \tag{4.46}$$

Then, the CSTR model (4.44) can be expressed by

$$\begin{aligned}
\dot{x}_1 =& -\bar{k}_1(\bar{x}_3)x_1 - \bar{k}_3(\bar{x}_3)x_1^2 + (c_{A0} - x_1)u_1 \\
\dot{x}_2 =& -u_1x_2 + \bar{k}_0(\bar{x}_3)(x_1 - x_2)\bar{x}_3 \\
\dot{\bar{x}}_3 =& -\bar{x}_3\frac{[\ln\frac{\bar{x}_3}{k_n}]^2}{E}\Big[u_1(\nu_0 - \frac{E}{\ln\frac{\bar{x}_3}{k_n}}) - \frac{C_1E}{\ln\frac{\bar{x}_3}{k_n}} \\
&- \frac{1}{\rho C_p}(\bar{k}_1(\bar{x}_3)(x_1d_1 + x_2d_2) + \bar{k}_3(\bar{x}_3)x_1^2d_3)\Big] - \bar{x}_3\frac{C_1}{E}[\ln\frac{\bar{x}_3}{k_n}]^2x_4 \\
\dot{x}_4 =& \frac{C_2}{m}(\frac{E}{\ln\frac{\bar{x}_3}{k_n}} - x_4) + \frac{1}{m}u_2
\end{aligned} \tag{4.47}$$

where

$$\begin{aligned}
&\bar{k}_1(\bar{x}_3) = k_{10}(\frac{\bar{x}_3}{k_n})^{\frac{E_1}{E}},\ \bar{k}_3(\bar{x}_3) = k_{30}(\frac{\bar{x}_3}{k_n})^{\frac{E_3}{E}}, \\
&\bar{k}_0(\bar{x}_3) = \frac{k_{10}}{k_n}(\frac{\bar{x}_3}{k_n})^{\frac{E_1-E}{E}}.
\end{aligned} \tag{4.48}$$

Since $E_i$ are negative constants and $E$ is chosen such as $E_1 < E$, $\bar{k}_i(\bar{x}_3)$ , $i = 0, 1, 3$ are bounded for all $x_3 > 0$.

The control objective here is to regulate the concentration $x_2$ as well as regulating the value of the ratio $x_2/x_1$.

**Remark 4.3.** *Above assumptions are reasonable from the viewpoint in physical and practical situations. However, the controller design considered here will not yield a global result under these assumptions.*

### 4.5.2 Adaptive Controller Design

The controller design is divided into two parts. In the first part, the control input $u_1$ is determined so as to have $x_1$ track the command signal $x_{r1}$ by applying the proposed method to $x_1$-system. In the second part, the control input $u_2$ is designed so as to have $x_2$ track the command signal $x_{r2}$. The proposed procedure will be applied to the remaining third order $(x_2, \bar{x}_3, x_4)$-system with input $u_1$ as known and available exogenous signal.

It is easy to confirm that $x_1$-system in (4.47) can be expressed by the form:

$$\dot{x}_1 = f_1(x) + (c_{A0} - x_1)u_1 \tag{4.49}$$

and the remaining $(x_2, \bar{x}_3, x_4)$-system with input $u_1$ as known and available exogenous signal can be expressed by

$$\begin{aligned} \dot{x}_2 &= f_2(x, t) + g_2(x)(x_1 - x_2)\bar{x}_3 \\ \dot{\bar{x}}_3 &= f_3(x, t) + g_3(x)x_4 \\ \dot{x}_4 &= f_4(x) + g_4(x)u_2. \end{aligned} \tag{4.50}$$

From the assumptions imposed on the controlled system (4.44) $f_i, g_i$ in (4.49) and (4.50) satisfy the assumptions 4.1~4.3.

*Part I* : From assumption (4.45) we have $(c_{A0} - x_1) \neq 0$. the control input $u_1$ can be designed as follows:

$$\begin{aligned} u_1 &= -\widehat{k}_1 z_1/(c_{A0} - x_1) \qquad (4.51) \\ \widehat{k}_1 &= \widehat{k}_{1I} + \widehat{k}_{1P} \\ \dot{\widehat{k}}_{1I} &= \gamma_{1I} D(z_1) z_1^2, \ \widehat{k}_{1I}(0) \geq 0, \ \gamma_{1I} > 0 \\ \widehat{k}_{1P} &= \gamma_{1P} f_{M1}^2, \ f_{M1} = x_1 + x_1^2, \ \gamma_{1P} > 0 \end{aligned}$$

where $z_1 = x_1 - x_{1r}$.

*Part II* : From assumption (4.45) we have $(x_1 - x_2) \neq 0$, the control input $u_2$ can be designed for $(x_2, \bar{x}_3, x_4)$-system as follows:

$$\begin{aligned} u_2 &= -\widehat{k}_4 z_4 \qquad (4.52) \\ \widehat{k}_4 &= \widehat{k}_{4I} + \widehat{k}_{4P} \\ \dot{\widehat{k}}_{4I} &= \gamma_{4I} z_4^2, \ \widehat{k}_{4I}(0) \geq 0, \ \gamma_{4I} > 0 \\ \widehat{k}_{4P} &= \gamma_{4P} \eta_4, \ \gamma_{4P} > 0 \\ \eta_4 &= f_{M4}^2 + \sum_{i=1}^{2} (\frac{\partial \alpha_3}{\partial x_i} f_{Mi})^2 + (\frac{\partial \alpha_3}{\partial x_1} \widehat{k}_1 z_1)^2 + (g_{M3} z_3)^2 + (\frac{\partial \alpha_3}{\partial x_2}(x_1 - x_2)\bar{x}_3)^2 \\ &\quad + \sum_{i=1}^{4} (\frac{\partial \alpha_3}{\partial \bar{x}_3} f_{M3i})^2 + (\frac{\partial \alpha_3}{\partial \bar{x}_3} g_{M3} x_4)^2 + \sum_{i=1}^{3} (\frac{\partial \alpha_3}{\partial \widehat{k}_{iI}} z_i^2)^2 + \sum_{i=1}^{2} (\frac{\partial \alpha_3}{\partial x_{ir}})^2 \end{aligned}$$

$$
\begin{aligned}
\alpha_3 &= -\widehat{k}_3 z_3 \qquad (4.53)\\
\widehat{k}_3 &= \widehat{k}_{3I} + \widehat{k}_{3P}\\
\dot{\widehat{k}}_{3I} &= \gamma_{3I} z_3^2,\ \widehat{k}_{3I}(0) \geq 0,\ \gamma_{3I} > 0\\
\widehat{k}_{3P} &= \gamma_{3P}\eta_3,\ \gamma_{3P} > 0\\
\eta_3 &= \sum_{i=1}^{4}(f_{M3i})^2 + \sum_{i=1}^{2}(\frac{\partial\alpha_2}{\partial x_i} f_{Mi})^2 + (\frac{\partial\alpha_2}{\partial x_1}\widehat{k}_1 z_1)^2\\
&+ (\frac{\partial\alpha_2}{\partial x_2}(x_1 - x_2)\bar{x}_3)^2 + \sum_{i=1}^{2}(\frac{\partial\alpha_2}{\partial \widehat{k}_{iI}} z_i)^2\\
&+ \sum_{i=1}^{2}(\frac{\partial\alpha_2}{\partial x_{ir}})^2 + ((x_1 - x_2)z_2)^2
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2 &= -\widehat{k}_2 z_2/(x_1 - x_2) \qquad (4.54)\\
\widehat{k}_2 &= \widehat{k}_{2I} + \widehat{k}_{2P}\\
\dot{\widehat{k}}_{2I} &= \gamma_{2I} z_2^2,\ \widehat{k}_{2I}(0) \geq 0,\ \gamma_{2I} > 0\\
\widehat{k}_{2P} &= \gamma_{2P}\eta_2,\ \eta_2 = f_{M2}^2,\ \gamma_{2P} > 0
\end{aligned}
$$

where $z_2 = x_2 - x_{r2}$, $z_3 = \bar{x}_3 - \alpha_2$, $z_4 = x_4 - \alpha_3$ and

$$
\begin{aligned}
&f_{M2} = u_1 x_2,\ f_{M31} = \bar{x}_3(\frac{1}{x_3})^2 u_1,\ f_{M32} = \frac{\bar{x}_3}{x_3} u_1\\
&f_{M33} = \frac{\bar{x}_3}{x_3},\ f_{M34} = \bar{x}_3(\frac{1}{x_3})^2(x_1 + x_2 + x_1^2)\\
&f_{M4} = x_3 - x_4,\ g_{M3} = \bar{x}_3(\frac{1}{x_3})^2.
\end{aligned}
$$

### 4.5.3 Simulation Results

In this simulation, we set $c_{A0} = 5.10(mol/l)$ and $\nu_0 = 378.05(K)$ and give command signals $x_{r1}$ and $x_{r2}$ by the following reference models:

$$
\begin{aligned}
x_{r1} &= 2.14 + G_1(s)[r_1] \qquad (4.55)\\
G_1(s) &= \frac{0.02 \times 0.03}{(s + 0.02)(s + 0.03)},\ r_1 = \begin{cases} 0 & \text{for } 0 \leq t < 3500\\ -0.03 & \text{for } 3500 \leq t \end{cases}\\
x_{r2} &= 1.09 + G_2(s)[r_2] \qquad (4.56)\\
G_2(s) &= \frac{0.02 \times 0.03}{(s + 0.02)(s + 0.03)},\ r_2 = \begin{cases} 0 & \text{for } 0 \leq t < 500\\ 0.03 & \text{for } 500 \leq t < 1500\\ -0.03 & \text{for } 1500 \leq t \end{cases}
\end{aligned}
$$

Furthermore to demonstrate the robustness of the proposed method under increment of the inflow concentration $c_{A0}$ and temperature $\nu_0$, we add the increment of +10% for $c_{A0}$ from $t = 5000$ to $t = 5500$ and +15% for $\nu_0$ from $t = 5500$ to $t = 6000$ respectively.

The design parameters of the controller are given as follows:

$$\gamma_{1I} = 10^5,\ \gamma_{2I} = 3 \times 10^4,\ \gamma_{3I} = 5 \times 10^3,\ \gamma_{4I} = 0.05$$
$$\gamma_{1P} = 1,\ \gamma_{2P} = \gamma_{3P} = \gamma_{4P} = 10^{-8},\ \delta_z = 10^{-4}$$
$$\widehat{k}_{1I}(0) = 0,\ \widehat{k}_{2I}(0) = 60,\ \widehat{k}_{3I}(0) = 6 \times 10^3,\ \widehat{k}_{4I}(0) = 0$$

and we set the initial value of states as $x_1(0) = 2.14, x_2(0) = 1.09, x_3(0) = 387.2, x_4(0) = 387.2$ that are the same value of parameters given in Reference[111]. Further we set the parameters of $\bar{x}_3$ such as $E = -9,500$ , $k_n = 3.575 \times 10^5$.

Fig. 4.1 to 4.8 show the simulation results of applying proposed controller. It can be seen that the proposed controller gives us good control performances in spite of almost of all the model parameters are unknown and some parameters vary in the simulation. For the comparison, simulation results with PI controller are shown in Fig 4.9 to 4.12. It was difficult to get good control performances by PI controller.

## 4.6 Conclusion

In this chapter, a robust adaptive controller design for uncertain nonlinear system with nonparametric uncertainties in control input term was proposed based on backstepping procedure. The method estimates only feedback gain for each subsystem and has high gain feedback mechanism to get a robust performance for uncertain nonlinearities which satisfy the so-called triangular bounds condition. The effectiveness of the proposed method was confirmed through a numerical simulation for a CSTR model.

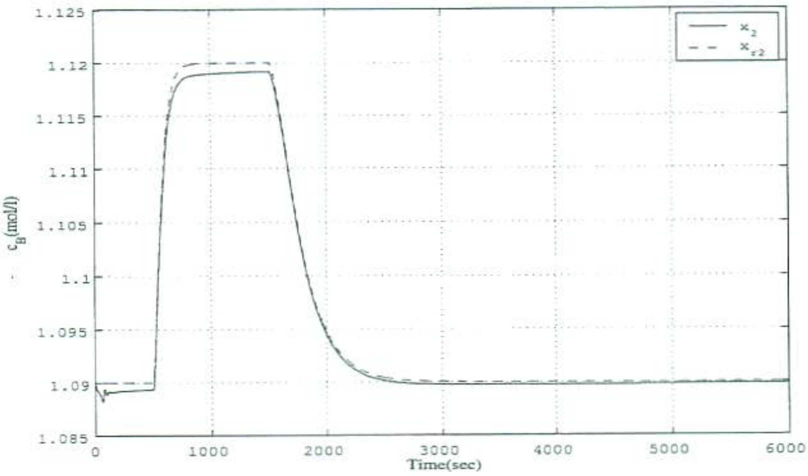


Figure 4.1: Tracking control result of $x_2$ by the proposed controller

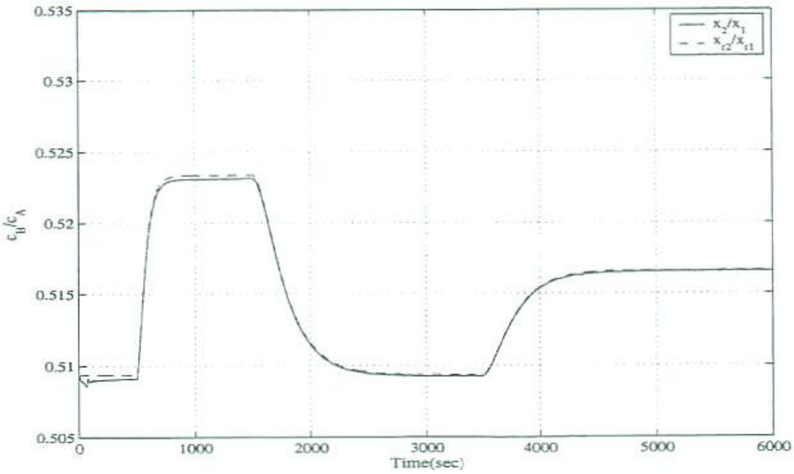


Figure 4.2: Tracking control result of $x_2/x_1$ by the proposed controller

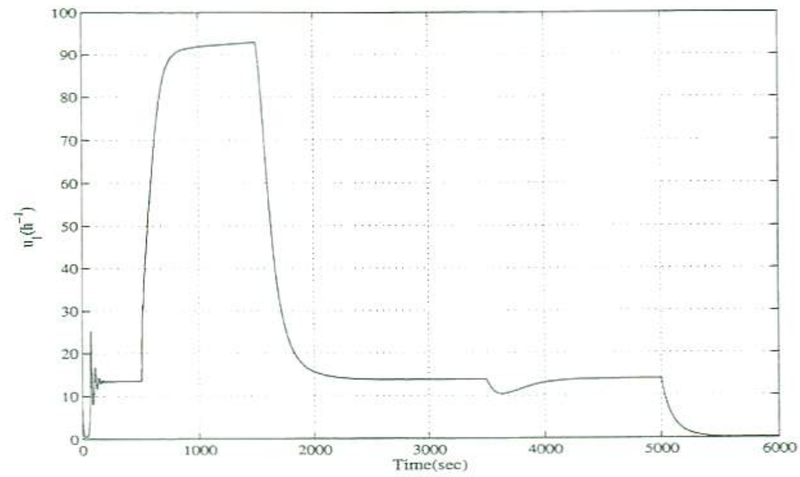


Figure 4.3: Control input $u_1$ of the proposed controller

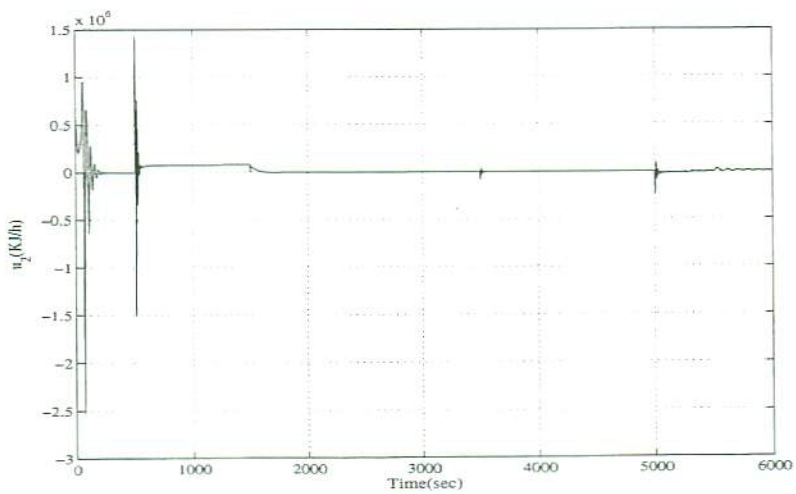


Figure 4.4: Control input $u_2$ of the proposed controller

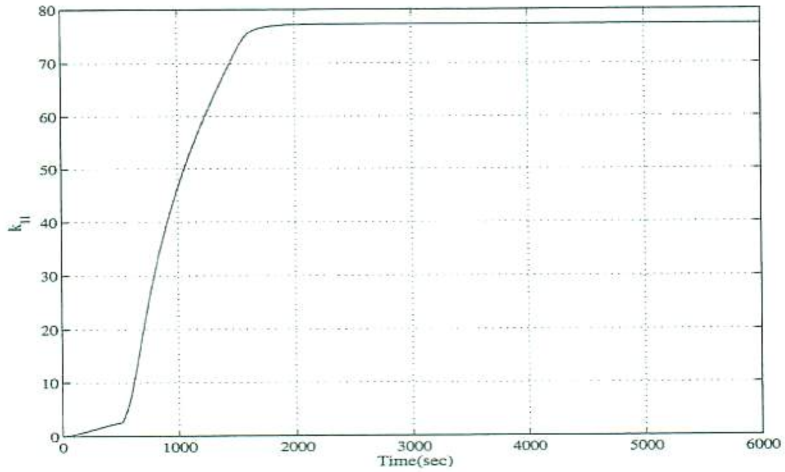


Figure 4.5: Adaptive feedback gain $\widehat{k}_{1I}$

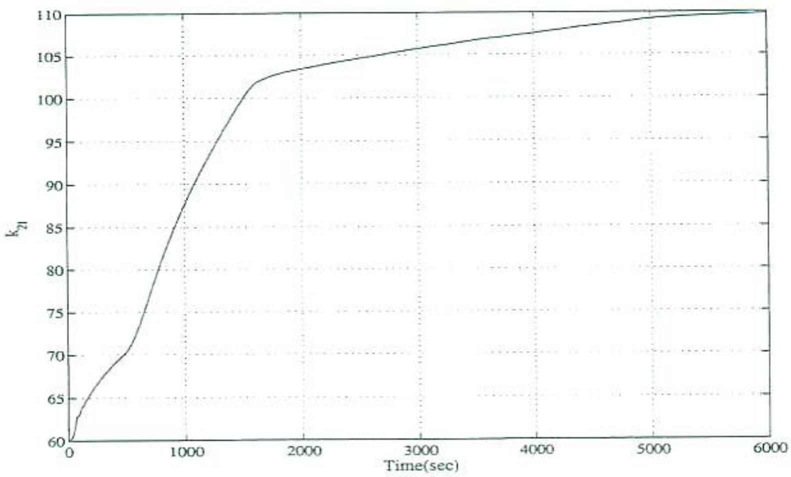


Figure 4.6: Adaptive feedback gain $\widehat{k}_{2I}$

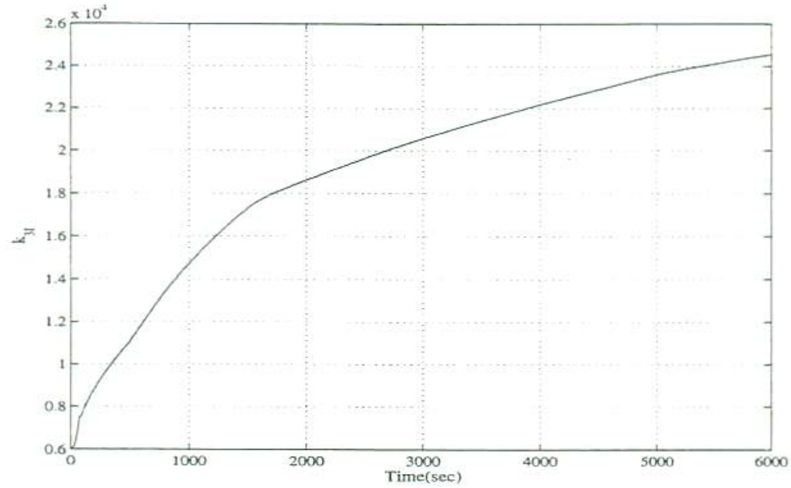


Figure 4.7: Adaptive feedback gain $\widehat{k}_{3I}$

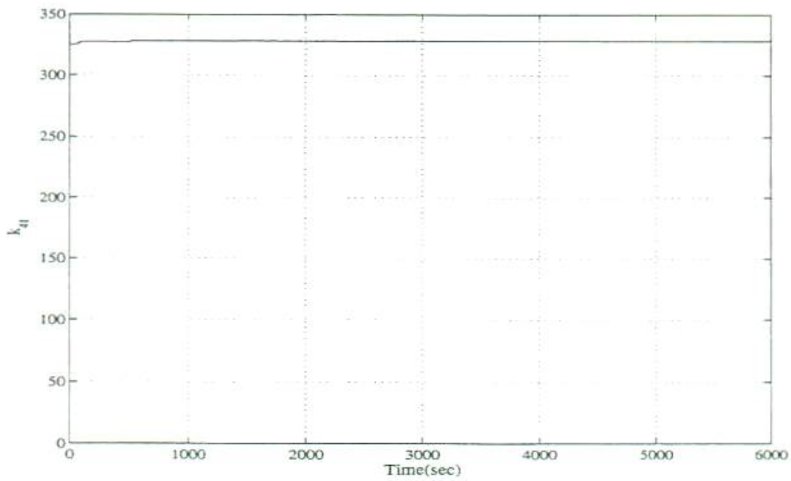


Figure 4.8: Adaptive feedback gain $\widehat{k}_{4I}$

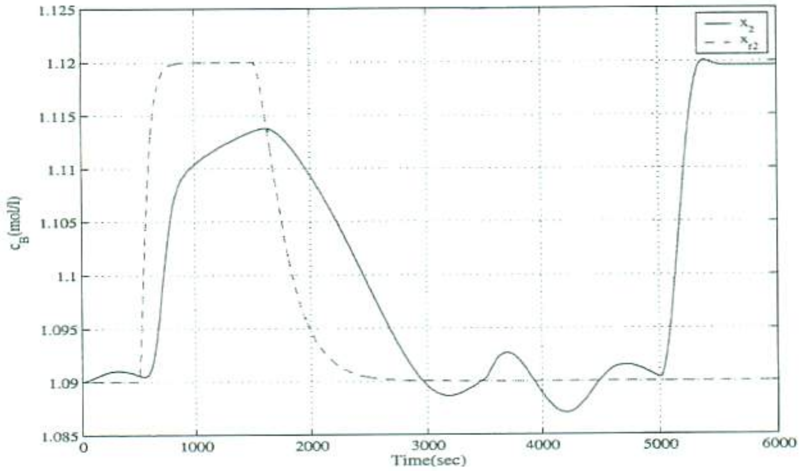


Figure 4.9: Tracking control result of $x_2$ by PI controller

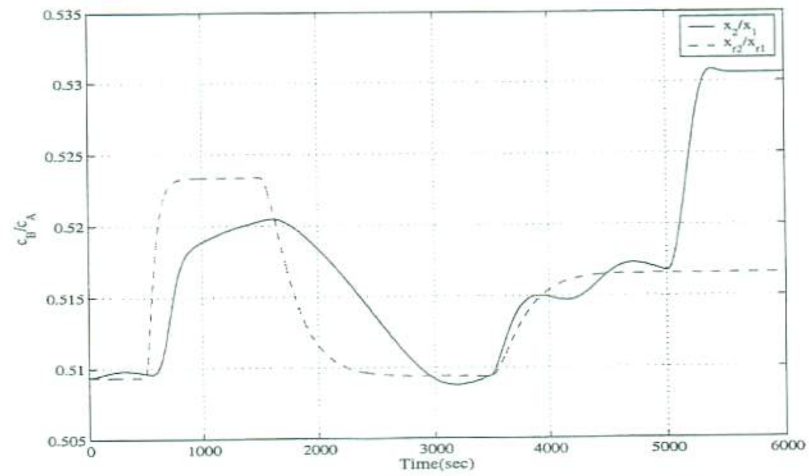


Figure 4.10: Tracking control result of $x_2/x_1$ by PI controller

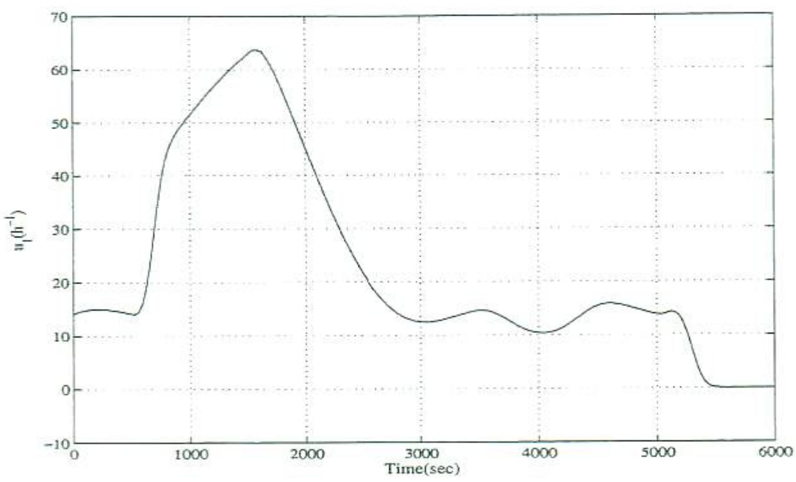


Figure 4.11: Control input $u_1$ of PI controller

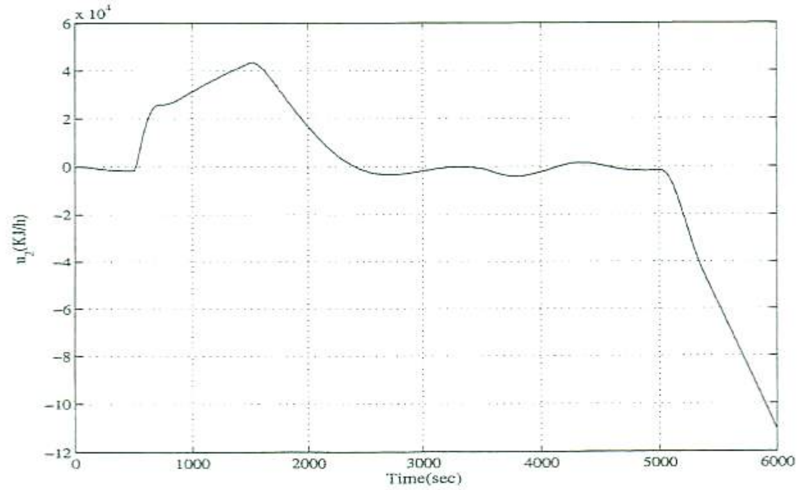


Figure 4.12: Control input $u_2$ of PI controller

# Chapter 5

# Design of High Gain Adaptive Output Feedback Control System for Uncertain Nonlinear Systems with a Higher Order Relative Degree

## 5.1 Introduction

In chapter 3, the robust high gain adaptive output feedback controller has been designed for non-OFEP nonlinear systems with non-Lipschitz nonlinearities and a relative degree of 1. However it might be difficult to apply this control strategy to practical systems since most practical systems have a relative degree of greater than 2. As mentioned in chapter 4, several robust adaptive controllers including the method presented in chapter 4[77–86] have been proposed for nonlinear systems of triangular form with nonparametric and time-varying uncertainties and with a higher order relative degree. Unfortunately these methods, however, require the state variables in order to design the controllers. In the case where the state variables are not available, one has to design an adaptive observer, this may however require a complex controller structure.

In this chapter, a robust adaptive controller design scheme based on high gain adaptive output feedback control will be proposed for non-OFEP nonlinear systems with non-Lipschitz nonlinearities and a higher order relative degree by introducing a virtual control input filter and applying backstepping procedure without introducing an adaptive observer. The methods with a virtual control input filter was initiated by Marino and Tomei[92] and several robust adaptive control designs using the virtual control input filter have been proposed[93–96]. One can design a adaptive controller without the use of state variables and/or a state observer by introducing a virtual filter for the control input since the actual control input can be designed through a backstepping strategy applied to the virtual filter. However most of previous methods dealt with controlled systems with unknown but constant or linear combination of unknown constants and known functions.

Here a class of uncertain time-varying nonlinear systems of triangular form with nonparametric uncertainties and unknown time-varying functions in control input terms, and with a higher order relative degree is considered. The robust adaptive control methods proposed in chapter 3 and 4 are extended. Further, it is also shown that an appropriate choice of design parameters guarantees the convergence of the output tracking error to any given bound without prior information about the size of the uncertainties.

## 5.2 Problem Statement

Consider the following $n$th order nonlinear systems with a relative degree of $r$ $(1 \leq i \leq r-1,\ 2 \leq r \leq n)$

$$\begin{aligned} \dot{x}_i &= f_i(x,t) + g_i(t)x_{i+1} \\ \dot{x}_r &= f_n(x,t) + g_r(t)u + b(t)^T\eta \\ \dot{\eta} &= f_\eta(x,t) + q(y,\eta) \\ y &= x_1 \end{aligned} \tag{5.1}$$

where $x = [x_1, \cdots, x_n]^T \in R^n$, $\eta^T = [x_{r+1}, \cdots, x_n]^T \in R^{n-r}$ are state vectors and $u, y \in R$ are an input and an output, respectively. $f_1(x,t), \cdots, f_r(x,t)$, $f_\eta(x,t) = [f_{r+1}(x,t), \cdots, f_n(x,t)]$ are uncertain nonlinearities and $g_1(t), \cdots, g_r(t)$, $b(t) = [b_{r+1}(t), \cdots, b_n(t)]^T$ are unknown time-varying functions.

Here we impose the following assumptions on the system (5.1).

**Assumption 5.1.** *The uncertain nonlinearities $f_i(x,t)$, $f_\eta(x,t)$ can be evaluated by*

$$\begin{aligned} |f_i(x,t)| &\leq d_{1i}|\psi_i(y)| + d_{0i} \ (1 \leq i \leq r) \\ \|f_\eta(x,t)\| &\leq d_{1\eta}|\psi_\eta(y)| + d_{0\eta} \end{aligned} \tag{5.2}$$

*with unknown positive constants $d_{1i}, d_{1\eta}, d_{0i}, d_{0\eta}$ and known smooth functions $\psi_i(y), \psi_\eta(y)$ which have the following properties for any variables $y_1$ and $y_2$ such that*

$$\begin{aligned} |\psi_i(y_1 + y_2)| &\leq |\psi_{1i}(y_1, y_2)||y_1| + |\psi_{2i}(y_2)| \\ |\psi_\eta(y_1 + y_2)| &\leq |\psi_{1\eta}(y_1, y_2)||y_1| + |\psi_{2\eta}(y_2)| \end{aligned} \tag{5.3}$$

*with known smooth functions $\psi_{1i}, \psi_{1\eta}$ and functions $\psi_{2i}, \psi_{2\eta}$ which are bounded for all bounded $y_2$.*

**Assumption 5.2.** *Unknown functions $g_i(t)(1 \leq i \leq r)$ are smooth and bounded with bounded derivative for any $t \geq 0$ and there exists an unknown positive constant $g_m$ such that*

$$g_{1,r}(t) := \prod_{i=1}^{r} g_i(t) \geq g_m > 0. \tag{5.4}$$

**Assumption 5.3.** *Unknown functions $b_{r+1}(t), \cdots, b_n(t)$ are bounded for all $t \geq 0$.*

**Assumption 5.4.** *The function $q(y,\eta)$ is globally Lipschitz with respect to $(y,\eta)$, i.e., there exists a positive constant $L_1$ such that*

$$\|q(y_1, \eta_1) - q(y_2, \eta_2)\| \leq L_1(|y_1 - y_2| + \|\eta_1 - \eta_2\|). \tag{5.5}$$

**Assumption 5.5.** *Nominal part of the system* (5.1) *is exponential minimum-phase. That is, the zero dynamics of the nominal system:*

$$\dot{\eta}(t) = q(0, \eta) \tag{5.6}$$

*is exponentially stable.*

Under these assumptions the control objective is to achieve the goal:

$$\lim_{t\to\infty} |y(t) - y^*(t)| \le \delta \tag{5.7}$$

for a given positive constant $\delta$ and a smooth reference signal $y^*$ such as

$$|y^*(t)| \le d_0,\ |\dot{y}^*(t)| \le d_1 \quad \forall t \in [0, \infty) \tag{5.8}$$

with positive constants $d_0$ and $d_1$.

## 5.3 Adaptive Controller Design

### 5.3.1 Virtual System

For the controlled system (5.1) we introduce the following $(r-1)$th order virtual control input filter.

$$\begin{aligned} \dot{u}_{f_i} &= -\lambda_i u_{f_i} + u_{f_{i+1}}\ (1 \le i \le r-2) \\ \dot{u}_{f_{r-1}} &= -\lambda_{r-1} u_{f_{r-1}} + u \\ \lambda_j &> 0,\ (1 \le j \le r-1) \end{aligned} \tag{5.9}$$

The concerning the virtual system, which is obtained by considering $u_{f_1}$ given from a virtual filter as the control input, the following proposition is comprised.

**Proposition 5.1.** *For the system* (5.1) *with a relative degree $r \le n$, consider the following variable transformation using the filtered signal $u_{f_i}$ given in* (5.9)

$$\xi_k = \bar{g}_{k,r} x_k - u_{f_{k-1}} - \sum_{d=1}^{k-1} \chi_{k,d} x_{k-d},\ (2 \le k \le r) \tag{5.10}$$

*where*

$$g_{m,n} = \prod_{i=m}^{n} g_i,\ \bar{g}_{m,n} = 1/g_{m,n},\ \bar{g}_m = 1/g_m$$

*and*

$$\begin{aligned} \chi_{r,1} &= \bar{g}_{r-1}(\lambda_{r-1}\bar{g}_r + \dot{\bar{g}}_r) \\ \chi_{r,d+1} &= \bar{g}_{r-d-1}(-\lambda_{r-1}\chi_{r,d} - \dot{\chi}_{r,d}),\ (1 \le d \le r-2) \\ \chi_{k,1} &= \bar{g}_{k-1}(\lambda_{k-1}\bar{g}_{k,r} + \dot{\bar{g}}_{k,r} + \chi_{k+1,1}) \\ \chi_{k,d+1} &= \bar{g}_{k-d-1}(-\lambda_{k-1}\chi_{k,d} - \dot{\chi}_{k,d} + \chi_{k+1,d+1}),\ (2 \le k \le r-1,\ 1 \le d \le k-2) \end{aligned}$$

*Then the system* (5.1) *can be expressed by the form:*

$$\begin{aligned} \dot{y} &= a(y,\xi,t) + g_{1,r}(t) u_{f_1} + f_1(y,\xi,\eta,t) \\ \dot{\xi} &= A_\xi \xi + a_\xi(t) y + B_\xi(t)\eta + F(y,\xi,\eta,t) \\ \dot{\eta} &= q(y,\eta) + f_\eta(y,\xi,\eta,t) \end{aligned} \tag{5.11}$$

*where*

$$a(y, \xi, t) = g_{1,r}\chi_{2,1}y + g_{1,r}\xi_2$$

$$A_\xi = \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & -\lambda_i & \ddots & 0 \\ \vdots & & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & 0 & -\lambda_{r-1} \end{bmatrix}, \quad B_\xi(t) = \begin{bmatrix} 0 \\ \bar{g}_r b^T \end{bmatrix}$$

$$a_\xi(t) = [a_{\xi_2}, \cdots, a_{\xi_k}, \cdots, a_{\xi_r}]^T$$

$$a_{\xi_k} = -\lambda_{k-1}\chi_{k,k-1} - \dot{\chi}_{k,k-1} + \chi_{k+1,k}, \ (2 \le k \le r-1)$$

$$a_{\xi_r} = -\lambda_{r-1}\chi_{r,r-1} - \dot{\chi}_{r,r-1}$$

$$F(y, \xi, \eta, t) = [f_{\xi_2}, \cdots, f_{\xi_k}, \cdots, f_{\xi_r}]^T$$

$$f_{\xi_k} = \bar{g}_{k,r} f_k - \sum_{d=1}^{k-1} \chi_{k,d} f_{k-d}, \ (2 \le k \le r)$$

*Proof.* Since it follows from (5.10) that

$$\xi_2 = \bar{g}_{2,r}x_2 - u_{f_1} - \chi_{2,1}x_1$$

we have

$$\begin{aligned} \dot{y} =& f_1 + g_1(g_{2,r}\xi_2 + g_{2,r}u_{f_1} + g_{2,r}\chi_{2,1}x_1) \\ =& a(y, \xi) + g_{1,r}u_{f_1} + f_1. \end{aligned} \tag{5.12}$$

Further for $k = 2, \cdots, r-1$ from (5.10) that

$$\xi_k = \bar{g}_{k,r}x_k - u_{f_{k-1}} - \sum_{d=1}^{k-1} \chi_{k,d}x_{k-d}$$

the time derivative of $\xi_k$ is obtained by

$$\begin{aligned} \dot{\xi}_k =& \dot{\bar{g}}_{k,r}x_k + \bar{g}_{k,r}(f_k + g_k x_{k+1}) - (-\lambda_{k-1}u_{f_{k-1}} + u_{f_k}) \\ & - \sum_{d=1}^{k-1} \dot{\chi}_{k,d}x_{k-d} - \sum_{d=1}^{k-1} \chi_{k,d}(f_{k-d} + g_{k-d}x_{k-d+1}). \end{aligned}$$

Here since it follows from (5.10) that

$$u_{f_k} = -\xi_{k+1} + \bar{g}_{k+1,r}x_{k+1} - \sum_{d=1}^{k} \chi_{k+1,d}x_{k-d+1}$$

$$u_{f_{k-1}} = -\xi_k + \bar{g}_{k,r}x_k - \sum_{d=1}^{k-1} \chi_{k,d}x_{k-d}$$

we have

$$\begin{aligned} \dot{\xi}_k =& (\xi_{k+1} + \sum_{d=1}^{k} \chi_{k+1,d}x_{k-d+1}) + \lambda_{k-1}(-\xi_k + \bar{g}_{k,r}x_k - \sum_{d=1}^{k-1} \chi_{k,d}x_{k-d}) \\ & + \dot{\bar{g}}_{k,r}x_k - \dot{\chi}_{k,k-1}x_1 - \sum_{d=1}^{k-2} \dot{\chi}_{k,d}x_{k-d} - \chi_{k,1}g_{k-1}x_k - \sum_{d=2}^{k-1} \chi_{k,d}g_{k-d}x_{k-d+1} + f_{\xi_k} \end{aligned}$$

Using the facts that

$$\sum_{d=1}^{k} \chi_{k+1,d} x_{k-d+1} = \chi_{k+1,1} x_k + \chi_{k+1,k} x_1 + \sum_{d=1}^{k-2} \chi_{k+1,d+1} x_{k-d}$$
$$\sum_{d=1}^{k-1} \dot{\chi}_{k,d} x_{k-d} = \dot{\chi}_{k,k-1} x_1 + \sum_{d=1}^{k-2} \dot{\chi}_{k,d} x_{k-d}$$

and

$$\sum_{d=2}^{k-1} \chi_{k,d} g_{k-d} x_{k-d+1} = \sum_{d=1}^{k-2} \chi_{k,d+1} g_{k-d-1} x_{k-d}$$

the time derivative of $\xi_k$ can be expressed as

$$\begin{aligned}
\dot{\xi}_k = &-\lambda_{k-1}\xi_k + \xi_{k+1} + \chi_{k+1,k} x_1 - \lambda_{k-1}\chi_{k,k-1} x_1 - \dot{\chi}_{k,k-1} x_1 + f_{\xi_k} \\
&- \chi_{k,1} g_{k-1} x_k + \chi_{k+1,1} x_k + \lambda_{k-1}\bar{g}_{k,r} x_k + \dot{\bar{g}}_{k,r} x_k \\
&- \sum_{d=1}^{k-2} \chi_{k,d+1} g_{k-d-1} x_{k-d} + \sum_{d=1}^{k-2} \chi_{k+1,d+1} x_{k-d} - \lambda_{k-1} \sum_{d=1}^{k-2} \chi_{k,d} x_{k-d} - \sum_{d=1}^{k-2} \dot{\chi}_{k,d} x_{k-d}.
\end{aligned}$$

Finally since it follows from the structure of $\chi_{k,1}$ and $\chi_{k,d+1}$, we obtain

$$\dot{\xi}_k = -\lambda_k \xi_k + \xi_{k+1} + a_{\xi_k} y + f_{\xi_k}. \tag{5.13}$$

As for $\xi_r$, since

$$\xi_r = \bar{g}_r x_r - u_{f_{r-1}} - \sum_{d=1}^{r-1} \chi_{r,d} x_{r-d}$$

the time derivative of $\xi_r$ can be obtained by using the same manner as in the former cases as follows:

$$\begin{aligned}
\dot{\xi}_r =& \dot{\bar{g}}_r x_r + \bar{g}_r(f_r + g_r u + b^T\eta) - (-\lambda_{r-1} u_{f_{r-1}} + u) \\
&- \sum_{d=1}^{r-1} \dot{\chi}_{r,d} x_{r-d} - \sum_{d=1}^{r-1} \chi_{r,d}(f_{r-d} + g_{r-d} x_{r-d+1}) \\
=& \dot{\bar{g}}_r x_r + \bar{g}_r b^T \eta + \lambda_{r-1}(-\xi + \bar{g}_r x_r - \sum_{d=1}^{r-1} \chi_{r,d} x_{r-d}) + f_{\xi_r} \\
&- \dot{\chi}_{r,r-1} x_1 - \sum_{d=1}^{r-2} \dot{\chi}_{r,d} x_{r-d} - \chi_{r,1} g_{r-1} x_r - \sum_{d=2}^{r-1} \chi_{r,d} g_{r-d} x_{r-d+1} \\
=& -\lambda_{r-1}\xi_r - \lambda_{r-1}\chi_{r,r-1} x_1 - \dot{\chi}_{r,r-1} x_1 + \bar{g}_r b^T \eta + f_{\xi_r} \\
&- \chi_{r,1} g_{r-1} x_r + \dot{\bar{g}}_r x_r + \lambda_{r-1} \bar{g}_r x_r \\
&- \sum_{d=1}^{r-2} \chi_{r,d+1} g_{r-d-1} x_{r-d} - \lambda_{r-1} \sum_{d=1}^{r-2} \chi_{r,d} x_{r-d} - \sum_{d=1}^{r-2} \dot{\chi}_{r,d} x_{r-d} \\
=& -\lambda_{r-1}\xi_r + a_{\xi_r} y + \bar{g}_r b^T \eta + f_{\xi_r}.
\end{aligned} \tag{5.14}$$

Thus we get the desired results. □

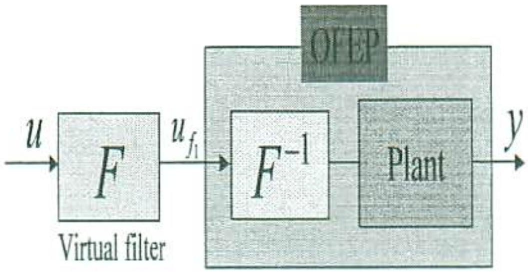


Figure 5.1: Augmented virtual system

From this proposition, we know that the system (5.1) with a relative degree of $r$ can be transformed into one of the form (5.11) having a relative degree of 1 with a filtered signal $u_{f_1}$ as the control input by an appropriate transformation using the filtered signal $u_{f_i}$.

For the obtained virtual system (5.11), it is easy to confirm from assumption 5.2 that $a(y, \boldsymbol{\xi}, t)$ is bounded for all $t \geq 0$ and Lipschitz with respect to $y$ and $\boldsymbol{\xi}$ so that there exists a positive constant $L_2$ such that

$$|a(y_1, \boldsymbol{\xi}_1) - a(y_2, \boldsymbol{\xi}_2)| \leq L_2(|y_1 - y_2| + \|\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2\|). \tag{5.15}$$

The uncertain vector function $\boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)$ can be evaluated from assumption 5.1 by

$$\|\boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)\| \leq p_1|\phi(y)| + p_0 \tag{5.16}$$

with unknown positive constants $p_1$ and $p_0$ and a known function $\phi(y)$ which has the following property for any variables $y_1$ and $y_2$:

$$|\phi(y_1 + y_2)| \leq |\phi_1(y_1, y_2)||y_1| + |\phi_2(y_2)| \tag{5.17}$$

with a known smooth function $\phi_1(y_1, y_2)$ and a function $\phi_2(y_2)$ which is bounded for all bounded $y_2 \in R$. Further, since $A_\xi$ is a stable matrix, there exists a positive symmetric matrix $P_\xi$ for any positive matrix $Q_\xi$ such as

$$P_\xi A_\xi + A_\xi^T P_\xi = -Q_\xi. \tag{5.18}$$

Moreover, the system (5.1) is exponential minimum-phase from assumption 5.5 so that there exist a positive definite function $W(\boldsymbol{\eta})$ and positive constants $\kappa_1$ to $\kappa_4$ from the converse theorem of Lyapunov[62, 66] such that

$$\begin{aligned} &\frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} \boldsymbol{q}(0, \boldsymbol{\eta}) \leq -\kappa_1 \|\boldsymbol{\eta}(t)\|^2, \quad \left\| \frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}} \right\| \leq \kappa_2 \|\boldsymbol{\eta}(t)\| \\ &\kappa_4 \|\boldsymbol{\eta}(t)\|^2 \leq W(\boldsymbol{\eta}) \leq \kappa_3 \|\boldsymbol{\eta}(t)\|^2 \end{aligned} \tag{5.19}$$

Seeing $f_1, \boldsymbol{F}$ and $\boldsymbol{f}_\eta$ as disturbances in system (5.11), the nominal part of the virtual system (5.11) is OFEP. Therefore, we can attain the control objective by designing the actual control input $u$ to make the filtered signal $u_{f_1}$ become the high gain adaptive output feedback input with robust properties for the disturbances.

Next, we design such the control system using backstepping strategy in the filter dynamics. Fig.5.1 shows a diagram of this control method.

### 5.3.2 Controller Design through Backstepping

**[Step 1]** Defining $\nu(t) = y(t) - y^*(t)$ as a tracking error to attain the control objective (5.7), the error system can be expressed from (5.11) as the following form:

$$\begin{aligned}
\dot{\nu} &= a(\nu + y^*, \boldsymbol{\xi}) + g_{1,r}u_{f_1} + f_1(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta}) - \dot{y}^* \\
\dot{\boldsymbol{\xi}} &= A_\xi \boldsymbol{\xi} + \boldsymbol{a}_\xi[\nu + y^*] + B_\xi \boldsymbol{\eta} + \boldsymbol{F}(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta}) \\
\dot{\boldsymbol{\eta}} &= \boldsymbol{q}(\nu + y^*, \boldsymbol{\eta}) + \boldsymbol{f}_\eta(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta})
\end{aligned} \tag{5.20}$$

For this system, a virtual control input $\alpha_1$ for the filtered signal $u_{f_1}$ in the error system (5.20) is designed based on a robust adaptive high gain feedback proposed in chapter 3 as follows:

$$\alpha_1(t) = -[k(t)\nu(t) + u_R(t)] \tag{5.21}$$
$$k(t) = k_I(t) + k_P(t) \tag{5.22}$$
$$\dot{k}_I(t) = \gamma_I D(\nu, \omega)\nu(t)^2, \quad k_I(0) \geq 0 \tag{5.23}$$
$$k_P(t) = \gamma_p[\phi_1(\nu, y^*)^4 + \psi_{1\eta}(\nu, y^*)^4]\nu(t)^2 \tag{5.24}$$
$$u_R(t) = \gamma_R \psi_1(y)^2 \nu(t) \tag{5.25}$$

where $\gamma_I, \gamma_p$ and $\gamma_R$ are arbitrary positive constants and $D(\nu, \omega)$ is defined for any positive constant $\delta_\nu$ that

$$\begin{aligned}
D(\nu, \omega) &= \begin{cases} 0 & \text{for } (\nu, \omega) \in \Omega_0 \\ 1 & \text{for } (\nu, \omega) \in \Omega_1 \end{cases} \\
\Omega_0 &= \{\nu \in R,\ \omega \in R^{r-1} \mid \nu^2 + \|\omega\|^2 \leq \delta_\nu^2\} \\
\Omega_1 &= \{\nu \in R,\ \omega \in R^{r-1} \mid \nu^2 + \|\omega\|^2 > \delta_\nu^2\}
\end{aligned} \tag{5.26}$$

where $\omega = [\omega_1, \omega_2, \cdots, \omega_{r-1}]^T$, $\omega_1 = u_{f_1} - \alpha_1$ and $\omega_i = u_{f_i} - \alpha_i$, $i = 2, \cdots, r-1$. The virtual input $\alpha_1$ is given in (5.21) and the virtual inputs $\alpha_i, (2 \leq i \leq r-1)$ will be designed in the following step $i$ by backstepping strategy.

Consider the following positive definite function $V_1(\nu, \boldsymbol{\xi}, \boldsymbol{\eta}, k_I)$ for $(\nu, \omega) \in \Omega_1$:

$$V_1(\nu, \boldsymbol{\xi}, \boldsymbol{\eta}, k) = \frac{1}{2}\nu^2 + \mu_0 \boldsymbol{\xi}^T P_\xi \boldsymbol{\xi} + \mu_1 W(\boldsymbol{\eta}) + \frac{g_m}{2\gamma_I}[k_I - k^*]^2 \tag{5.27}$$

where $\mu_0, \mu_1$ are any positive constants and $k^*$ is an ideal feedback gain for $k_I$ to be determined later. $g_m$ is a positive constant given in assumption 5.2. The time derivative of $V_1$ along the trajectories (5.20) and (5.23) yield that

$$\begin{aligned}
\dot{V}_1 =& \nu\big[a(\nu + y^*, \xi) - g_{1,r}[k\nu + u_R] + g_{1,r}[u_{f_1} - \alpha_1] + f_1(\nu + y^*) - \dot{y}^*\big] \\
&+ \mu_0 \boldsymbol{\xi}^T (A_\xi^T P_\xi + P_\xi A_\xi)\boldsymbol{\xi} + \mu_0\big[\boldsymbol{a}_\xi[\nu + y^*] + B_\xi \boldsymbol{\eta} + \boldsymbol{F}\big]^T P_\xi \boldsymbol{\xi} \\
&+ \mu_0 \boldsymbol{\xi}^T P_\xi \big[\boldsymbol{a}_\xi[\nu + y^*] + B_\xi \boldsymbol{\eta} + \boldsymbol{F}\big] \\
&+ \mu_1 \frac{\partial W}{\partial \boldsymbol{\eta}}\big[\boldsymbol{q}(\nu + y^*, \boldsymbol{\eta}) + \boldsymbol{f}_\eta(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta})\big] + g_m[k_I - k^*]\nu^2.
\end{aligned} \tag{5.28}$$

It follows from assumptions 5.1 and 5.4 and from (5.15) to (5.19) that $\dot{V}$ can be evaluated by

$$
\begin{aligned}
\dot{V}_1 \leq & L_2(|\nu| + |y^*| + \|\xi\|)|\nu| - g_{1,r}(k\nu + u_R)\nu + |\dot{y}^*||\nu| + g_{1,r}\nu\omega_1 \\
& + (d_{11}|\psi_1| + d_{01})|\nu| - \mu_0 \xi^T Q_\xi \xi \\
& + 2\mu_0 \left[a_{\xi M}(|\nu| + |y^*|) + B_{\xi M}\|\eta\|\right] \|P_\xi\| \|\xi\| \\
& + \mu_0 \|P_\xi\| \|\xi\| \left[p_1(|\phi_1||\nu| + |\phi_2|) + p_0\right] \\
& - \mu_1 \kappa_1 \|\eta\|^2 + \mu_1 \kappa_2 L_1(|\nu| + |y^*|)\|\eta\| \\
& + \mu_1 \kappa_2 \left[d_{1\eta}(|\psi_{1\eta}||\nu| + |\psi_{2\eta}|) + d_{0\eta}\right] \|\eta\| \\
& + g_m k_I \nu^2 - g_m k^* \nu^2
\end{aligned}
\tag{5.29}
$$

where $a_{\xi M}, B_{\xi M}$ are positive constants which satisfy $\|a_\xi(t)\| \leq a_{\xi M}, \|B_\xi(t)\| \leq B_{\xi M}$. Such positive constants exist from assumptions 5.2 and 5.3. Since we have

$$
\begin{aligned}
& - g_{1,r}(k\nu + u_R)\nu + g_m k_I \nu^2 \\
\leq & - g_m k \nu^2 - g_m \gamma_R \psi_1^2 \nu^2 + g_m k_I \nu^2 \\
= & - g_m k_p \nu^2 - g_m \gamma_R \psi_1^2 \nu^2
\end{aligned}
\tag{5.30}
$$

from the fact that $k(t) \geq 0$ which is obtained from (5.23) and (5.24) and from assumption 5.2, the time derivative of $V_1$ can be evaluated by

$$
\begin{aligned}
\dot{V}_1 \leq & - (g_m k^* - L_2)\nu^2 - \mu_0 \lambda_{min}[Q_\xi]\|\xi\|^2 - \mu_1 \kappa_1 \|\eta\|^2 \\
& + \mu_1 \kappa_2 L_1 |\nu| \|\eta\| + (d_0 L_2 + d_{01} + d_1)|\nu| \\
& + (L_2 + 2\mu_0 a_{\xi M}\|P_\xi\|)|\nu|\|\xi\| - g_m k_p \nu^2 + g_{1,r}\nu\omega_1 \\
& + \mu_0 p_1 \|P_\xi\| |\phi_1| |\nu| \|\xi\| + \mu_1 \kappa_2 d_{1\eta} |\psi_{1\eta}| |\nu| \|\eta\| \\
& + \mu_0 \|P_\xi\| (2 d_0 a_{\xi M} + p_1 |\phi_2| + p_0) \|\xi\| \\
& + \mu_1 \kappa_2 (d_0 L_1 + d_{1\eta} |\psi_{2\eta}| + d_{0\eta}) \|\eta\| \\
& + 2\mu_0 B_{\xi M} \|P_\xi\| \|\xi\| \|\eta\| - g_m \gamma_R \psi_1^2 \nu^2 + d_{11} |\psi_1| |\nu|
\end{aligned}
\tag{5.31}
$$

Further we have

$$
\begin{aligned}
& - g_m \gamma_R \psi_1^2 \nu^2 + d_{11} |\psi_1| |\nu| \\
= & - g_m \gamma_R (|\psi_1||\nu| - \frac{d_{11}}{2 g_m \gamma_R})^2 + \frac{d_{11}^2}{4 g_m \gamma_R} \\
\leq & \frac{d_{11}^2}{4 g_m \gamma_R}
\end{aligned}
\tag{5.32}
$$

and

$$
\begin{aligned}
&\rho_1\|\eta\|^2 - \rho_1\|\eta\|^2 + \mu_1\kappa_2 L_1|\nu|\|\eta\| \\
&\quad= \rho_1\|\eta\|^2 - \rho_1(\|\eta\| - \frac{\mu_1\kappa_2 L_1|\nu|}{2\rho_1})^2 + \frac{(\mu_1\kappa_2 L_1|\nu|)^2}{4\rho_1} \\
&\quad\le \rho_1\|\eta\|^2 + \frac{(\mu_1\kappa_2 L_1)^2}{4\rho_1}\nu^2 &(5.33)\\
&\rho_2\nu^2 - \rho_2\nu^2 + (d_0 L_2 + d_{01} + d_1)|\nu| \\
&\quad\le \rho_2\nu^2 + \frac{(d_0 L_2 + d_{01} + d_1)^2}{4\rho_2} &(5.34)\\
&\rho_3\|\xi\|^2 - \rho_3\|\xi\|^2 + (L_2 + 2\mu_0 a_{\xi M}\|P_\xi\|)|\nu|\|\xi\| \\
&\quad\le \rho_3\|\xi\|^2 + \frac{(L_2 + 2\mu_0 a_{\xi M}\|P_\xi\|)^2}{4\rho_3}\nu^2 &(5.35)\\
&\rho_4\|\xi\|^2 - \rho_4\|\xi\|^2 + \mu_0 p_1\|P_\xi\||\phi_1||\nu|\|\xi\| \\
&\quad\le \rho_4\|\xi\|^2 + \frac{(\mu_0 p_1\|P_\xi\|)^2}{4\rho_4}\phi_1^2\nu^2 &(5.36)\\
&\rho_5\|\eta\|^2 - \rho_5\|\eta\|^2 + \mu_1\kappa_2 d_{1\eta}|\psi_{1\eta}||\nu|\|\eta\| \\
&\quad\le \rho_5\|\eta\|^2 + \frac{(\mu_1\kappa_2 d_{1\eta})^2}{4\rho_5}\psi_{1\eta}^2\nu^2 &(5.37)\\
&\rho_6\|\xi\|^2 - \rho_6\|\xi\|^2 + \mu_0\|P_\xi\|(2d_0 a_{\xi M} + p_1|\phi_2| + p_0)\|\xi\| \\
&\quad\le \rho_6\|\xi\|^2 + \frac{[\mu_0\|P_\xi\|(2d_0 a_{\xi M} + p_1\phi_{2M} + p_0)]^2}{4\rho_6} &(5.38)\\
&\rho_7\|\eta\|^2 - \rho_7\|\eta\|^2 + \mu_1\kappa_2(d_0 L_1 + d_{1\eta}|\psi_{2\eta}| + d_{0\eta})\|\eta\| \\
&\quad\le \rho_7\|\eta\|^2 + \frac{[\mu_1\kappa_2(d_0 L_1 + d_{1\eta}\psi_{2\eta M} + d_{0\eta})]^2}{4\rho_7} &(5.39)\\
&\rho_8\|\eta\|^2 - \rho_8\|\eta\|^2 + 2\mu_0 B_{\xi M}\|P_\xi\|\|\xi\|\|\eta\| \\
&\quad\le \rho_8\|\eta\|^2 + \frac{(\mu_0 B_{\xi M}\|P_\xi\|)^2}{\rho_8}\|\xi\|^2 &(5.40)
\end{aligned}
$$

with any positive constants $\rho_1$ to $\rho_8$ and positive constants $\phi_{2M}$ and $\psi_{2\eta M}$ which satisfy $|\phi_2(y^*)| \le \phi_{2M}, |\psi_{2\eta}(y^*)| \le \psi_{2\eta M}$. Since $y^*$ is bounded, such constants exist from assumption 5.1 that $\phi_2(y_2)$ and $\psi_{2\eta}(y_2)$ are bounded for all bounded $y_2$. Moreover, since we have

$$
\begin{aligned}
&- g_m k_p\nu^2 + \frac{(\mu_0 p_1\|P_\xi\|)^2}{4\rho_4}\phi_1^2\nu^2 + \frac{(\mu_1\kappa_2 d_{1\eta})^2}{4\rho_5}\psi_{1\eta}^2\nu^2 \\
&= - g_m\gamma_p\phi_1^4\nu^4 + \frac{(\mu_0 p_1\|P_\xi\|)^2}{4\rho_4}\phi_1^2\nu^2 \\
&\quad - g_m\gamma_p\psi_{1\eta}^4\nu^4 + \frac{(\mu_1\kappa_2 d_{1\eta})^2}{4\rho_5}\psi_{1\eta}^2\nu^2 \\
&\le \frac{1}{g_m\gamma_p}\left[\frac{(\mu_0 p_1\|P_\xi\|)^2}{8\rho_4}\right]^2 + \frac{1}{g_m\gamma_p}\left[\frac{(\mu_1\kappa_2 d_{1\eta})^2}{8\rho_5}\right]^2 &(5.41)
\end{aligned}
$$

from (5.24), the time derivative of $V_1$ can be eventually evaluated by

$$
\begin{aligned}
\dot{V}_1 \le &- [g_m k^* - v_0]\nu^2 - [\mu_0\lambda_{min}[Q_\xi] - v_1]\|\xi\|^2 \\
&- [\mu_1\kappa_1 - v_2]\|\eta\|^2 + g_{1,r}\nu\omega_1 + R_1 &(5.42)
\end{aligned}
$$

where

$$
\begin{aligned}
v_0 =& L_2 + \frac{(\mu_1\kappa_2 L_1)^2}{4\rho_1} + \rho_2 + \frac{(L_2 + 2\mu_0 a_{\xi M}\|P_\xi\|)^2}{4\rho_3} \\
v_1 =& \rho_3 + \rho_4 + \rho_6 + \frac{(\mu_0 B_{\xi M}\|P_\xi\|)^2}{\rho_8} \\
v_2 =& \rho_1 + \rho_5 + \rho_7 + \rho_8 \\
R_1 =& \frac{d_{11}^2}{4g_m\gamma_R} + \frac{(d_0 L_2 + d_{01} + d_1)^2}{4\rho_2} \\
&+ \frac{[\mu_0\|P_\xi\|(2d_0 a_{\xi M} + p_1\phi_{2M} + p_0)]^2}{4\rho_6} \\
&+ \frac{[\mu_1\kappa_2(d_0 L_1 + d_{1\eta}\psi_{2\eta M} + d_{0\eta})]^2}{4\rho_7} \\
&+ \frac{1}{64 g_m\gamma_p}\left[\frac{(\mu_0 p_1\|P_\xi\|)^4}{\rho_4^2} + \frac{(\mu_1\kappa_2 d_{1\eta})^4}{\rho_5^2}\right].
\end{aligned}
$$

**[Step 2]** The control objective is achieved when the virtual control input $\alpha_1$ designed in Step 1 identifies the filtered signal $u_{f_1}$. In this step, we consider an error system, $\omega_1$-system, between $\alpha_1$ and $u_{f_1}$. The $\omega_1$-system is given from (5.9) as

$$\dot{\omega}_1 = -\lambda_1 u_{f_1} + u_{f_2} - \dot{\alpha}_1. \tag{5.43}$$

Here $\dot{\alpha}_1$ is given from (5.21) to (5.25) by

$$
\begin{aligned}
\dot{\alpha}_1 =& \frac{\partial\alpha_1}{\partial y}\dot{y} + \frac{\partial\alpha_1}{\partial y^*}\dot{y}^* + \frac{\partial\alpha_1}{\partial k_I}\dot{k}_I \\
=& \frac{\partial\alpha_1}{\partial y}[g_{1,r}\chi_{2,1}y + g_{1,r}\xi_2 + g_{1,r}u_{f_1} + f_1] + \frac{\partial\alpha_1}{\partial y^*}\dot{y}^* + \frac{\partial\alpha_1}{\partial k_I}\gamma_I D(\nu,\omega)\nu^2.
\end{aligned} \tag{5.44}
$$

Taking the form of $\dot{\alpha}_1$ for $(\nu,\omega) \in \Omega_1$ into consideration, we design the virtual control input $\alpha_2$ for the filtered signal $u_{f_2}$ in the $\omega_1$-system as follows:

$$\alpha_2 = -c_1\omega_1 + \lambda_1 u_{f_1} + \frac{\partial\alpha_1}{\partial k_I}\gamma_I\nu^2 - \epsilon_1\Psi_1\omega_1 \tag{5.45}$$

for all $\nu$ and $\omega$. $c_1$ is an adaptive feedback gain which is adaptively adjusted by the following parameter adjusting law:

$$\dot{c}_1 = \gamma_{c1} D(\nu,\omega)\omega_1^2,\ c_1(0) \geq 0 \tag{5.46}$$

with any positive constant $\gamma_{c1}$. $\Psi_1$ is given by

$$\Psi_1 = (\psi_1^2 + u_{f_1}^2 + l_1)\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + \left(\frac{\partial\alpha_1}{\partial y^*}\right)^2 \tag{5.47}$$

where $\epsilon_1$ and $l_1$ are any positive constants.

Consider the following positive definite function $V_2$:

$$V_2 = V_1 + \frac{1}{2}\omega_1^2 + \frac{1}{2\gamma_{c1}}[c_1 - c_1^*]^2 \tag{5.48}$$

where $c_1^*$ is an ideal feedback gain for $c_1$ to be determined later. The time derivative of $V_2$ for $(\nu, \omega) \in \Omega_1$ yields that

$$\begin{aligned}\dot{V}_2 =& \dot{V}_1 - \omega_1(\dot{u}_{f_1} - \dot{\alpha}_1)\\ =& \dot{V}_1 + \omega_1\Big[-\lambda_1 u_{f_1} + \omega_2 + \alpha_2 - \frac{\partial \alpha_1}{\partial y}[g_{1,r}\chi_{2,1}(\nu + y^*) + g_{1,r}\xi_2 + g_{1,r}u_{f_1} + f_1]\\ &- \frac{\partial \alpha_1}{\partial y^*}\dot{y}^* - \frac{\partial \alpha_1}{\partial k_I}\gamma_I\nu^2\Big] + [c_1 - c_1^*]\omega_1^2 \end{aligned} \tag{5.49}$$

where $\omega_2 = u_{f_2} - \alpha_2$. Considering $\alpha_2$ given in (5.45) and assumption 5.1, the derivative of $V_2$ can be evaluated by

$$\begin{aligned}\dot{V}_2 \le& -[g_m k^* - v_0]\nu^2 - [\mu_0\lambda_{m[Q_\xi]} - v_1]\|\xi\|^2 - [\mu_1\kappa_1 - v_2]\|\eta\|^2 + g_{1,r}\nu\omega_1 + R_1\\ &- c_1^*\omega_1^2 + \omega_1\omega_2 - \epsilon_1\Psi_1\omega_1^2 + |\omega_1|\left|\frac{\partial \alpha_1}{\partial y}\right|[\chi_M|\nu| + g_M\|\xi\| + g_M|u_{f_1}|\\ &+ d_{11}|\psi_1| + (d_0\chi_M + d_{01})] + d_1|\omega_1|\left|\frac{\partial \alpha_1}{\partial y^*}\right| \end{aligned} \tag{5.50}$$

where $g_M$ and $\chi_M$ are positive constants such as $g_M \ge g_{1,r}, \chi_M \ge g_{1,r}\chi_{2,1}$. Such positive constants exist from assumption 5.2. Here we have

$$\bar{\rho}_1\omega_1^2 - \bar{\rho}_1\omega_1^2 + g_{1,r}\nu\omega_1 \le \bar{\rho}_1\omega_1^2 + \frac{g_M^2}{4\bar{\rho}_1}\nu^2 \tag{5.51}$$

with any positive constant $\bar{\rho}_1$ and

$$-\epsilon_1\frac{l_1}{3}\left(\frac{\partial \alpha_1}{\partial y}\right)^2\omega_1^2 + \left|\frac{\partial \alpha_1}{\partial y}\right|\chi_M|\nu||\omega_1| \le \frac{3\chi_M^2}{4\epsilon_1 l_1}\nu^2 \tag{5.52}$$

$$-\epsilon_1\frac{l_1}{3}\left(\frac{\partial \alpha_1}{\partial y}\right)^2\omega_1^2 + \left|\frac{\partial \alpha_1}{\partial y}\right|g_M\|\xi\||\omega_1| \le \frac{3g_M^2}{4\epsilon_1 l_1}\|\xi\|^2 \tag{5.53}$$

$$-\epsilon_1\left(\frac{\partial \alpha_1}{\partial y}\right)^2 u_{f_1}^2\omega_1^2 + \left|\frac{\partial \alpha_1}{\partial y}\right|g_M|u_{f_1}||\omega_1| \le \frac{g_M^2}{4\epsilon_1} \tag{5.54}$$

$$-\epsilon_1\left(\frac{\partial \alpha_1}{\partial y}\right)^2 \psi_1^2\omega_1^2 + \left|\frac{\partial \alpha_1}{\partial y}\right|d_{11}|\psi_1||\omega_1| \le \frac{d_{11}^2}{4\epsilon_1} \tag{5.55}$$

$$-\epsilon_1\frac{l_1}{3}\left(\frac{\partial \alpha_1}{\partial y}\right)^2\omega_1^2 + \left|\frac{\partial \alpha_1}{\partial y}\right||\omega_1|(d_0\chi_M + d_{01}) \le \frac{3(d_0\chi_M + d_{01})^2}{4\epsilon_1 l_1} \tag{5.56}$$

$$-\epsilon_1\left(\frac{\partial \alpha_1}{\partial y^*}\right)^2\omega_1^2 + \left|\frac{\partial \alpha_1}{\partial y^*}\right||\omega_1|d_1 \le \frac{d_1^2}{4\epsilon_1} \tag{5.57}$$

from (5.47). In the result the time derivative of $V_2$ can be evaluated by

$$\begin{aligned}\dot{V}_2 \le& -[g_m k^* - v_0 - \frac{g_M^2}{4\bar{\rho}_1} - \frac{3\chi_M^2}{4\epsilon_1 l_1}]\nu^2 - [\mu_0\lambda_{min}[Q_\xi] - v_1 - \frac{3g_M^2}{4\epsilon_1 l_1}]\|\xi\|^2\\ &- [\mu_1\kappa_1 - v_2]\|\eta\|^2 - (c_1^* - \bar{\rho}_1)\omega_1^2 + \omega_1\omega_2 + R_1 + R_2 \end{aligned} \tag{5.58}$$

where

$$R_2 = \frac{g_M^2}{4\epsilon_1} + \frac{d_{11}^2}{4\epsilon_1} + \frac{3(d_0\chi_M + d_{01})^2}{4\epsilon_1 l_1} + \frac{d_1^2}{4\epsilon_1}.$$

[**Step i** $(3 \le i \le r-1)$] In step $i$, we consider the error system, $\omega_{i-1}$-system, in the same manner as step 2. $\omega_{i-1}$-system is given from (5.9) as

$$\dot{\omega}_{i-1} = -\lambda_{i-1}u_{f_{i-1}} + u_{f_i} - \dot{\alpha}_{i-1}. \tag{5.59}$$

The time derivative of $\alpha_{i-1}$ is given by

$$\begin{aligned}\dot{\alpha}_{i-1} =& \frac{\partial\alpha_{i-1}}{\partial y}[g_{1,r}\chi_{2,1}y + g_{1,r}\xi_2 + g_{1,r}u_{f_1} + f_1] + \frac{\partial\alpha_{i-1}}{\partial y^*}\dot{y}^* \\ &+ \frac{\partial\alpha_{i-1}}{\partial k_I}\gamma_I D(\nu,\omega)\nu^2 + \sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial c_k}\gamma_{ck}\omega_k^2 + \sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial u_{f_k}}[-\lambda_k u_{f_k} + u_{f_{k+1}}]. \end{aligned} \tag{5.60}$$

For this $\omega_{i-1}$-system, we design the virtual control input $\alpha_i$ for the filtered signal $u_{f_i}$ for all $\nu$ and $\omega$ as follows by taking the form $\dot{\alpha}_{i-1}$ for $(\nu,\omega) \in \Omega_1$ into account:

$$\begin{aligned}\alpha_i =& -c_{i-1}\omega_{i-1} + \lambda_{i-1}u_{f_{i-1}} + \frac{\partial\alpha_{i-1}}{\partial k_I}\gamma_I\nu^2 + \sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial c_k}\gamma_{ck}\omega_k^2 \\ &+ \sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial u_{f_k}}[-\lambda_k u_{f_k} + u_{f_{k+1}}] - \epsilon_{i-1}\Psi_{i-1}\omega_{i-1} \end{aligned} \tag{5.61}$$

$$\dot{c}_{i-1} = \gamma_{ci-1}D(\nu,\omega)\omega_{i-1}^2,\ c_{i-1}(0) \ge 0 \tag{5.62}$$

$$\Psi_{i-1} = (\psi_1^2 + u_{f_1}^2 + l_{i-1})\left(\frac{\partial\alpha_{i-1}}{\partial y}\right)^2 + \left(\frac{\partial\alpha_{i-1}}{\partial y^*}\right)^2 \tag{5.63}$$

where $\gamma_{ci-1}, \epsilon_{i-1}$ and $l_{i-1}$ are any positive constants. Here, consider the following positive definite function $V_i$:

$$V_i = V_{i-1} + \frac{1}{2}\omega_{i-1}^2 + \frac{1}{2\gamma_{ci-1}}[c_{i-1} - c_{i-1}^*]^2 \tag{5.64}$$

with an ideal feedback gain $c_{i-1}^*$ for $c_{i-1}$ which is determined later. The time derivative of $V_i$ for $(\nu,\omega) \in \Omega_1$ yields that

$$\begin{aligned}\dot{V}_i =& \dot{V}_{i-1} + \omega_{i-1}\Bigg[-\lambda_{i-1}u_{f_{i-1}} + \omega_i + \alpha_i - \frac{\partial\alpha_{i-1}}{\partial y}[g_{1,r}\chi_{2,1}(\nu + y^*) + g_{1,r}\xi_2 + g_{1,r}u_{f_1} + f_1] \\ &- \frac{\partial\alpha_{i-1}}{\partial y^*}\dot{y}^* - \frac{\partial\alpha_{i-1}}{\partial k_I}\gamma_I\nu^2 - \sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial c_k}\gamma_{ck}\omega_k^2 - \sum_{k=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial u_{f_k}}[-\lambda_k u_{f_k} + u_{f_{k+1}}]\Bigg] \\ &+ [c_{i-1} - c_{i-1}^*]\omega_{i-1}^2 \end{aligned} \tag{5.65}$$

where $\omega_i = u_{f_i} - \alpha_i$. Since considering the virtual control input $\alpha_i$ given in (5.61) and applying the same calculations as (5.51) to (5.57) in step 2, the time derivative of $V_i$ can be evaluated by

$$\begin{aligned}\dot{V}_i \le& -[g_m k^* - v_0 - \frac{g_M^2}{4\bar{\rho}_1} - \sum_{k=1}^{i-1}\frac{3\chi_M^2}{4\epsilon_k l_k}]\nu^2 - [\mu_0\lambda_{min}[Q_\xi] - v_1 - \sum_{k=1}^{i-1}\frac{3g_M^2}{4\epsilon_k l_k}]\|\xi\|^2 \\ &- [\mu_1\kappa_1 - v_2]\|\eta\|^2 - \sum_{k=1}^{i-2}(c_k^* - \bar{\rho}_k - \frac{1}{4\bar{\rho}_{k+1}})\omega_k^2 \\ &- (c_{i-1}^* - \bar{\rho}_{i-1})\omega_{i-1}^2 + \omega_{i-1}\omega_i + \sum_{k=1}^{i} R_k \end{aligned} \tag{5.66}$$

where $\bar{\rho}_2, \cdots, \bar{\rho}_{i-1}$ are any positive constants and

$$R_i = \frac{g_M^2}{4\epsilon_{i-1}} + \frac{d_{11}^2}{4\epsilon_{i-1}} + \frac{3(d_0\chi_M + d_{01})^2}{4\epsilon_{i-1}l_{i-1}} + \frac{d_1^2}{4\epsilon_{i-1}}.$$

**[Step r]** This is the final step. In this step, the actual control input $u$ is designed as follows:

$$u = \alpha_r \tag{5.67}$$

using $\alpha_i$ given in (5.45) and (5.61). In the final step, we consider the following positive definite function $V_r$:

$$V_r = V_{r-1} + \frac{1}{2}\omega_{r-1}^2 + \frac{1}{2\gamma_{cr-1}}[c_{r-1} - c_{r-1}^*]^2. \tag{5.68}$$

The time derivative of $V_r$ can be evaluated by

$$\begin{aligned}
\dot{V}_r \leq &- [g_m k^* - v_0 - \frac{g_M^2}{4\bar{\rho}_1} - \sum_{k=1}^{r-1} \frac{3\chi_M^2}{4\epsilon_k l_k}]\nu^2 \\
&- [\mu_0\lambda_{min}[Q_\xi] - v_1 - \sum_{k=1}^{r-1} \frac{3g_M^2}{4\epsilon_k l_k}]\|\xi\|^2 \\
&- [\mu_1\kappa_1 - v_2]\|\eta\|^2 - \sum_{k=1}^{r-2}(c_k^* - \bar{\rho}_k - \frac{1}{4\bar{\rho}_{k+1}})\omega_k^2 \\
&- (c_{r-1}^* - \bar{\rho}_{r-1})\omega_{r-1}^2 + \sum_{k=1}^{r} R_k
\end{aligned} \tag{5.69}$$

$$R_r = \frac{g_M^2}{4\epsilon_{r-1}} + \frac{d_{11}^2}{4\epsilon_{r-1}} + \frac{3(d_0\chi_M + d_{01})^2}{4\epsilon_{r-1}l_{r-1}} + \frac{d_1^2}{4\epsilon_{r-1}}$$

for $(\nu, \omega) \in \Omega_1$ by using the same manner as shown in the previous steps.

By setting $\rho_1 = \rho_5 = \rho_7 = \frac{\mu_1\kappa_1}{12}$, $\rho_3 = \rho_4 = \rho_6 = \frac{\mu_0\lambda_{min}[Q_\xi]}{6}$, $\rho_8 = \frac{6\mu_0(B_{\xi M}\|P_\xi\|)^2}{\lambda_{min}[Q_\xi]}$ and considering $\mu_0$ and $\mu_1$ such as $\mu_0 = \sum_{k=1}^{r-1} \frac{3g_M^2}{\lambda_{min}[Q_\xi]\epsilon_k l_k}$ and $\mu_1 = \frac{9\mu_0(B_{\xi M}\|P_\xi\|)^2}{\lambda_{min}[Q_\xi]\kappa_1}$, we have

$$\dot{V}_r \leq -K^*(\nu^2 + \sum_{k=1}^{r-1}\omega_k^2) - \frac{1}{12}\left[\mu_0\lambda_{min}[Q_\xi]\|\xi\|^2 - \mu_1\kappa_1\|\eta\|^2\right] + R_T \tag{5.70}$$

where $R_T = \sum_{k=1}^{r} R_k$ and

$$K^* = \min\left[g_m k^* - v_0', c_1^* - \bar{\rho}_1 - \frac{1}{4\bar{\rho}_2}, \cdots, c_{r-2}^* - \bar{\rho}_{r-2} - \frac{1}{4\bar{\rho}_{r-1}}, c_{r-1}^* - \bar{\rho}_{r-1}\right] \tag{5.71}$$

$$v_0' = v_0 + \frac{g_M^2}{2\bar{\rho}_1} + \sum_{i=1}^{r-1}\frac{3\chi_M^2}{4\epsilon_k l_k}.$$

## 5.4 Boundedness and Convergence Analysis

For the designed control system, the following theorem concerning the boundedness of all the signals in the control system and convergence of the tracking error is given.

**Theorem 5.1.** *Under assumptions* 5.1 *to* 5.5 *on the controlled system* (5.1), *all the signals in the resulting closed-loop system with controller* (5.67) *designed according to each step are bounded. Further, the tracking error* $\nu$ *converges to any given bound*

$$\lim_{t\to\infty} |\nu| \le \delta. \tag{5.72}$$

*Proof.* Consider the following positive and continuous function $V$:

$$V = \begin{cases} \frac{1}{2}\delta_\nu^2 + \frac{g_m}{2\gamma_I}\Delta k_I^2 + \sum_{k=1}^{r-1}\frac{1}{2\gamma_{ck}}\Delta c_k^2 + \delta_{V_v}^2 \\ \qquad \text{for } (\nu,\omega)\in\Omega_0 \text{ and } (\xi,\eta)\in\Omega_{V_0} \\ \frac{1}{2}\delta_\nu^2 + \frac{g_m}{2\gamma_I}\Delta k_I^2 + \sum_{k=1}^{r-1}\frac{1}{2\gamma_{ck}}\Delta c_k^2 + V_v \\ \qquad \text{for } (\nu,\omega)\in\Omega_0 \text{ and } (\xi,\eta)\in\Omega_{V_1} \\ \frac{1}{2}\nu^2 + \frac{g_m}{2\gamma_I}\Delta k_I^2 + \sum_{k=1}^{r-1}\frac{1}{2}(\frac{1}{\gamma_{ck}}\Delta c_k^2 + \omega_k^2) + \delta_{V_v}^2 \\ \qquad \text{for } (\nu,\omega)\in\Omega_1 \text{ and } (\xi,\eta)\in\Omega_{V_0} \\ \qquad V_r \\ \qquad \text{for } (\nu,\omega)\in\Omega_1 \text{ and } (\xi,\eta)\in\Omega_{V_1} \end{cases} \tag{5.73}$$

where

$$V_v = \mu_0\xi^T P_\xi \xi + \mu_1 W(\eta),$$
$$\Delta k_I = k_I - k^*,\ \Delta c_k = c_k - c_k^*$$

and $\delta_\nu$ is a positive constant given in (5.26). Further $\delta_{V_v}$ is a positive constant such that

$$\delta_{V_v}^2 \ge \bar{R}/\bar{\alpha}_v \tag{5.74}$$

with positive constants $\bar{\alpha}_v$ and $\bar{R}$ that are determined later, and $\Omega_{V_0}$ and $\Omega_{V_1}$ are defined by

$$\Omega_{V_0} = \{\xi\in R^{r-1},\ \eta\in R^{n-r} |\ V_v \le \delta_{V_v}^2\}$$
$$\Omega_{V_1} = \{\xi\in R^{r-1},\ \eta\in R^{n-r} |\ V_v > \delta_{V_v}^2\}$$

In this function $V$, we consider the ideal feedback gains $k^*$ and $c_i^*$ such that the following inequality for $K^*$ given in (5.71) is satisfied

$$K^* \ge (R_{\Omega_1} + \gamma_\delta)/\delta_\nu^2 \tag{5.75}$$

where $\gamma_\delta$ is any positive constant, $R_{\Omega_1} = \max[R_T, \bar{R}']$ and

$$\bar{R}' = \frac{d_{11}^2}{4g_m\gamma_R} + \frac{(d_0L_2 + d_{01} + d_1 + \frac{L_2\delta_{V_v}}{\sqrt{\mu_0\lambda_{min}[P_\xi]}})^2}{4\rho_2} + \sum_{k=1}^{r-1}\frac{3(g_M\delta_{V_v})^2}{4\mu_0\epsilon_k l_k\lambda_{min}[P_\xi]} + \sum_{k=2}^{r}R_k.$$

The time derivative of $V$ given in (5.73) for $(\nu,\omega)\in\Omega_0$ and $(\xi,\eta)\in\Omega_{V_0}$ is given by

$$\dot{V} = 0 \tag{5.76}$$

Further, since we have

$$\begin{aligned}\dot{V} =\dot{V}_v &= \mu_0\xi^T(A_\xi^T P_\xi + P_\xi A_\xi)\xi \\ &+ \mu_0\big[a_\xi[\nu + y^*] + B_\xi\eta + F\big]^T P_\xi\xi \\ &+ \mu_0\xi^T P_\xi\big[a_\xi[\nu + y^*] + B_\xi\eta + F\big] \\ &+ \mu_1\frac{\partial W}{\partial\eta}\big[q(\nu + y^*, \eta) + f_\eta(\nu + y^*, \xi, \eta)\big]\end{aligned} \tag{5.77}$$

the time derivative of $V$ for $(\nu, \omega) \in \Omega_0$ and $(\xi, \eta) \in \Omega_{V_1}$ can be evaluated by considering fact that $|\nu| \leq \delta_\nu$ as follows:

$$\begin{aligned} \dot{V} &= \dot{V}_v \leq -\bar{\alpha}_v V_v + \bar{R} \\ \bar{\alpha}_v &= \min\left[\frac{2\lambda_{min}[Q_\xi]}{3\lambda_{max}[P_\xi]}, \frac{\kappa_1}{4\kappa_3}\right] \end{aligned} \tag{5.78}$$

where

$$\begin{aligned} \bar{R} =& \frac{[\mu_0 \|P_\xi\| \{a_{\xi M}(\delta_\nu + d_0) + p_1(\phi_{1M}\delta_\nu + \phi_{2M}) + p_0\}]^2}{\rho_6} \\ &+ \frac{[\mu_1 \kappa_2 \{L_1(\delta_\nu + d_0) + d_{1\eta}(\psi_{1\eta M}\delta_\nu + \psi_{2\eta M}) + d_{0\eta}\}]^2}{4\rho_7}. \end{aligned}$$

$\phi_{1M}$ and $\psi_{1\eta M}$ are positive constants that satisfy $|\phi_1(\nu, y^*)| \leq \phi_{1M}$ and $|\psi_{1\eta}(\nu, y^*)| \leq \psi_{1\eta M}$. Such positive constants surely exist from the relations $|\nu| \leq \delta_\nu$ and $|y^*(t)| \leq d_0$. Since $V_v > \delta^2_{V_v}$ for $(\nu, \omega) \in \Omega_0$ and $(\xi, \eta) \in \Omega_{V_1}$, the time derivative of $V$ can be evaluated from (5.74) by

$$\dot{V} \leq 0 \tag{5.79}$$

In the case where $(\nu, \omega) \in \Omega_1$ and $(\xi, \eta) \in \Omega_{V_0}$, the time derivative of $V$ yields that

$$\begin{aligned} \dot{V} =& \nu[a(\nu + y^*, \xi) - g_{1,r}[k\nu + u_R] + g_{1,r}[u_{f_1} - \alpha_1] \\ &+ f_1(\nu + y^*) - \dot{y}^*] + g_m[k_I - k^*]\nu^2 + \sum_{k=1}^{r-1}(\Delta c_k \omega_k^2 + \omega_k \dot{\omega}_k). \end{aligned} \tag{5.80}$$

Thus, considering that $V_v \leq \delta^2_{V_v}$, we have

$$\dot{V} \leq -K^*(\nu^2 + \sum_{k=1}^{r-1}\omega_k^2) + \bar{R}'. \tag{5.81}$$

Since $K^*$ satisfies the condition (5.75), the time derivative of $V$ can be evaluated by

$$\dot{V} \leq -\gamma_\delta < 0. \tag{5.82}$$

Furthermore, the time derivative of $V$ can be evaluated from (5.70) and (5.75) as follows:

$$\dot{V} \leq -\gamma_\delta < 0 \tag{5.83}$$

for $(\nu, \omega) \in \Omega_1$ and $(\xi, \eta) \in \Omega_{V_1}$.

Thus the time derivative of $V$ can be evaluated as $\dot{V} \leq 0$ for all $t$ so we can conclude that all the signals in the control system are bounded.

Next, we analyze the convergence of the tracking error $\nu$. Suppose that there exists a time $t_0$ such that $\nu^2 + \|\omega\|^2 > \delta_\nu^2$ for all $t \geq t_0$. This implies that $V \geq \frac{1}{2}\delta_\nu^2 + \delta^2_{V_v}$, $\forall t \geq t_0$. Since $\dot{V} \leq -\gamma_\delta < 0$ for $(\nu, \omega) \in \Omega_1$ from (5.82) and (5.83), we have

$$V(t) = V(t_0) + \int_{t_0}^{t} \dot{V}(\tau)d\tau \leq V(t_0) - \gamma_\delta(t - t_0). \tag{5.84}$$

The inequality (5.84) contradicts the fact that $V \geq \frac{1}{2}\delta_\nu^2 + \delta^2_{V_v}$, $\forall t \geq t_0$, because the right hand side of (5.84) will eventually become negative $t \to \infty$. This means that the interval

$(t_0, t_1)$ in which $(\nu, \omega) \in \Omega_1$ is finite. Let $(t_2, t_3)$ be a finite interval during $(\nu, \omega) \in \Omega_0$ and $(t_3, t_4)$ be a finite interval during $(\nu, \omega) \in \Omega_1$. Since $\dot{V} \leq 0$ from (5.76) and (5.79) for $(\nu, \omega) \in \Omega_0$ and $\dot{V} \leq -\gamma_\delta < 0$ from (5.82) and (5.83) for $(\nu, \omega) \in \Omega_1$, it follows that for the interval $(t_2, t_3)$ during $(\nu, \omega) \in \Omega_0$

$$V(t_3) \leq V(t_2) \tag{5.85}$$

and that for the interval $(t_3, t_4)$ during $(\nu, \omega) \in \Omega_1$

$$V(t_4) < V(t_3). \tag{5.86}$$

Thus the positive function $V$ decreases a finite amount time $(\nu, \omega)$ leaves $\Omega_0$ and re-enters into $\Omega_0$ in finite time and $V$ does not increase during that $(\nu, \omega) \in \Omega_0$. Finally we conclude that there exists a finite time $T > 0$ such that $V$ converges to a constant for all $t \geq T$, *i.e.*, $(\nu, \omega) \in \Omega_0$ for all $t \geq T$. This fact gives that

$$\lim_{t \to \infty} |\nu| \leq \delta_\nu. \tag{5.87}$$

The control objective (5.7) is attained by setting the positive constant $\delta_\nu$ as $\delta_\nu = \delta$ after all. □

**Remark 5.1.** *Since the proposed method is designed an output feedback controller by considering OFEP property of the controlled systems, the stability of the control system is not lost even if adaptive feedback gains $k_I$ and $c_i$ are estimated as too large. Further, the proposed method can be applied for the system with disturbances, which satisfy the assumption* 5.1. *In the case where there is noise in the output signal, the stability of the control system is ensured if the noise satisfies the condition* (5.8) *for the reference signal $y^*$. However, we have to pay attention to apply the controller through the proposed method for practical systems, because the effects of noise appear directly in the output signal since the proposed controller is designed for noised output error : $\bar{\nu} = y + n - y^*$, ($n$ : noise) and controls to make the output error with noise converge to the objective small bound.*

## 5.5 Numerical Simulations

### 5.5.1 Example 1: 5th Order Nonlinear System

Consider the following SISO nonlinear system:

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2) + g_1(t)x_2 \\
\dot{x}_2 &= f_2(x_1, t) + g_2(t)x_3 \\
\dot{x}_3 &= f_3(x_1) + g_3(t)u + b(t)^T\eta \\
\dot{\eta} &= f_\eta(x_1, \eta, t) + q(x_1, \eta) \\
y &= x_1
\end{aligned} \tag{5.88}$$

where

$$\begin{aligned}
&f_1 = x_1^2 \sin 0.1x_2,\ f_2 = x_1^2 \cos 0.5t,\ f_3 = x_1^2 \\
&g_1 = 2(1/(t+2) + 1),\ g_2 = 5(1 + \exp(-t)), \\
&g_3 = 0.7(\cos 0.02t + 1.2),\ b = \begin{bmatrix} 1 \\ \sin t + 1.5 \end{bmatrix}, \\
&f_\eta = \begin{bmatrix} 0.03x_1^2 \sin 4\eta_2^2 \\ 0.2x_1^2 \sin 3t \end{bmatrix},\ q = \begin{bmatrix} x_1 - \eta_1 \\ -\eta_2 \end{bmatrix}
\end{aligned}$$

The controlled system given in (5.88) has a relative degree 3 and the nominal part of (5.88) is exponential minimum-phase. In this simulation, it is supposed that we have prior information about the controlled system such as the nonlinearity $q(y,\eta)$ is Lipschitz in $(y,\eta)$ and nonlinear functions $f_i (i = 1, 2, 3)$ and $f_\eta$ are not Lipschitz but can be evaluated by

$$\begin{aligned} |f_1| \le & d_{11}|\psi_1|, \ |f_2| \le d_{12}|\psi_2|, \ |f_3| \le d_{13}|\psi_3| \\ & \psi_1 = \psi_2 = \psi_3 = y^2 \end{aligned} \tag{5.89}$$

and

$$\|f_\eta\| \le d_{1\eta}|\psi_\eta|, \ \psi_\eta = y^2. \tag{5.90}$$

For the controlled system (5.88), we introduce second order virtual filter given in (5.9) and apply the variable transformation (5.10), the system (5.88) is represented by

$$\begin{aligned} \dot{y} &= a(y,\xi,t) + g_{1,3}(t)u_{f_1} + f_1(y) \\ \dot{\xi} &= A_\xi \xi + a_\xi(t)y + B_\xi(t)\eta + F(y,t) \\ \dot{\eta} &= q(y,\eta) + f_\eta(y,t) \\ \dot{u}_{f_1} &= -\lambda_1 u_{f_1} + u_{f_2} \\ \dot{u}_{f_2} &= -\lambda_2 u_{f_2} + u \end{aligned} \tag{5.91}$$

where

$$F = \begin{bmatrix} F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} \bar{g}_2 f_2 - \chi_{2,2}\bar{g}_{1,3} f_1 \\ \bar{g}_3 f_3 - \chi_{3,1}\bar{g}_{2,3} f_2 - \chi_{3,2}\bar{g}_{1,3} f_1 \end{bmatrix} \tag{5.92}$$

Since $g_1, g_2, g_3$ are bounded for all $t > 0$, $F$ can be evaluated by

$$\|F\| \le |F_2| + |F_3| \le p_1|\phi|, \ \phi = y^2 \tag{5.93}$$

with $\phi_1 = y$ and a positive constant $p_1$.

Considering above evaluations, we design a robust adaptive controller through the proposed procedure as follows:

$$u = -c_2\omega_2 + \lambda_2 u_{f_2} + \frac{\partial \alpha_2}{\partial k_I}\gamma_I \nu^2 - \epsilon_2 \Psi_2 \omega_2 + \frac{\partial \alpha_2}{\partial u_{f_1}}[-\lambda_k u_{f_1} + u_{f_2}] \tag{5.94}$$

$$\begin{aligned} \dot{c}_2 &= \gamma_{c2} D(\nu,\omega)\omega_2^2 \\ \Psi_2 &= (\psi_1^2 + u_{f_1}^2 + l_2)\left(\frac{\partial \alpha_2}{\partial y}\right)^2 + \left(\frac{\partial \alpha_2}{\partial y^*}\right)^2 \\ \alpha_2 &= -c_1\omega_1 + \lambda_1 u_{f_1} + \frac{\partial \alpha_1}{\partial k_I}\gamma_I \nu^2 - \epsilon_1 \Psi_1 \omega_1 \\ \dot{c}_1 &= \gamma_{c1} D(\nu,\omega)\omega_1^2 \\ \Psi_1 &= (\psi_1^2 + u_{f_1}^2 + l_1)\left(\frac{\partial \alpha_1}{\partial y}\right)^2 + \left(\frac{\partial \alpha_1}{\partial y^*}\right)^2 \end{aligned} \tag{5.95}$$

$$\begin{aligned} \alpha_1 &= -[k\nu + u_R] \\ k &= k_I + k_P, \ \dot{k}_I = \gamma_I D(\nu,\omega)\nu^2 \\ k_P &= \gamma_p[\phi_1^4 + \psi_{1\eta}^4]\nu^2, \ u_R = \gamma_R \psi_1^2 \nu \end{aligned} \tag{5.96}$$

where

$$\begin{aligned} &\psi_1 = y^2, \psi_{1\eta} = \phi_1 = y, \ \nu = y - y^*, \\ &\omega_1 = u_{f_1} - \alpha_1, \ \omega_2 = u_{f_2} - \alpha_2. \end{aligned}$$

In this simulation the reference signal is give as $y^*(t) = \sin 2t$ and controller parameters are set as follows:

$$\gamma_I = 60,\ \gamma_p = \gamma_R = 0.01,\ \gamma_{c_1} = \gamma_{c_2} = 1,\ \epsilon_1 = \epsilon_2 = 1,\ \delta_\nu = 0.05$$
$$\lambda_1 = \lambda_2 = 3,\ l_1 = l_2 = 3,\ k_I(0) = 1,\ c_1(0) = c_2(0) = 0.$$

Fig.5.2 to 5.8 show the simulation results. However the control input is rather large during the transient period of parameters adjusting, the output error converges into the objective bound $\delta = 0.05$ after the parameter estimations finished.

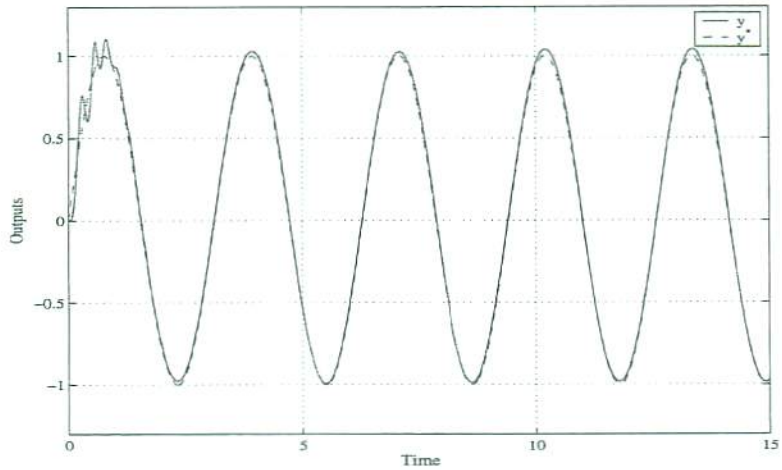


Figure 5.2: System output and reference signal: $y, y^*$

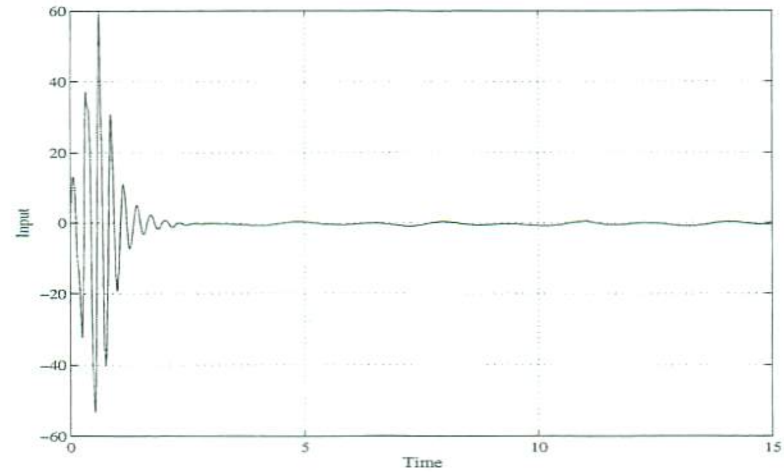


Figure 5.3: Control input: $u$

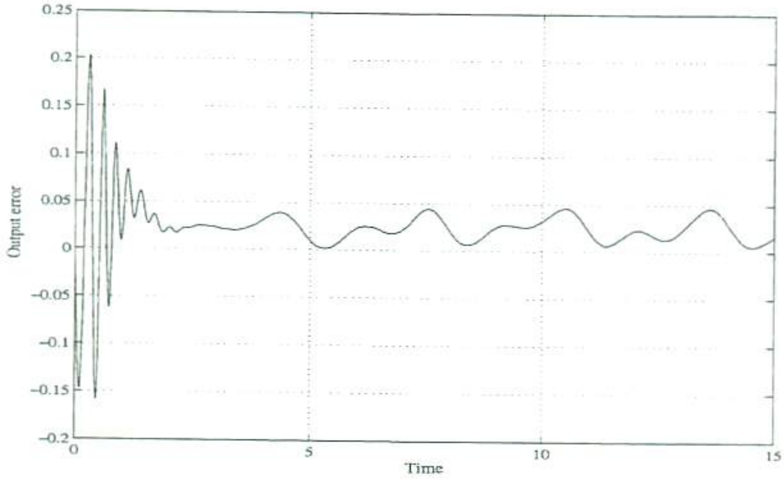


Figure 5.4: Output error: $\nu$

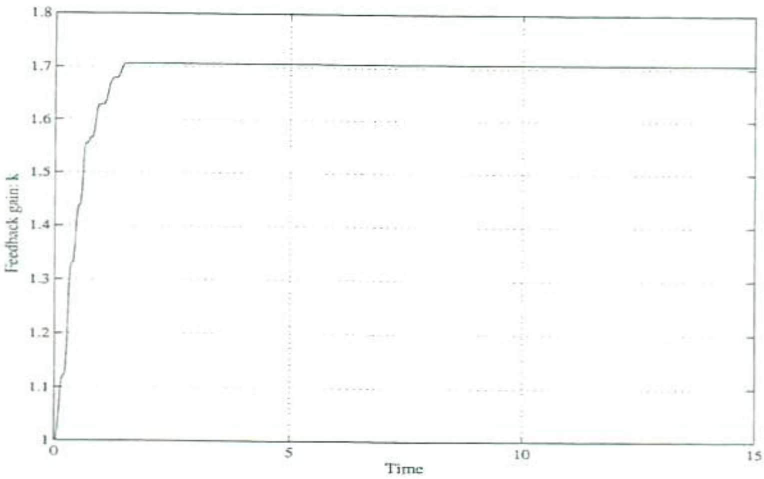


Figure 5.5: Adaptive feedback gain: $k = k_I + k_p$

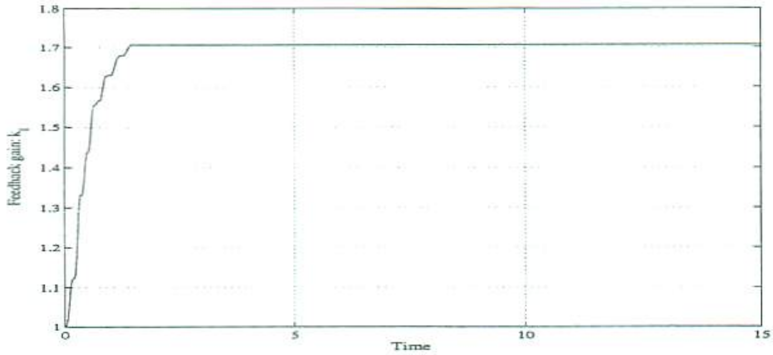


Figure 5.6: Adaptive feedback gain: $k_I$

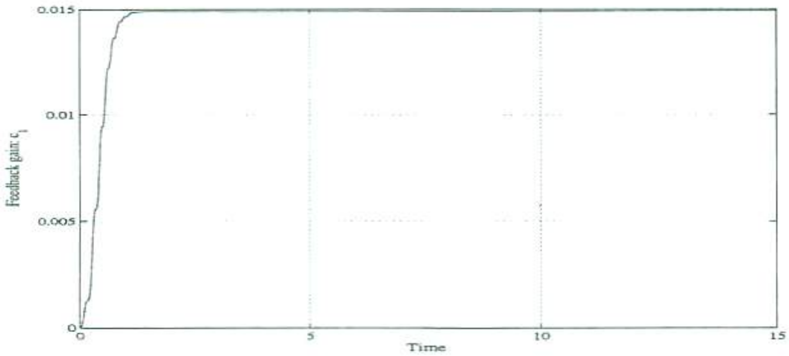


Figure 5.7: Adaptive feedback gain: $c_1$

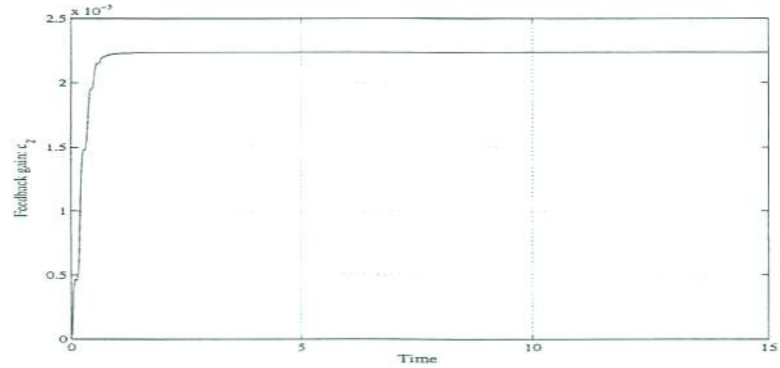


Figure 5.8: Adaptive feedback gain: $c_2$

### 5.5.2 Example 2: DC Motor

#### DC Motor Model

Consider the following DC motor model:

$$\begin{aligned}
&\frac{d}{dt}(L(t)i(t)) + R(t)i(t) + K_b w(t) = v(t) \\
&J\frac{dw}{dt} + T(t) + F(w) = K_t i(t)
\end{aligned} \tag{5.97}$$

where $v(t)$ is the armature voltage, $i(t)$ is the armature current, $w(t)$ is the rotational velocity of the motor. $R(t)$ and $L(t)$ are the resistance of armature winding and the self-inductance, respectively. $J$ is the rotor inertia, $T(t)$ is the load torque, $K_b$ and $K_t$ are the back-emf parameter and the torque motor parameter, respectively. $F(w)$ is the friction given in the following form[112]

$$\begin{aligned}
&F(w) = F_0(\omega) + \theta w \\
&F_0(w) = F_c\mathrm{sgn}(w) + (F_s - F_c)\exp\left\{-\left(\frac{w}{v_s}\right)^2\right\}\mathrm{sgn}(w)
\end{aligned} \tag{5.98}$$

Here we consider the armature voltage $v$ as the control input $u$, the rotational velocity of the motor $w$ as the output $y$ and $x = [x_1, x_2]^T = [w, i]^T$, the DC motor model (5.97) is represented by

$$\begin{aligned}
\dot{x}_1 &= -\frac{F(x_1)}{J} + \frac{K_t}{J}x_2 - \frac{T(t)}{J} \\
\dot{x}_2 &= -\frac{K_b}{L(t)}x_1 - \frac{(R(t) + \frac{dL}{dt})}{L(t)}x_2 + \frac{1}{L(t)}u(t) \\
y &= x_1
\end{aligned} \tag{5.99}$$

The parameters of the DC motor and friction model are shown in Table 5.1.

In order to apply the proposed method, we consider the following nonsingular transformation for the system (5.99):

$$\begin{aligned}
z_1 &= x_1 \\
z_2 &= \frac{J(R(t) + \frac{dL}{dt})}{K_t L(t)}x_1 + x_2
\end{aligned} \tag{5.100}$$

Table 5.1: Motor and friction model parameters

| Symbol | Volues | unites |
|---|---|---|
| J | 0.2 | $kgm^2$ |
| $K_t$ | 0.306 | Nm/A |
| $K_b$ | 3.15 | Vs/rad |
| $\theta$ | 0.4 | Ns/m |
| $v_s$ | 0.001 | rad/s |
| $F_c$ | 1 | Nm |
| $F_s$ | 1.5 | Nm |

so that the system (5.99) can be represented by

$$
\begin{aligned}
\dot{z}_1 &= f_1(z_1,t) + g_1 z_2 \\
\dot{z}_2 &= f_2(z_1,t) + g_2(t)u \\
y &= z_1
\end{aligned}
\tag{5.101}
$$

where

$$
\begin{aligned}
f_1(z_1,t) &= -\frac{1}{J}\left[F_0(z_1) + \theta z_1 + \frac{J(R(t)+\frac{dL}{dt})}{L(t)} z_1\right] - \frac{T(t)}{J} \\
f_2(z_1,t) &= -\frac{(R(t)+\frac{dL}{dt})}{K_t L(t)} F_0(z_1) + \left[\frac{J}{K_t}\cdot\frac{d}{dt}\left\{\frac{(R(t)+\frac{dL}{dt})}{L(t)}\right\}\right. \\
&\quad \left. - \frac{\theta(R(t)+\frac{dL}{dt})}{K_t L(t)} - \frac{K_b}{L(t)}\right] z_1 - \frac{T(t)(R(t)+\frac{dL}{dt})}{K_t L(t)} \\
g_1 &= \frac{K_t}{J}\ ,\ g_2(t) = \frac{1}{L(t)}.
\end{aligned}
$$

**Adaptive Controller Design**

Suppose that uncertain nonlinearities $f_1(z_1,t), f_2(z_1,t)$ can be evaluated by

$$
\begin{aligned}
|f_1(z_1,t)| &\le d_{11}|\psi_1(y)| + d_{01} \\
|f_2(z_1,t)| &\le d_{12}|\psi_2(y)| + d_{02}
\end{aligned}
\tag{5.102}
$$

where $\psi_1(y) = \psi_2(y) = y$ are known functions. Further since the system (5.101) has a relative degree 2, we introduce first order virtual filter:

$$
\dot{u}_{f_1} = -\lambda_1 u_{f_1} + u \tag{5.103}
$$

and design the control input $u$ as follows

$$
\begin{aligned}
u &= -c_1\omega_1 + \lambda_1 u_{f_1} + \frac{\partial \alpha_1}{\partial k_I}\gamma_I \nu^2 - \epsilon_1 \Psi_1 \omega_2 \\
\dot{c}_1 &= \gamma_{c_1} D(\nu,\omega_1)\omega_1^2 \\
\Psi_1 &= \left(\psi_1^2 + u_{f1}^2 + l_1\right)\left(\frac{\partial \alpha_1}{\partial y}\right)^2 + \left(\frac{\partial \alpha_1}{\partial y^*}\right)^2 \\
\alpha_1 &= -[k\nu + u_R] \\
k &= k_I + k_p \\
\dot{k}_I &= \gamma_I D(\nu,\omega_1)\nu^2 \\
k_p &= \gamma_p\left[\phi_1^4 + \psi_{1\eta}^4\right]\nu^2 \\
u_R &= \gamma_R \psi_1 \nu.
\end{aligned}
\tag{5.104}
$$

In this simulation the controller parameters are set as follows:

$$
\begin{aligned}
&\gamma_I = 7.5\times 10^5,\ \gamma_p = \gamma_R = 6.0\times 10^3,\ \gamma_{c_1} = 10,\ \delta_\nu = 5.0\times 10^{-3} \\
&\lambda_1 = 20,\ \epsilon_1 = 1,\ l_1 = 3,\ k_I(0) = 40,\ c_1(0) = 0.
\end{aligned}
$$

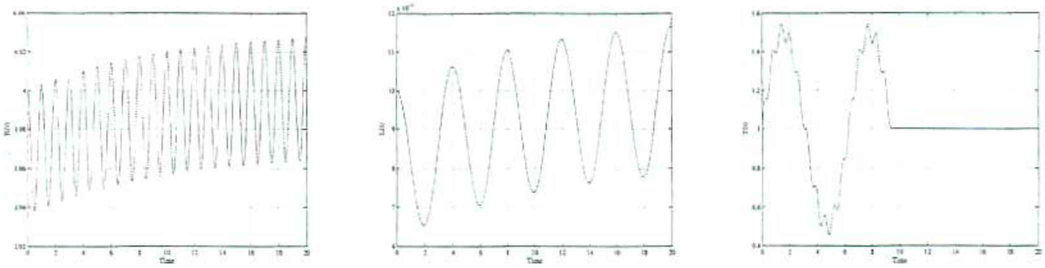

Figure 5.9: Left : $R(t)$ Middle : $L(t)$ Right : $T(t)$

and the reference signal $y^*(t)$ is given by

$$
\begin{aligned}
&y^* = y_1^* + y_2^* \\
&y_1^* = 0.25(1 - e^{-4t}) \ , \ y_2^* = \begin{cases} 0 & (t < 5) \\ 0.05\sin(t-5) + 0.25 & (t \geq 5) \end{cases}
\end{aligned}
\tag{5.105}
$$

Further, we assume that the resistance $R(t)$ and the self-inductance $L(t)$ vary related to temperature in time and we give the load torque $T(t)$. These changes are shown in Fig. 5.9.

Fig. 5.10 to 5.15 show the simulation results. A good control results are obtained in spite of the controlled system having unknown nonlinearities and the time-varying unknown coefficient in control input term.

## 5.6 Conclusion

In this chapter, a design method for robust adaptive controller based on high gain output feedback for uncertain nonlinear systems with a higher order relative degree and unknown time-varying functions in the control input terms has proposed. The proposed method designs the output feedback based controller by introducing a virtual control input filter and applying backstepping procedure in this filter without introducing a state estimator. The effectiveness of the proposed method has confirmed through numerical simulations.

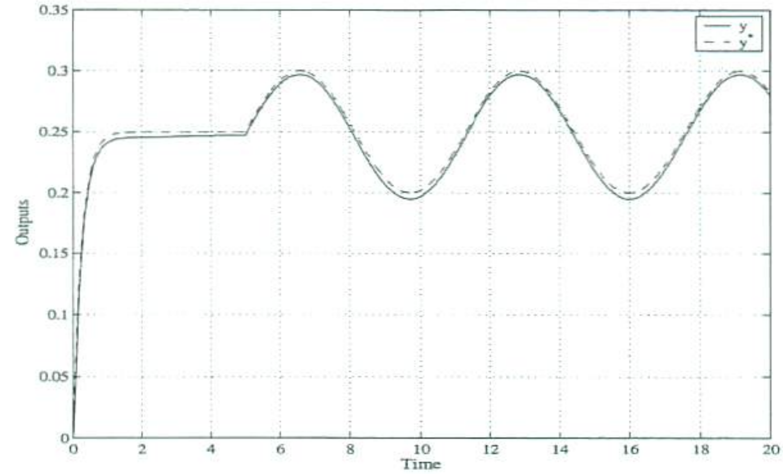


Figure 5.10: System output and reference signal: $y, y^*$

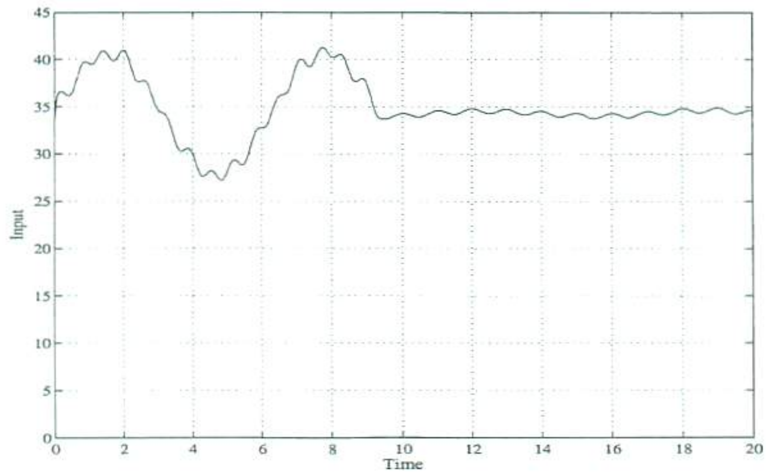


Figure 5.11: Control input: $u$

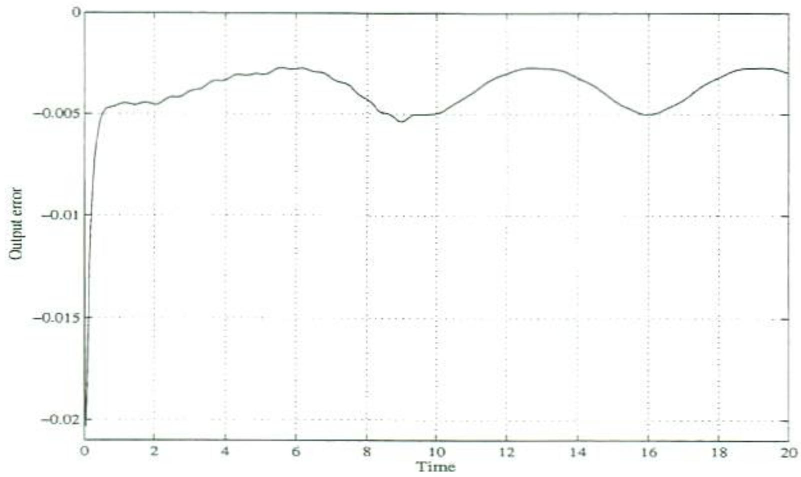


Figure 5.12: Output error: $\nu$

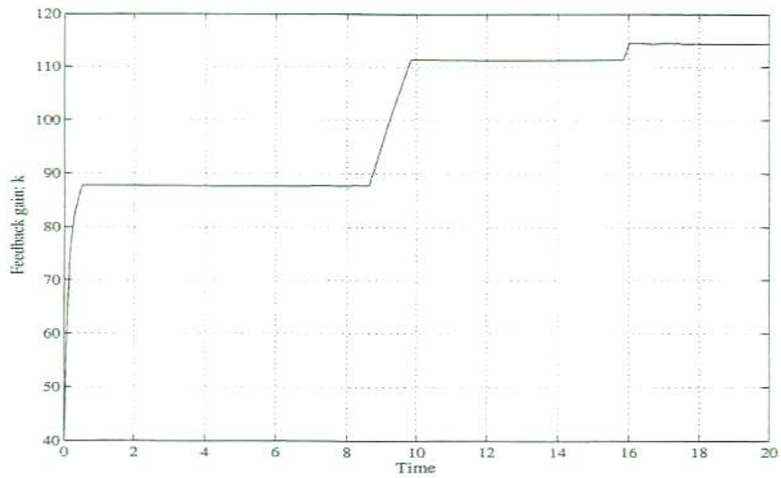


Figure 5.13: Adaptive feedback gain: $k = k_I + k_p$

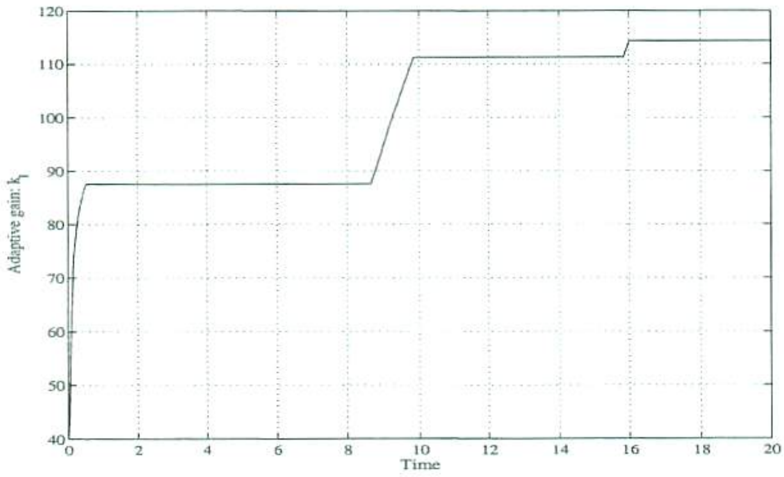


Figure 5.14: Adaptive feedback gain: $k_I$

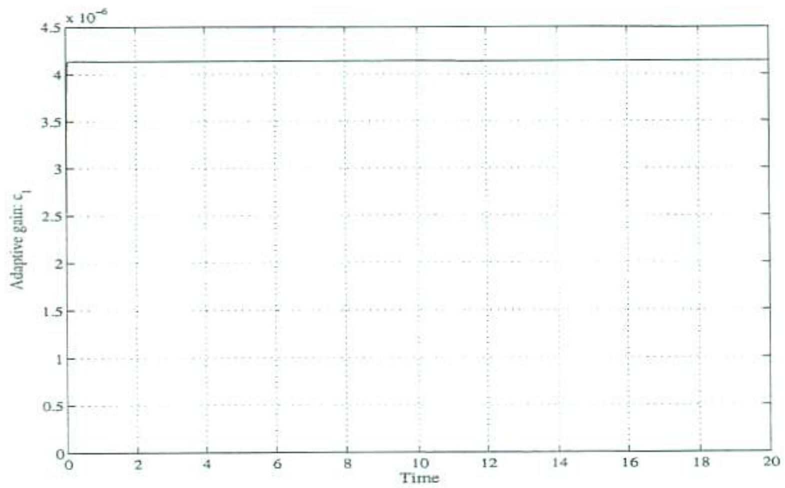


Figure 5.15: Adaptive feedback gain: $c_1$

# Chapter 6

# Design of Adaptive Output Feedback Control System by One-step Backstepping

## 6.1 Introduction

In chapter 5, a robust adaptive control system based on high gain output feedback is designed for nonlinear systems with non-Lipschitz nonlinearities and a higher order relative degree by introducing a virtual control input filter and applying backstepping procedure to the filter. In this method, however, the structure of the control system becomes complex for systems with a higher order relative degree because the recursive design of backstepping depends on the order of the relative degree of the controlled system.

In this chapter, a new design method of an adaptive controller based on high gain output feedback for uncertain nonlinear and linear systems is proposed. Even when the controlled system has a higher order relative degree, the proposed method allows us to design an adaptive controller through backstepping of only one step by introducing a PFC, which creates an augmented virtual control input filter with a relative degree of 1. Since the PFC is put in parallel with the virtual filter, the bias effect from PFC does not appear directly in the output of the controlled system. Therefore, it is possible to show that the tracking error converges to any given bound. Thus, we can design a simple structural robust adaptive controller based on high gain feedback for uncertain nonlinear systems with a higher order relative degree.

## 6.2 Problem Statement

Consider the $n$th order nonlinear system with a relative degree of $r$, $(1 \leq i \leq r-1,\ 2 \leq r \leq n)$:

$$
\begin{aligned}
\dot{x}_i &= f_i(x,t) + g_i(t)x_{i+1} \\
\dot{x}_r &= f_n(x,t) + g_r(t)u + b(t)^T\eta \\
\dot{\eta} &= f_\eta(x,t) + q(y,\eta) \\
y &= x_1.
\end{aligned}
\tag{6.1}
$$

This system is same as the controlled system in chapter 5. Further, we assume that this controlled system (6.1) satisfies the assumptions 5.1 to 5.5.

Under theses assumptions the objective is same as in chapter 5, which is to achieve the goal:

$$\lim_{t\to\infty} |y(t) - y^*(t)| \le \delta. \tag{6.2}$$

## 6.3 Adaptive Controller Design

### 6.3.1 Virtual System

For the system (6.1), we introduce a $r-1$th order stable virtual control input filter:

$$\begin{aligned} \dot{u}_f &= A_{u_f} u_f + b_{u_f} u \\ y_{u_f} &= c_{u_f}^T u_f \end{aligned} \tag{6.3}$$

where $u_f = [u_{f_1}, \cdots, u_{f_{r-1}}]^T$ and

$$A_{u_f} = \begin{bmatrix} 0 & I_{r-2,r-2} \\ -\beta_1 & \cdots - \beta_{r-1} \end{bmatrix}, \; b_{u_f} = \begin{bmatrix} 0 \\ b_u \end{bmatrix}, \; c_{u_f}^T = [1, 0, \cdots, 0], \; b_u > 0$$

Then concerning the virtual system, which is obtained by considering $u_{f_1}$ given in (6.3) as the control input, the following proposition is given.

**Proposition 6.1.** *For the system* (6.1) *with a relative degree $r \le n$, consider the following variable transformation using the filtered signals $u_{f_i}$ given in* (6.3) :

$$\xi_k = b_u \bar{g}_{k,r} x_k - u_{f_{k-1}} - \sum_{d=1}^{k-1} \chi_{k,d} x_{k-d} \tag{6.4}$$

*where*

$$g_{m,n} = \prod_{i=m}^{n} g_i, \; \bar{g}_{m,n} = 1/g_{m,n}, \; \bar{g}_m = 1/g_m, \; g_0(t) = 1$$

*and*

$$\begin{aligned} \chi_{r,1} &= b_u \bar{g}_{r-1} (\beta_{r-1} \bar{g}_r + \dot{\bar{g}}_r) \\ \chi_{r,k} &= \bar{g}_{r-k} \Big( - \sum_{d=1}^{k-1} \beta_{r+d-k} \chi_{r+d-k+1,d} - \dot{\chi}_{r,k-1} + b_u \beta_{r-k} \bar{g}_{r-k+1,r} \Big), \\ & \qquad\qquad\qquad\qquad (2 \le k \le r-1) \\ \chi_{r,r} &= - \sum_{d=1}^{r-1} \beta_d \chi_{d+1,d} - \dot{\chi}_{r,r-1} \\ \chi_{k,1} &= \bar{g}_{k-1} (\chi_{k+1,1} + b_u \dot{\bar{g}}_{k,r}), \; (2 \le k \le r-1) \\ \chi_{k,d+1} &= \bar{g}_{k-d-1} (\chi_{k+1,d+1} - \dot{\chi}_{k,d}), \; (2 \le d \le k-1). \end{aligned}$$

*Then the system* (6.1) *can be expressed by the form:*

$$\begin{aligned} \dot{y} &= a(y, \xi, t) + g'_{1,r}(t) u_{f_1} + f_1(y, \xi, \eta, t) \\ \dot{\xi} &= A_{u_f} \xi + a_\xi(t) y + B_\xi(t) \eta + F(y, \xi, \eta, t) \\ \dot{\eta} &= q(y, \eta) + f_\eta(y, \xi, \eta, t) \end{aligned} \tag{6.5}$$

*where*

$$a(y,\xi,t) = g'_{1,r}(\chi_{2,1}y + \xi_2), \ g'_{1,r} = \frac{g_{1,r}}{b_u}$$

$$a_\xi(t) = \begin{bmatrix} \chi_{2.2} \\ \vdots \\ \chi_{r,r} \end{bmatrix}, \ B_\xi(t) = \begin{bmatrix} 0 \\ b_u \bar{g}_r b^T \end{bmatrix}$$

$$F(y,\xi,\eta,t) = [f_{\xi_2}, \cdots, f_{\xi_k}, \cdots, f_{\xi_r}]^T$$

$$f_{\xi_k} = b_u \bar{g}_{k,r} f_k - \sum_{d=1}^{k-1} \chi_{k,d} f_{k-d}, \ (2 \le k \le r).$$

*Proof.* Since it follows from (6.4) that

$$\xi_2 = b_u \bar{g}_{2,r} x_2 - u_{f_1} - \chi_{2.1} x_1$$

we have

$$\begin{aligned} \dot{y} =& f_1 + g_1 \frac{g_{2,r}}{b_u}(\xi_2 + u_{f_1} + \chi_{2.1} x_1) \\ =& a(y,\xi) + g'_{1,r} u_{f_1} + f_1. \end{aligned} \tag{6.6}$$

Further for $k = 2, \cdots, r-1$, since it follows from (6.4) that

$$\xi_k = b_u \bar{g}_{k,r} x_k - u_{f_{k-1}} - \sum_{d=1}^{k-1} \chi_{k,d} x_{k-d}$$

the time derivative of $\xi_k$ is obtained by

$$\begin{aligned} \dot{\xi}_k =& b_u \dot{\bar{g}}_{k,r} x_k + b_u \bar{g}_{k,r}(f_k + g_k x_{k+1}) - u_{f_k} - \sum_{d=1}^{k-1} \dot{\chi}_{k,d} x_{k-d} \\ & - \sum_{d=1}^{k-1} \chi_{k,d}(f_{k-d} + g_{k-d} x_{k-d+1}). \end{aligned}$$

Here we have from (6.4)

$$\xi_{k+1} = b_u \bar{g}_{k+1,r} x_{k+1} - u_{f_k} - \sum_{d=1}^{k} \chi_{k+1,d} x_{k-d+1}$$

the time derivative of $\xi_k$ is expressed by

$$\dot{\xi}_k = (\xi_{k+1} + \sum_{d=1}^{k} \chi_{k+1,d} x_{k-d+1}) + b_u \dot{\bar{g}}_{k,r} x_k - \sum_{d=1}^{k-1} \dot{\chi}_{k,d} x_{k-d} - \sum_{d=1}^{k-1} \chi_{k,d} g_{k-d} x_{k-d+1} + f_{\xi_k}.$$

Additionally, we have

$$\sum_{d=1}^{k} \chi_{k+1,d} x_{k-d+1} = \chi_{k+1,1} x_k + \chi_{k+1,k} x_1 + \sum_{d=1}^{k-2} \chi_{k+1,d+1} x_{k-d}$$

$$\sum_{d=1}^{k-1} \dot{\chi}_{k,d} x_{k-d} = \dot{\chi}_{k,k-1} x_1 + \sum_{d=1}^{k-2} \dot{\chi}_{k,d} x_{k-d}$$

and

$$\sum_{d=1}^{k-1} \chi_{k,d} g_{k-d} x_{k-d+1} = \chi_{k,1} g_{k-1} x_k + \sum_{d=1}^{k-2} \chi_{k,d+1} g_{k-d-1} x_{k-d}$$

the time derivative of $\xi_k$ is represented by

$$\begin{aligned}\dot{\xi}_k =& \xi_{k+1} + (\chi_{k+1,k} - \dot{\chi}_{k,k-1}) x_1 + f_{\xi_k} \\ & - \chi_{k,1} g_{k-1} x_k + \chi_{k+1,1} x_k + b_u \dot{\bar{g}}_{k,r} x_k \\ & - \sum_{d=1}^{k-2} \chi_{k,d+1} g_{k-d-1} x_{k-d} + \sum_{d=1}^{k-2} \chi_{k+1,d+1} x_{k-d} - \sum_{d=1}^{k-2} \dot{\chi}_{k,d} x_{k-d}.\end{aligned}$$

Considering the structures of $\chi_{k,1}$ and $\chi_{k,d+1}$, we have eventually

$$\dot{\xi}_k = \xi_{k+1} + \chi_{k,k} y + f_{\xi_k} \tag{6.7}$$

As for $\xi_r$ given in (6.4), since

$$\xi_r = b_u \bar{g}_r x_r - u_{f_{r-1}} - \sum_{d=1}^{r-1} \chi_{r,d} x_{r-d}$$

the time derivative of $\xi_r$ is obtained by

$$\begin{aligned}\dot{\xi}_r =& b_u \dot{\bar{g}}_r x_r + b_u \bar{g}_r (f_r + g_r u + b^T \eta) - (-\sum_{d=1}^{r-1} \beta_d u_{f_d}) \\ & - \sum_{d=1}^{r-1} \dot{\chi}_{r,d} x_{r-d} - \sum_{d=1}^{r-1} (f_{r-d} + g_{r-d} x_{r-d+1}).\end{aligned}$$

Here considering the facts that we have

$$\begin{aligned} u_{f_1} &= -\xi_2 + b_u \bar{g}_{2,r} x_2 - \chi_{2,1} x_1 \\ u_{f_2} &= -\xi_3 + b_u \bar{g}_{3,r} x_3 - \sum_{d=1}^{2} \chi_{3,d} x_{3-d} \\ & \vdots \\ u_{f_{r-1}} &= -\xi_r + b_u \bar{g}_r x_r - \sum_{d=1}^{r-1} \chi_{r,d} x_{r-d} \end{aligned}$$

from (6.4) and

$$\begin{aligned} \sum_{d=1}^{r-1} \dot{\chi}_{r,d} x_{r-d} &= \dot{\chi}_{r,r-1} x_1 + \sum_{d=1}^{r-2} \dot{\chi}_{r,d} x_{r-d} \\ \sum_{d=1}^{r-1} \dot{\chi}_{r,d} g_{r-d} x_{r-d+1} &= \chi_{r,1} g_{r-1} x_r + \sum_{d=1}^{r-2} \dot{\chi}_{r,d+1} g_{r-d-1} x_{r-d} \end{aligned}$$

the time derivative of $\xi_r$ is expressed by

$$\begin{aligned}
\dot{\xi}_r =& \beta_1(-\xi_2 + b_u\bar{g}_{2,r}x_2 - \chi_{2,1}x_1) + \beta_2(-\xi_3 + b_u\bar{g}_{3,r}x_3 - \sum_{d=1}^{2}\chi_{3,d}x_{3-d}) \\
&+ \cdots + \beta_{r-1}(-\xi_r + b_u\bar{g}_r x_r - \sum_{d=1}^{r-1}\chi_{r,d}x_{r-d}) + b_u\dot{\bar{g}}_r x_r + b_u\bar{g}_r b^T\eta + f_{\xi r} \\
&- \dot{\chi}_{r,r-1}x_1 - \chi_{r,1}r_{r-1}x_r - \sum_{d=1}^{r-2}(\dot{\chi}_{r,d} + \chi_{r,d+1}g_{r-d-1})x_{r-d} \\
=& -\sum_{d=1}^{r-1}\beta_d\xi_{d+1} + b_u\bar{g}_r b^T\eta + f_{\xi r} \\
&+ (-d_1\chi_{2,1} - \beta_2\chi_{3,2} - \cdots - \beta_{r-1}\chi_{r,r-1} - \dot{\chi}_{r,r-1})x_1 \\
&+ (-\beta_2\chi_{3,1} - \beta_3\chi_{4,2} - \cdots - \beta_{r-1}\chi_{r,r-2} - \dot{\chi}_{r,r-2} + b_u\beta_1\bar{g}_{2,r})x_2 \\
&+ \cdots + (-\beta_{r-1}\chi_{r,1} - \dot{\chi}_{r,1} + b_u\beta_{r-2}\bar{g}_{r-1,r})x_{r-1} \\
&+ b_u\beta_{r-1}\bar{g}_{r,r}x_r + b_u\dot{\bar{g}}_{r,r}x_r - \chi_{r-1}g_{r-1}x_r \\
&- (\chi_{r,r-1}g_1x_2 + \chi_{r,r-2}g_2x_3 + \cdots + \chi_{r,2}g_{r-2}x_{r-1}).
\end{aligned}$$

Further considering the structures of $\chi_{r,1}, \chi_{r,k}$ and $\chi_{r,r}$, finally we have

$$\dot{\xi}_r = -\sum_{d=1}^{r-1}\beta_d\xi_{d+1} + \chi_{r,r}y + b_u\bar{g}_r b^T\eta + f_{\xi r}. \tag{6.8}$$

Thus we get the desired results. □

For the obtained virtual system (6.5), it is easy to confirm from assumption 5.2 that $a(y, \xi, t)$ is bounded for all $t$ and Lipschitz with respect to $y$ and $\xi$ so that there exists a positive constant $L_2$ such that

$$|a(y_1, \xi_1) - a(y_2, \xi_2)| \leq L_2(|y_1 - y_2| + \|\xi_1 - \xi_2\|). \tag{6.9}$$

The uncertain vector function $F(y, \xi, \eta, t)$ can be valuated from assumption 5.1 by

$$\|F(y, \xi, \eta, t)\| \leq p_1|\phi(y)| + p_0 \tag{6.10}$$

with unknown positive constants $p_1$ and $p_0$ and a known function $\phi(y)$ which has the following property for any variables $y_1$ and $y_2$

$$|\phi(y_1 + y_2)| \leq |\phi_1(y_1, y_2)||y_1| + |\phi_2(y_2)| \tag{6.11}$$

with a known smooth function $\phi_1(y_1, y_2)$ and a function $\phi_2(y_2)$ which is bounded for all bounded $y_2 \in R$. Further since $A_\xi$ is a stable matrix, there exists a positive symmetric matrix $P_\xi$ for any positive matrix $Q_\xi$ such as

$$P_\xi A_\xi + A_\xi^T P_\xi = -Q_\xi. \tag{6.12}$$

Moreover, the system (6.1) is exponential minimum-phase from assumption 5.5 so that there exist a positive definite function $W(\eta)$ and positive constants $\kappa_1$ to $\kappa_4$ from converse theorem of Lyapunov[62, 66] such that

$$\begin{aligned}
&\frac{\partial W(\eta)}{\partial \eta}q(0, \eta) \leq -\kappa_1\|\eta(t)\|^2, \quad \left\|\frac{\partial W(\eta)}{\partial \eta}\right\| \leq \kappa_2\|\eta(t)\| \\
&\kappa_4\|\eta(t)\|^2 \leq W(\eta) \leq \kappa_3\|\eta(t)\|^2
\end{aligned} \tag{6.13}$$

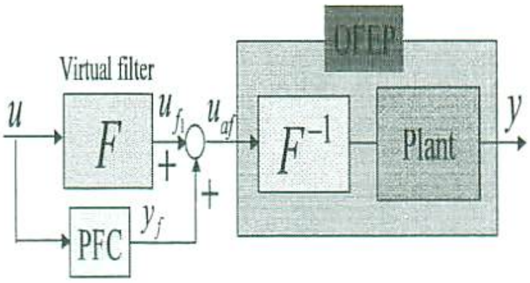


Figure 6.1: Augmented virtual system

Seeing $f_1, \boldsymbol{F}$ and $\boldsymbol{f}_\eta$ as disturbances, the nominal part of the virtual system (6.5) has a relative degree 1 for the filtered signal $u_{f_1}$ and exponential minimum-phase, *i.e.*, the virtual system is OFEP. Therefore the control objective can be attained by designing the actual control input $u$ to make the filtered signal $u_{f_1}$ converge to a high gain adaptive output feedback control input with robustness for disturbances. In chapter 5, we designed such control system by applying backstepping procedure in the virtual filter, but the structure of the controller becomes complex for a system with a higher order relative degree.

In the following, we introduce a PFC in parallel to the virtual filter so that the augmented virtual filter has a relative degree 1 and design a robust adaptive controller through backstepping of only one step in the augmented virtual filter.

### 6.3.2 Augmented Virtual System

Consider a stable PFC with relative degree of 1 and minimum phase:

$$\begin{aligned} \dot{y}_f &= -a_{f_1} y_f + \boldsymbol{a}_{f_2}^T \boldsymbol{\eta}_f + b_a u \\ \dot{\boldsymbol{\eta}}_f &= A_f \boldsymbol{\eta}_f + \boldsymbol{b}_f y_f \end{aligned} \tag{6.14}$$

where $y_f \in R$ is the PFC output and $\boldsymbol{\eta}_f \in R^{n_f-1}$ is the state variables of PFC. $a_f$ is any positive constant and $A_f$ is a stable matrix.

Suppose that the PFC (6.14) is designed such that the augmented virtual filter is ASPR(Almost Strictly Positive Real), that is, the augmented virtual filter has a relative degree 1 and is minimum-phase. Since the augmented virtual filter has a relative degree 1, there exists a nonsingular transformation $[u_{a_f}, \boldsymbol{\eta}_a^T]^T = \Phi_{f_1}[\boldsymbol{u}_f^T, y_f, \boldsymbol{\eta}_f^T]^T$ such that the augmented virtual filter can be rewritten as follows[58]:

$$\begin{aligned} \dot{u}_{a_f} &= a_{a1} u_{af} + \boldsymbol{a}_{a2}^T \boldsymbol{\eta}_a + b_a u \\ \dot{\boldsymbol{\eta}}_a &= A_a \boldsymbol{\eta}_a + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_{a_f} \end{aligned} \tag{6.15}$$

where $u_{a_f} = u_{f_1} + y_f$, $b_a = \boldsymbol{c}_f^T \boldsymbol{b}_f$ and $A_a$ is the system matrix corresponding to the zero dynamics of the augmented virtual filter. Since the augmented virtual filter is minimum-phase, $A_a$ is a stable matrix.

The virtual system (6.5) with the augmented virtual filter output $u_{a_f}$ as the control input can be represented as follows:

$$
\begin{aligned}
\dot{y} &= a(y,\xi,t) + g'_{1,r}(t)(u_{a_f} - y_f) + f_1(y,\xi,\eta,t) \\
\dot{\xi} &= A_{u_f}\xi + a_\xi(t)y + B_\xi(t)\eta + F(y,\xi,\eta,t) \\
\dot{\eta} &= q(y,\eta) + f_\eta(y,\xi,\eta,t).
\end{aligned}
\tag{6.16}
$$

The diagram is shown in Fig.6.1.

### 6.3.3 Adaptive Controller Design through One-step Backstepping

**[Pre-step]** Considering the output tracking error: $\nu(t) = y(t) - y^*(t)$, the augmented virtual system (6.16) can be represented as the following error system:

$$
\begin{aligned}
\dot{\nu} &= a(\nu + y^*, \xi) + g'_{1,r}(u_{a_f} - y_f) + f_1(\nu + y^*, \xi, \eta) - \dot{y}^* \\
\dot{\xi} &= A_\xi\xi + a_\xi[\nu + y^*] + B_\xi\eta + F(\nu + y^*, \xi, \eta) \\
\dot{\eta} &= q(\nu + y^*, \eta) + f_\eta(\nu + y^*, \xi, \eta).
\end{aligned}
\tag{6.17}
$$

For this system, we design a virtual control input $\alpha_1$ based on the robust adaptive high gain feedback proposed in chapter 3 for the augmented virtual filter output $u_{a_f}$ in the error system (6.17) as follows:

$$\alpha_1(t) = -[k(t)\nu(t) + u_R(t)] + \Psi_0(y_f) \tag{6.18}$$

$$k(t) = k_I(t) + k_P(t) \tag{6.19}$$

$$\dot{k}_I(t) = \gamma_I D(\nu)\nu(t)^2, \;\; k_I(0) \geq 0 \tag{6.20}$$

$$k_P(t) = \gamma_p[\phi_1(\nu, y^*)^4 + \psi_{1\eta}(\nu, y^*)^4]\nu(t)^2 \tag{6.21}$$

$$u_R(t) = \gamma_R\psi_1(y)^2\nu(t) \tag{6.22}$$

$$\dot{\Psi}_0(y_f) = D(y_f)[-a_{f_1}\Psi_0 + b_a u] \tag{6.23}$$

where $\gamma_I, \gamma_p$ and $\gamma_R$ are arbitrary positive constants and $D(x)$ is defined such that for any given positive constant $\delta_x$:

$$
D(x) = \begin{cases} 0 & \text{for } x \in \Omega_{x0} \\ 1 & \text{for } x \in \Omega_{x_1} \end{cases}
\tag{6.24}
$$

$$\Omega_{x_0} = \{x \in R \mid |x| \leq \delta_x\}$$

$$\Omega_{x_1} = \{x \in R \mid |x| > \delta_x\}$$

Consider the following positive definite function $V_1(\nu, \xi, \eta, k_I)$ for $\nu \in \Omega_{\nu_1}$

$$V_1(\nu, \xi, \eta, k) = \frac{1}{2}\nu^2 + \mu_0\xi^T P_\xi\xi + \mu_1 W(\eta) + \frac{g'_m}{2\gamma_I}[k_I - k^*]^2 \tag{6.25}$$

where $\mu_0$ and $\mu_1$ are any positive constants, $g'_m = \frac{g_m}{b_u}$ and $k^*$ is an ideal feedback gain for $k_I$ to be determined later. The time derivative of $V_1$ along the trajectories of (6.17) and (6.20) yields that

$$
\begin{aligned}
\dot{V}_1 =& \nu\big[a(\nu + y^*, \xi) - g'_{1,r}[k\nu + u_R + \Psi_0] + g_{1,r}[u_{a_f} - \alpha_1 - y_f] + f_1(\nu + y^*) - \dot{y}^*\big] \\
&+ \mu_0\xi^T(A_\xi^T P_\xi + P_\xi A_\xi)\xi + \mu_0\big[a_\xi[\nu + y^*] + B_\xi\eta + F\big]^T P_\xi\xi \\
&+ \mu_0\xi^T P_\xi\big[a_\xi[\nu + y^*] + B_\xi\eta + F\big] \\
&+ \mu_1\frac{\partial W}{\partial\eta}\big[q(\nu + y^*, \eta) + f_\eta(\nu + y^*, \xi, \eta)\big] + g_m[k_I - k^*]\nu^2
\end{aligned}
\tag{6.26}
$$

By applying the same evaluations of step 1 in chapter 5, we have

$$\begin{aligned}\dot{V}_1 \leq &- [g'_m k^* - v_0]\nu^2 - [\mu_0 \lambda_{min}[Q_\xi] - v_1]\|\xi\|^2 \\ &- [\mu_1 \kappa_1 - v_2]\|\eta\|^2 + g'_{1,r}\nu\omega_1 \\ &- g'_{1,r}[y_f - \Psi_0]\nu + R_1 \end{aligned} \tag{6.27}$$

where $\omega_1 = u_{a_f} - \alpha_1$ and

$$\begin{aligned} v_0 =& L_2 + \frac{(\mu_1\kappa_2 L_1)^2}{4\rho_1} + \rho_2 + \frac{(L_2 + 2\mu_0 a_{\xi M}\|P_\xi\|)^2}{4\rho_3} \\ v_1 =& \rho_3 + \rho_4 + \rho_6 + \frac{(\mu_0 B_{\xi M}\|P_\xi\|)^2}{\rho_8} \\ v_2 =& \rho_1 + \rho_5 + \rho_7 + \rho_8 \\ R_1 =& \frac{d_{11}^2}{4g'_m \gamma_R} + \frac{(d_0 L_2 + d_{01} + d_1)^2}{4\rho_2} \\ &+ \frac{[\mu_0\|P_\xi\|(2d_0 a_{\xi M} + p_1\phi_{2M} + p_0)]^2}{4\rho_6} \\ &+ \frac{[\mu_1\kappa_2(d_0 L_1 + d_{1\eta}\psi_{2\eta M} + d_{0\eta})]^2}{4\rho_7} \\ &+ \frac{1}{64 g'_m \gamma_p}\left[\frac{(\mu_0 p_1\|P_\xi\|)^4}{\rho_4^2} + \frac{(\mu_1\kappa_2 d_{1\eta})^4}{\rho_5^2}\right]. \end{aligned}$$

**[Step 1]** Consider the error system, $\omega_1$-system, between $u_{a_f}$ and $\alpha_1$. $\omega_1$-system is given from (6.15) that

$$\dot{\omega}_1 = a_{a1}u_{a_f} + a_{a2}^T\eta_a + b_a u - \dot{\alpha}_1 \tag{6.28}$$

where the derivative of $\alpha_1$ is given by

$$\begin{aligned} \dot{\alpha}_1 =& \frac{\partial\alpha_1}{\partial y}[a(y,\xi) + g'_{1,r}u_{f_1} + f_1(y,\xi,\eta)] + \frac{\partial\alpha_1}{\partial y^*}\dot{y}^* \\ &+ \frac{\partial\alpha_1}{\partial k_I}\gamma_I D(\nu)\nu^2 + \frac{\partial\alpha_1}{\partial\Psi_0}D(y_f)[-a_{f_1}\Psi_0 + b_a u]. \end{aligned} \tag{6.29}$$

Taking (6.28) and (6.29) into consideration, the actual control input $u$ is designed as follows:

$$u = \begin{cases} -\frac{1}{b_a}[c_1\omega_1 + \epsilon_0(u_{a_f}^2 + \|\eta_a\|^2)\omega_1 + \epsilon_1\Psi_1\omega_1] & \text{if } y_f \in \Omega_{y_{f_0}} \\ -\frac{\omega_1}{b_a y_f}[c_1\omega_1 + \epsilon_0(u_{a_f}^2 + \|\eta_a\|^2)\omega_1 + \epsilon_1\Psi_1\omega_1] & \\ -\frac{1}{b_a}[\gamma_f y_f + \epsilon_2\|\eta_f\|^2 y_f] - \frac{\epsilon_3}{b_a y_f}\Psi_0^2 & \text{if } y_f \in \Omega_{y_{f_1}} \end{cases} \tag{6.30}$$

where $\epsilon_0$ to $\epsilon_3$ are any positive constants, $\Psi_1$ is given with any positive constant $l_1$ as

$$\Psi_1 = (l_1 + u_{f_1}^2 + \psi_1^2)\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + \left(\frac{\partial\alpha_1}{\partial y^*}\right)^2 + \left(\frac{\partial\alpha_1}{\partial k_I}\right)^2\nu^4 \tag{6.31}$$

and $c_1$ and $\gamma_f$ are positive constants such that

$$c_1 > \frac{a_{f_1}^2}{2\epsilon_3},\ \gamma_f \geq \frac{\|a_{f_2}\|^2}{4\epsilon_2\delta_{y_f}^2}. \tag{6.32}$$

### 6.3.4 Boundedness and Convergence Analysis

For the designed control system, the following theorem concerning the boundedness of all the signals in the control system and convergence of the tracking error is given.

**Theorem 6.1.** *Under assumption* 5.1 *to* 5.5 *on the controlled system* (6.1), *all the signals in the resulting closed-loop system with the controller* (6.30) *are bounded. Further, the tracking error* $\nu$ *converges to any given bound*

$$\lim_{t\to\infty} |\nu| \leq \delta. \tag{6.33}$$

*Proof.* Consider the following positive and continuous function $V$:

$$V = \begin{cases} \frac{1}{2}\delta_\nu^2 + V_a, & \nu \in \Omega_{\nu_0} \\ \frac{1}{2}\nu^2 + V_a, & \nu \in \Omega_{\nu_1} \end{cases} \tag{6.34}$$

where

$$V_a = \begin{cases} \frac{1}{2}\delta_{y_f}^2 + \delta_{V_v}^2 + \frac{g_m'}{2\gamma_I}\Delta k_I^2, & y_f \in \Omega_{y_{f_0}}, (\xi, \eta, \omega_1) \in \Omega_{v_0} \\ \frac{1}{2}\delta_{y_f}^2 + V_v + \frac{g_m'}{2\gamma_I}\Delta k_I^2, & y_f \in \Omega_{y_{f_0}}, (\xi, \eta, \omega_1) \in \Omega_{v_1} \\ \frac{1}{2}y_f^2 + \delta_{V_v}^2 + \frac{g_m'}{2\gamma_I}\Delta k_I^2, & y_f \in \Omega_{y_{f_1}}, (\xi, \eta, \omega_1) \in \Omega_{v_0} \\ \frac{1}{2}y_f^2 + V_v + \frac{g_m'}{2\gamma_I}\Delta k_I^2, & y_f \in \Omega_{y_{f_1}}, (\xi, \eta, \omega_1) \in \Omega_{v_1} \end{cases}$$

$$V_v = \mu_0 \xi^T P_\xi \xi + \mu_1 W(\eta) + \frac{1}{2}\omega_1^2, \ \Delta k_I = k_I - k^*$$

and

$$\Omega_{v_0} = \{(\xi, \eta, \omega_1) \in R^{r-1\times n-r\times 1} \mid V_v \leq \delta_{V_v}^2\}$$
$$\Omega_{v_1} = \{(\xi, \eta, \omega_1) \in R^{r-1\times n-r\times 1} \mid V_v > \delta_{V_v}^2\}$$

with a positive constant $\delta_{V_v}$. The positive constant $\delta_{V_v}$ is determined by

$$\delta_{V_v}^2 \geq \bar{R}/\bar{\alpha}_v \tag{6.35}$$

where

$$\bar{\alpha}_v = \min\left[\frac{\lambda_{min}[Q_\xi] - v_1'/\mu_0}{\lambda_{max}[P_\xi]}, \frac{\kappa_1 - v_2/\mu_1}{\kappa_3}, 2(c_1 - \rho_9 - \frac{a_{f_1}^2}{2\epsilon_3})\right]$$

for positive constants $\mu_0, \mu_1$ and $\rho_9$ that satisfy

$$\mu_0\lambda_{min}[Q_\xi] - v_1' > 0, \ \mu_1\kappa_1 - v_2 > 0,$$
$$c_1 - \rho_9 - \frac{a_{f_1}^2}{2\epsilon_3} > 0, \ v_1' = v_1 + \frac{L_2^2}{\epsilon_1 l_1}$$

and

$$\begin{aligned}\bar{R} =& R_1 + \frac{1}{4\epsilon_0}(|a_{a1}|^2 + \|a_{a2}\|^2) + \frac{\|a_{f_2}\|^2}{4\epsilon_2} + \frac{(L_2\delta_\nu)^2}{\epsilon_1 l_1} \\ &+ \frac{1}{4\epsilon_1}(\frac{4(L_2 d_0)^2}{l_1} + \frac{4d_{01}^2}{l_1} + g_M'^2 + d_{11}^2 + d_1^2 + \gamma_I^2) \\ &+ \frac{[\mu_0\|P_\xi\|\{\|a_\xi\|(d_0 + \delta_\nu) + p_1(\phi_{1M}\delta_\nu + \phi_{2M}) + p_0\}]^2}{\rho_6} \\ &+ \frac{[\mu_1\kappa_1\{L_1(d_0 + \delta_\nu) + d_{1\eta}(\psi_{1\eta M}\delta_\nu + \phi_{2\eta M}) + d_{0\eta}\}]^2}{4\rho_7}\end{aligned}$$

where $g'_M = \frac{g_M}{b_u}$, $\phi_{1M}$ and $\psi_{1\eta M}$ are positive constants that satisfy $|\phi_1(\nu, y^*)| \le \phi_{1M}$ and $|\psi_{1\eta}(\nu, y^*)| \le \psi_{1\eta M}$ for $y$ such that $|y| \le \delta_\nu + d_0$.

Further, in the function $V$, we consider an ideal feedback gain $k^*$ such that the following inequality is satisfied

$$-(g'_m k^* - v'_0)\delta_\nu^2 + R_2 \le -\gamma_\nu < 0 \tag{6.36}$$

for

$$v'_0 = v_0 + \frac{g'^2_M}{4\rho_9} + \rho_{10} + \frac{g'^2_M}{4a_{f_1}} + \frac{g'^2_M}{2\epsilon_3} - \frac{L_2^2}{\epsilon_1 l_1}$$
$$R_2 = \bar{R} + \frac{2\delta^2_{V_v} g'^2_M}{4\rho_{10}}$$

where $\gamma_\nu$ and $\rho_{10}$ are any positive constants.

First, we consider the time derivative of $V$ for $\nu \in \Omega_{\nu_0}$.

$(a-1)$ For the case $y_f \in \Omega_{y_{f_0}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_0}$:

Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\delta_\nu^2 + \delta_{V_v}^2 + \frac{g'_m}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}\delta_{y_f}^2 \tag{6.37}$$

we have $\dot{V} = 0$.

$(a-2)$ For the case $y_f \in \Omega_{y_{f_0}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_1}$:

Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\delta_\nu^2 + V_v + \frac{g'_m}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}\delta_{y_f}^2 \tag{6.38}$$

the time derivative of $V$ yields that

$$\begin{aligned}\dot{V} = \dot{V}_v &= \mu_0\xi^T(A_{u_f}^T P_\xi + P_\xi A_{u_f})\xi + 2\mu_0\xi^T P_\xi a_\xi y + 2\mu_0\xi^T P_\xi B_\xi \eta \\ &+ 2\mu_0 \xi P_\xi F + \mu_1 \frac{\partial W}{\partial \eta}(q(y, \eta) + f_\eta) \\ &+ \omega_1\left[a_{a_1}u_{a_f} + a_{a_2}^T\eta_a + b_a u - \frac{\partial \alpha_1}{\partial y}(a(y, \xi) + g'_{1,r}u_{f_1} + f_1) - \frac{\partial \alpha_1}{\partial y^*}\dot{y}^*\right].\end{aligned} \tag{6.39}$$

Considering (6.30) and the fact that $|\nu| \le \delta_\nu$, the time derivative of $V$ can be evaluated by

$$\begin{aligned}\dot{V} = \dot{V}_v &\le -(\mu_0\lambda_{min}[Q] - v'_1)\|\xi\|^2 - (\mu_1\kappa_1 - v_2)\|\eta\|^2 - c_1\omega_1^2 + \bar{R} \\ &\le -\bar{\alpha}_v V_v + \bar{R}.\end{aligned} \tag{6.40}$$

Thus we have $\dot{V} \le 0$ from (6.35).

$(a-3)$ For the case $y_f \in \Omega_{y_{f_1}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_0}$:

Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\delta_\nu^2 + \delta_{V_v}^2 + \frac{g'_m}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}y_f^2 \tag{6.41}$$

the time derivative of $V$ can be evaluated by

$$\begin{aligned}\dot{V} &= y_f(-a_{f_1}y_f + a_{f_2}^T\eta_f + b_a u) \\ &\le -\gamma_f y_f^2 + \frac{\|a_{f_2}\|^2}{4\epsilon_2}\end{aligned} \tag{6.42}$$

therefore we have $\dot{V} \le 0$ from (6.32).
$(a-4)$ For the case $y_f \in \Omega_{y_{f_1}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_1}$:
Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\delta_\nu^2 + V_v + \frac{g_m'}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}y_f^2 \tag{6.43}$$

the time derivative of $V$ yields that

$$\begin{aligned}\dot{V} = \dot{V_v} = &\mu_0\xi^T(A_{u_f}^T P_\xi + P_\xi A_{u_f})\xi + 2\mu_0\xi^T P_\xi a_\xi y + 2\mu_0\xi^T P_\xi B_\xi \eta \\ &+ 2\mu_0 \xi P_\xi F + \mu_1\frac{\partial W}{\partial \eta}(q(y,\eta) + f_\eta) \\ &+ \omega_1\left[a_{a_1}u_{a_f} + a_{a_2}^T\eta_a + b_a u - \frac{\partial \alpha_1}{\partial y}(a(y,\xi) + g_{1,r}' u_{f_1} + f_1) - \frac{\partial \alpha_1}{\partial y^*}\dot{y}^*\right. \\ &\left.- \frac{\partial \alpha_1}{\partial \Psi_0}(-a_{f_1}\Psi_0 + b_a u)\right] + y_f(-a_{f_1}y_f + a_{f_2}^T\eta_f + b_a u).\end{aligned} \tag{6.44}$$

Considering $\frac{\partial \alpha_1}{\partial \Psi_0} = 1$ and (6.30), the time derivative of $\dot{V}$ can be evaluated by

$$\dot{V} \le -\bar{\alpha}_v V_v - \gamma_f y_f^2 + \bar{R}. \tag{6.45}$$

Thus, we have $\dot{V} \le 0$ from (6.32) and (6.35).

Next we consider the time derivative of $V$ for $\nu \in \Omega_{\nu_1}$.
$(b-1)$ For the case $y_f \in \Omega_{y_{f_1}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_0}$:
Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\nu^2 + \delta_{V_v}^2 + \frac{g_m'}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}y_f^2 \tag{6.46}$$

the time derivative of $V$ yields that

$$\begin{aligned}\dot{V} = &\nu\left[a(\nu + y^*, \xi) - g_{1,r}'[k\nu + u_R + \Psi_0] + g_{1,r}'[\omega_1 - y_f] + f_1(\nu + y^*) - \dot{y}^*\right] \\ &+ g_m \Delta k_I \nu^2 + y_f(-a_{f_1}y_f + a_{f_2}^T\eta_f + b_a u).\end{aligned} \tag{6.47}$$

Considering (6.18) and (6.30), the time derivative of $V$ can be evaluated by

$$\dot{V} \le -(g_m' k^* - v_0')\nu^2 - \gamma_f y_f^2 + R_2 \tag{6.48}$$

therefore we have $\dot{V} \le -\gamma_\nu$ from (6.32) and (6.36).
$(b-2)$ For the case $y_f \in \Omega_{y_{f_1}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_1}$:
Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\nu^2 + V_v + \frac{g_m'}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}y_f^2 \tag{6.49}$$

the time derivative of $V$ can be evaluated by

$$\begin{aligned}\dot{V} \le &-(g_m' k^* - v_0)\nu^2 - (\mu_0\lambda_{min}[Q_\xi] - v_1)\|\xi\|^2 - (\mu_1\kappa_1 - v_2)\|\eta\|^2 + g_{1,r}'\nu\omega_1 \\ &- g_{1,r}'(y_f - \Psi_0)\nu + \omega_1\left[a_{a_1}u_{a_f} + a_{a_2}^T\eta_a + b_a u - \frac{\partial \alpha_1}{\partial y}(a(y,\xi) + g_{1,r}' u_{f_1} + f_1)\right. \\ &\left.- \frac{\partial \alpha_1}{\partial y^*}\dot{y}^* - \frac{\partial \alpha_1}{\partial k_I}\gamma_I\nu^2 - \frac{\partial \alpha_1}{\partial \Psi_0}(-a_{f_1}\Psi_0 + b_a u)\right] + y_f(-a_{f_1}y_f + a_{f_2}^T\eta_f + b_a u) + R_1\end{aligned} \tag{6.50}$$

Further considering (6.30). the time derivative of $\dot{V}$ can be evaluated by

$$\dot{V} \leq -(g'_m k^* - v'_0)\nu^2 - \bar{\alpha}_v V_v - \gamma_f y_f^2 + \bar{R}. \tag{6.51}$$

Thus we have $\dot{V} \leq -\gamma_\nu$ from (6.35) and (6.36).

We can see from $(a-1)$ to $(a-4)$ and $(b-1),(b-2)$ that the PFC output $y_f$ is bounded. Furthermore it follows from (6.14) that the PFC state $\eta_f$ is bounded. As a consequence, since the signal $y_f - \Psi_0$ is given by

$$\frac{d}{dt}(y_f - \Psi_0) = -a_{f_1}(y_f - \Psi_0) + a_{f_2}^T \eta_f \tag{6.52}$$

for $y_f \in \Omega_{y_{f_1}}$, $y_f - \Psi_0$ is also bounded. Thus there exists a positive constant $\Psi_{0M}$ such that

$$|y_f - \Psi_0| \leq \Psi_{0M} \tag{6.53}$$

for the both regions $\Omega_{y_{f_0}}$ and $\Omega_{y_{f_1}}$. Here we consider the ideal feedback gain $k^*$ again. The ideal feedback gain is satisfied (6.36) and

$$-(g'_m k^* - v'_0)\delta_v^2 + \max(R_2, R_3) \leq -\gamma_\nu < 0 \tag{6.54}$$

for

$$\begin{aligned} R_3 =& \frac{\sigma_5^2}{4\rho_2} + \frac{d_{11}}{4g'_m \gamma_R} \\ \sigma_5 =& L_2 d_0 + d_{01} + d_1 + \frac{L_2 \delta_{V_v}}{\sqrt{\mu_0 \lambda_{min}[P_\xi]}} + \sqrt{2} g'_M \delta_{V_v} + g'_M \Psi_{0M} \end{aligned}$$

$(b-3)$ For the case $y_f \in \Omega_{y_{f_0}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_0}$:

Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\nu^2 + \delta_{V_v}^2 + \frac{g'_m}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}\delta_{y_f}^2 \tag{6.55}$$

the time derivative of $V$ yields that

$$\begin{aligned} \dot{V} =& \nu\big[a(\nu + y^*, \xi) - g'_{1,r}[k\nu + u_R + \Psi_0] + g'_{1,r}[\omega_1 - y_f] \\ &+ f_1(\nu + y^*) - \dot{y}^*\big] + g'_m \Delta k_I \nu^2. \end{aligned} \tag{6.56}$$

Here we have

$$\|\xi\| \leq \frac{\delta_{V_v}}{\sqrt{\mu_0 \lambda_{min}[P_\xi]}}, \quad |\omega_1| \leq \sqrt{2}\delta_{V_v}$$

from $V_v \leq \delta_{V_v}^2$, the time derivative of $\dot{V}$ can be evaluated by

$$\dot{V} \leq -(g'_m k^* - v'_0)\nu^2 + R_3. \tag{6.57}$$

Thus we have $\dot{V} \leq -\gamma_\nu$ from (6.54).

$(b-4)$ For the case $y_f \in \Omega_{y_{f_0}}$, $(\xi, \eta, \omega_1) \in \Omega_{v_1}$:

Since $V$ is expressed from (6.34) as

$$V = \frac{1}{2}\nu^2 + V_v + \frac{g'_m}{2\gamma_I}\Delta k_I^2 + \frac{1}{2}\delta_{y_f}^2 \tag{6.58}$$

the time derivative of $V$ can be evaluated by

$$\begin{aligned}\dot{V} \leq & -(g_m' k^* - v_0')\nu^2 - (\mu_0 \lambda_{min}[Q] - v_1')\|\xi\|^2 \\ & -(\mu_1 \kappa_1 - v_2)\|\eta\|^2 - (c_1 - \rho_9 - \frac{a_{f_1}^2}{2\epsilon_3})\omega_1^2 + \bar{R} + R_3 \\ \leq & -(g_m' k^* - v_0')\nu^2 - \bar{\alpha}_v V_v + \bar{R} + R_3. \end{aligned} \tag{6.59}$$

Therefore, we have $\dot{V} \leq -\gamma_\nu$ from (6.35) and (6.54).

Finally, we have

$$\begin{aligned} &\dot{V} \leq 0, && \text{if } \nu \in \Omega_{\nu_0} \\ &\dot{V} \leq -\gamma_\nu < 0, && \text{if } \nu \in \Omega_{\nu_1} \end{aligned} \tag{6.60}$$

Thus the time derivative of $V$ can be evaluated as $\dot{V} \leq 0$ for all $t$, so we can conclude that all the signals in the control system are bounded.

Next, we analyze the convergence of the tracking error $\nu$. Suppose that there exists a time $t_0$ such that $\nu^2 > \delta_\nu^2$ for all $t \geq t_0$. This implies that $V \geq \frac{1}{2}\delta_\nu^2$, $\forall t \geq t_0$. Since $\dot{V} \leq -\gamma_\delta < 0$ for $(\nu, \omega) \in \Omega_1$ from (6.60), we have

$$V(t) = V(t_0) + \int_{t_0}^{t} \dot{V}(\tau) d\tau \leq V(t_0) - \gamma_\delta (t - t_0). \tag{6.61}$$

The inequality (6.61) contradicts the fact that $V \geq \frac{1}{2}\delta_\nu^2$, $\forall t \geq t_0$, because the right hand side of (6.61) will eventually become negative $t \to \infty$. This means that the interval $(t_0, t_1)$ in which $(\nu, \omega) \in \Omega_1$ is finite. Let $(t_2, t_3)$ be a finite interval during $(\nu, \omega) \in \Omega_0$ and $(t_3, t_4)$ be a finite interval during $(\nu, \omega) \in \Omega_1$. Since $\dot{V} \leq 0$ for $(\nu, \omega) \in \Omega_0$ and $\dot{V} \leq -\gamma_\delta < 0$ for $(\nu, \omega) \in \Omega_1$ from (6.60), it follows that for the interval $(t_2, t_3)$ during $(\nu, \omega) \in \Omega_0$

$$V(t_3) \leq V(t_2) \tag{6.62}$$

and that for the interval $(t_3, t_4)$ during $(\nu, \omega) \in \Omega_1$

$$V(t_4) < V(t_3). \tag{6.63}$$

Thus the positive function $V$ decreases a finite amount time $(\nu, \omega)$ leaves $\Omega_0$ and re-enters into $\Omega_0$ in finite time and $V$ does not increase during that $(\nu, \omega) \in \Omega_0$. Finally we conclude that there exists a finite time $T > 0$ such that $V$ converges to a constant for all $t \geq T$, *i.e.*, $(\nu, \omega) \in \Omega_0$ for all $t \geq T$. This fact gives that

$$\lim_{t \to \infty} |\nu| \leq \delta_\nu. \tag{6.64}$$

The control objective (6.2) is attained by setting the positive constant $\delta_\nu$ as $\delta_\nu = \delta$ after all. □

## 6.4 Controller Design for Linear Systems

In this section, we design a robust adaptive controller for uncertain linear systems by one-step backstepping.

### 6.4.1 Problem Statement

Consider the following $n$th order SISO linear system:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + bu(t) + g(x, t)\\ y(t) &= c^T x(t)\end{aligned} \tag{6.65}$$

where $x = [x_1, \cdots, x_n]^T \in R^n$ is a state variable and $u, y \in R$ are an input and an output, respectively. $A \in R^{n\times n}$ is an unknown matrix, $b, c \in R^n$ are unknown vectors and $g(x, t)$ is an uncertain time-varying function.

Suppose that the controlled system (6.65) satisfy the following assumptions.

**Assumption 6.1.** *The system* (6.65) *is minimum-phase.*

**Assumption 6.2.** *The system* (6.65) *has a relative degree $r$ ($r \leq n$).*

**Assumption 6.3.** *The high frequency gain is positive, i.e., $c^T A^{\gamma-1} b > 0$.*

**Assumption 6.4.** *The uncertain function $g(x, t)$ is bounded for all $x$ and $t$.*

Since the system (6.65) has a relative degree $r$ from assumption 6.2, there exists a smooth nonsingular variable transformation $\Phi_l x = [z^T, \eta^T]^T$ such that the system (6.65) can be transformed to the form[58]:

$$\begin{aligned}\dot{z} &= A_z z + b_z u + \begin{bmatrix} 0 \\ c_z^T \end{bmatrix} \eta + g_{c1}\\ \dot{\eta} &= Q_\eta \eta + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z_1 + g_{c2}\\ y &= z_1 = [1, 0, \cdots, 0] z\end{aligned} \tag{6.66}$$

where

$$\begin{aligned}A_z &= \begin{bmatrix} 0 & I_{\gamma-1\times\gamma-1} \\ -a_0 & \cdots - a_{\gamma-1} \end{bmatrix},\\ b_z^T &= [0, \cdots, b_z],\ b_z = c^T A^{\gamma-1} b,\\ (\Phi_l g)^T &= [g_{c1}^T, g_{c2}^T] = [g_{c1,1}, \cdots, g_{c1,r}, g_{c2,1}, \cdots, g_{c2,n-r}]\end{aligned}$$

and $c_z \in R^{n-r}$ is a appropriate vector. From assumption 6.1, $Q_\eta$ is a stable matrix because $\dot{\eta} = Q_\eta \eta$ denotes the zero dynamics of system (6.65).

### 6.4.2 Controller Design through One-step Backstepping

#### Augmented Virtual System

We introduce $r-1$th order stable virtual filter (6.3) to the controlled system (6.66) as well as the case of nonlinear system, then the following proposition is given.

**Proposition 6.2.** *For the system* (6.66) *with a relative degree $r \leq n$, consider the following variable transformation using the filtered signal $u_{f_i}$ given in* (6.3)*:*

$$\xi_i = -b_z' u_{f_{i-1}} + z_i + \sum_{j=1}^{i-1} c_{\xi j} z_{i-j} \tag{6.67}$$

*where* $b'_z = \frac{b_z}{b_u}$ *and*

$$\begin{aligned}
c_{\xi k} &= a_{r-k} - \theta_k, \ (1 \le k \le r-1) \\
c_{\xi r} &= a_0 - \sum_{j=1}^{r-1} \beta_{j-1} c_{\xi j} \\
\theta_1 &= \beta_{r-1} \\
\theta_k &= \beta_{r-k} + \sum_{j=1}^{k} \beta_{r-k+j} c_{\xi j}.
\end{aligned}$$

*Then the system* (6.66) *can be expressed by the form:*

$$\begin{aligned}
\dot{y} &= -c_{\xi 1} y + \xi_2 + b_z u_{f_1} + g_{c1,1} \\
\dot{\xi} &= A_{u_f} \xi + \begin{bmatrix} 0 \\ 1 \end{bmatrix} c_z^T \eta - c_\xi y + g_\xi \\
\dot{\eta} &= Q_\eta \eta + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y + g_{c2}
\end{aligned} \tag{6.68}$$

*where* $y = z_1 = x_1$, $\xi^T = [\xi_2, \cdots, \xi_i, \cdots, \xi_r]$, $c_\xi^T = [c_{\xi 2}, \cdots, c_{\xi i}, \cdots, c_{\xi r}]$, $g_\xi^T = [g_{\xi 2}, \cdots, g_{\xi i}, \cdots, g_{\xi r}]$ *and*

$$g_{\xi i} = g_{c1,i} + \sum_{j=1}^{i-1} c_{\xi j} g_{c1,i-j}.$$

(*The proof is shown in Appendix A.*1.)

For the obtained virtual system (6.68) given from the variable transformation (6.67), seeing the filtered signal $u_{f_1}$ as the control input for this virtual system, the virtual system has a relative degree 1. Further, since the zero dynamics of the nominal part of this virtual system (6.68) seeing $g_{c1,1}, g_\xi$ and $g_{c2}$ as disturbances is given by

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} = A_1 \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \left[\begin{array}{c|c} A_{u_f} & \begin{array}{c} \mathbf{0} \\ c_z^T \end{array} \\ \hline \mathbf{0} & Q_\eta \end{array}\right] \begin{bmatrix} \xi \\ \eta \end{bmatrix} \tag{6.69}$$

the zero dynamics of the nominal system is stable from the facts that $Q_\eta$ and $A_{u_f}$ are stable matrices from assumption 6.1 and (6.3). Thus the nominal part of this virtual system (6.68) is ASPR with the control input $u_{f_1}$. Moreover the disturbances $g_{c1,1}, g_\xi$ and $g_{c2}$ are bounded from the assumption 6.4.

Here we introduce the PFC (6.14) in the same manner of the nonlinear case, the virtual system (6.68) can be represented by

$$\begin{aligned}
\dot{y} &= a_y y + \xi_2 + b'_z (u_{a_f} - y_f) + g_{c1,1} \\
\dot{\eta}_\xi &= A_1 \eta_\xi + b_1 y + g_\eta
\end{aligned} \tag{6.70}$$

where $\eta_\xi^T = [\xi^T, \eta^T]$, $b_1^T = [-c_\xi^T, 0, \cdots, 0, 1]$, $a_y = -c_{\xi 1}$, $g_\eta^T = [g_\xi^T, g_{c2}^T]$. Since $A_1$ is a stable matrix, there exists a symmetric positive definite matrix $P_1$ for any positive definite matrix $Q_1$ such that

$$A_1^T P_1 + P_1 A_1 = -Q_1. \tag{6.71}$$

Further $g_{c1,1}$ and $g_\eta$ can be evaluated by

$$|g_{c1,1}| \le m_0, \ \|g_\eta\| \le m_\eta \tag{6.72}$$

with positive constants $m_0$ and $m_\eta$ because $g_{c1,1}$ and $g_\eta$ are bounded from the assumption 6.4.

### Adaptive Controller Design through One-step Backstepping

For the augmented virtual system (6.70), we design the actual control input $u$ as follows:

$$u = \begin{cases} -\frac{1}{b_a}\left[c_1\omega_1 + \epsilon_0(u_{a_f}^2 + \|\eta_a\|^2)\omega_1 + \epsilon_1\Psi_1\omega_1\right] & \text{if } y_f \in \Omega_{f_0} \\ -\frac{\omega_1}{b_a y_f}\left[c_1\omega_1 + \epsilon_0(u_{a_f}^2 + \|\eta_a\|^2)\omega_1 + \epsilon_1\Psi_1\omega_1\right] & \\ -\frac{1}{b_a}[\gamma_f y_f + \epsilon_2\|\eta_f\|^2 y_f] - \frac{\epsilon_3}{b_a y_f}\Psi_0^2 & \text{if } y_f \in \Omega_{f_1} \end{cases} \tag{6.73}$$

$$\forall \epsilon_0, \epsilon_1, \epsilon_2 > 0, \ c_1 > \frac{a_{f_1}^2}{2\epsilon_3}, \ \gamma_f \ge \frac{\|a_{f_2}\|^2}{4\epsilon_2\delta_{y_f}^2} \tag{6.74}$$

$$\Psi_1 = (l_1 + u_{f_1}^2)\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + \left(\frac{\partial\alpha_1}{\partial y^*}\right)^2 + \left(\frac{\partial\alpha_1}{\partial k}\right)^2 \nu^4, \ \forall l_1 > 0 \tag{6.75}$$

$$\alpha_1(t) = -k(t)\nu(t) + \Psi_0(y_f), \tag{6.76}$$

$$\dot{k}(t) = \gamma_l D(\nu)\nu(t)^2, \ \forall \gamma_l > 0, \ k(0) \ge 0 \tag{6.77}$$

$$\dot{\Psi}_0 = D(y_f)[-a_{f_1}\Psi_0 + b_a u] \tag{6.78}$$

in the same manner of the nonlinear case. Then the following theorem concerning the boundedness of all the signals in the control system and convergence of the tracking error is given.

**Theorem 6.2.** *Under the assumptions* 6.1 *to* 6.4 *on the controlled system* (6.65), *all the signals in the resulting closed-loop system with the controller* (6.73) *are bounded. Further, the tracking error $\nu$ converges to any given bound $\delta$ such that*

$$\lim_{t\to\infty} |\nu| \le \delta. \tag{6.79}$$

(*The proof is shown in Appendix A.2.*)

## 6.5 Numerical Simulations

### 6.5.1 Example 1: 5th Order Nonlinear System

Consider the same nonlinear system in the section 5.5.1.

Since the nonlinear system has a relative degree 3, we introduce a second order virtual filter:

$$\begin{aligned} \dot{u}_{f_1} &= u_{f_2} \\ \dot{u}_{f_2} &= -\beta_1 u_{f_1} - \beta_2 u_{f_2} + b_u u \end{aligned} \tag{6.80}$$

and the first order PFC:

$$\dot{y}_f = -a_f y_f + b_f u. \tag{6.81}$$

For the nonlinear system with the virtual filter and the PFC, we design the control input $u$ as (6.30) with

$$\omega_1 = u_{f_a} - \alpha_1 \ , \psi_1 = y^2, \ \psi_{1\eta} = \phi_1 = y, \ \nu = y - y^*.$$

In this simulation the reference signal is give as $y^*(t) = \sin 2t$ and controller parameters are set as follows:

$$\gamma_I = 1000, \ \gamma_p = \gamma_R = \gamma_f = 10, \ \epsilon_0 = \epsilon_1 = \epsilon_2 = \epsilon_3 = 10, \ c_1 = 1, \ \delta_\nu = 0.05, \ l_1 = 10$$
$$k_I(0) = 1, \ \Psi_0(0) = 0, \ \beta_1 = 40, \ \beta_2 = 10, \ b_u = 1, \ a_f = 1, \ b_f = 0.1, \ \delta_{y_f} = 5$$

Fig.6.2 to 6.5 show the simulation results. The proposed method gave us a good tracking performance even the structure of the controller is simpler than the one given in chapter 5.

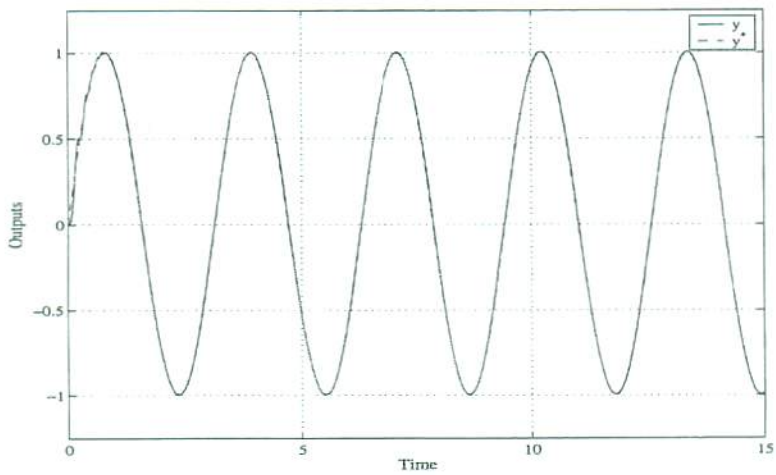


Figure 6.2: System output and reference signal: $y, y^*$

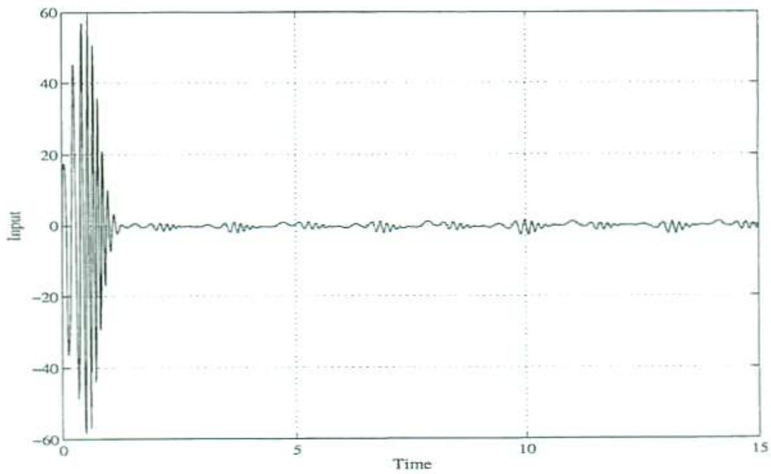


Figure 6.3: Control input: $u$

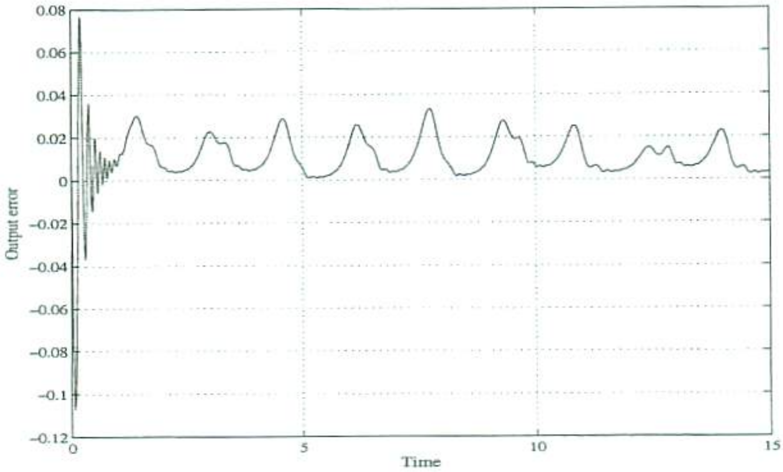


Figure 6.4: Output error: $\nu$

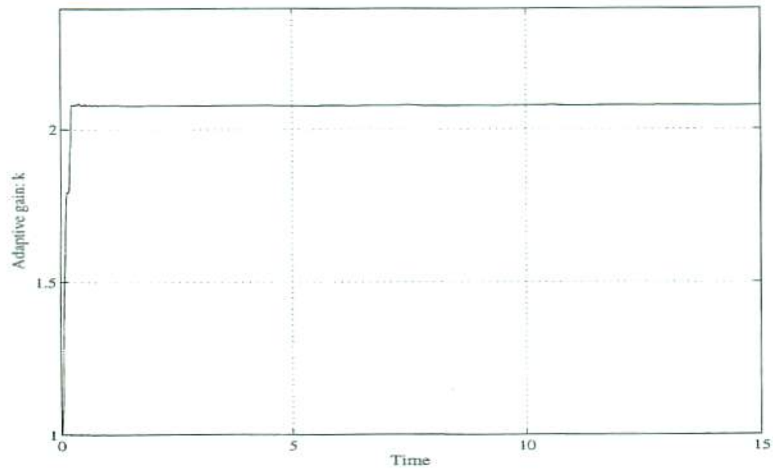


Figure 6.5: Adaptive feedback gain: $k = k_I + k_p$

### 6.5.2 Example 2: One Link Robot Arm

Consider a model of the one link robot arm with a nonrigid actuator shaft[58].

$$\begin{aligned} J_1\ddot{q}_1 + F_1\dot{q}_1 + \frac{K}{N}(q_2 - \frac{q_1}{N}) &= T \\ J_2\ddot{q}_2 + F_2\dot{q}_2 + K(q_2 - \frac{q_1}{N}) + mgd\cos q_2 &= 0 \end{aligned} \tag{6.82}$$

where $q_1$ and $q_2$ denote the angular positions of the link and the shaft, respectively. $J_1, J_2$ and $F_1, F_2$ represent inertia and viscous friction constants of the link and the shaft and $K$ is the elasticity constant of the spring which represents the elastic coupling with the joint. $N$ is the transmission gear ratio, $T$ is the torque of the actuator and $m, d$ are mass and the position of the link's center of gravity, respectively. Plant parameters in this simulation are shown in table 6.1.

Table 6.1: Plant parameters

| | | |
|---|---|---|
| $J_1 : 2.5 \times 10^{-5}$ | $J_2 : 8.0 \times 10^{-2}$ | $[Nms^2/rad]$ |
| $F_1 : 2.0 \times 10^{-4}$ | $F_2 : 2 \times 10^{-4}$ | $[Nms/rad]$ |
| | $K : 1.0 \times 10^4$ | $[Nm/rad]$ |
| $m : 1\ [kg]$ | $d : 2.5 \times 10^{-1}\ [m]$ | |
| $g : 9.8\ [m/s^2]$ | $N : 0.1$ | |

Setting $x = [q_2, \dot{q}_2, q_1, \dot{q}_1]$, $u = T$ as a control input and $y = x_1 = q_1$ as an output, the system (6.82) can be expressed by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{K}{J_2}x_1 - \frac{F_2}{J_2}x_2 + \frac{K}{J_2N}x_3 - \frac{mgd}{J_2}\cos x_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{K}{J_1N}x_1 - \frac{K}{J_1N^2}x_3 - \frac{F_1}{J_1}x_4 + \frac{1}{J_1}u. \end{aligned} \tag{6.83}$$

It is easy to know that this system has a relative degree 4. Here seeing $-\frac{mgd}{J_2}\cos x_1$ as a kind of disturbance, which is bounded for all $x$ and $t$, we can apply the control scheme given in 6.4 for designing a adaptive controller for the system (6.82).

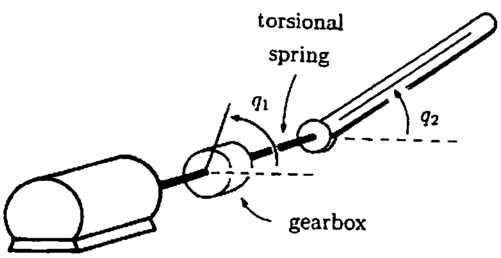


Figure 6.6: One link robot arm with a nonrigid actuator shaft

Since the system has a relative degree 4, we introduce third order virtual filter:

$$\begin{aligned}\dot{u}_{f_1} &= u_{f_2}\\ \dot{u}_{f_2} &= u_{f_3}\\ \dot{u}_{f_3} &= -\beta_1 u_{f_1} - \beta_2 u_{f_2} - \beta_3 u_{f_3} + b_u u\end{aligned} \tag{6.84}$$

and first order PFC:

$$\dot{y}_f = -a_f y_f + b_f u. \tag{6.85}$$

For the system with the virtual filter and the PFC, we design the control input $u$ as (6.73).

In this simulation, we gave the reference signal as

$$y^*(t) = \begin{cases} \sin \frac{\pi}{8} t & 0 \le t < 4 \\ 1 & 4 \le t < 10 \\ 1 + 0.2 \sin \pi (t - 10) & 10 \le t < 20 \end{cases} \tag{6.86}$$

and set the controller parameters as follows:

$$\gamma_I = 2500,\ \gamma_f = 10,\ \epsilon_0 = \epsilon_1 = \epsilon_2 = \epsilon_3 = 20,\ c_1 = 10,\ \delta_\nu = 0.02,\ \delta_{y_f} = 1,\ l_1 = 10$$
$$k_I(0) = 1,\ \Psi_0(0) = 0,\ \beta_1 = 50,\ \beta_2 = 25,\ \beta_3 = 50,\ b_u = 1,\ a_f = 10,\ b_f = 0.05.$$

Fig.6.7 to 6.10 show the simulation results. The proposed controller with simple structure gave us the desired tracking performance even the system has a relative degree 4.

## 6.6 Conclusion

In this chapter, we proposed a novel one-step backstepping design scheme for designing a robust adaptive high gain output feedback controller for uncertain linear systems and nonlinear systems. The proposed method can be applied to uncertain systems with a higher order relative degree and the designed control system has a simple controller structure because the controller is designed with only one step of backstepping. Further the effectiveness of the proposed method was confirmed by the numerical simulations.

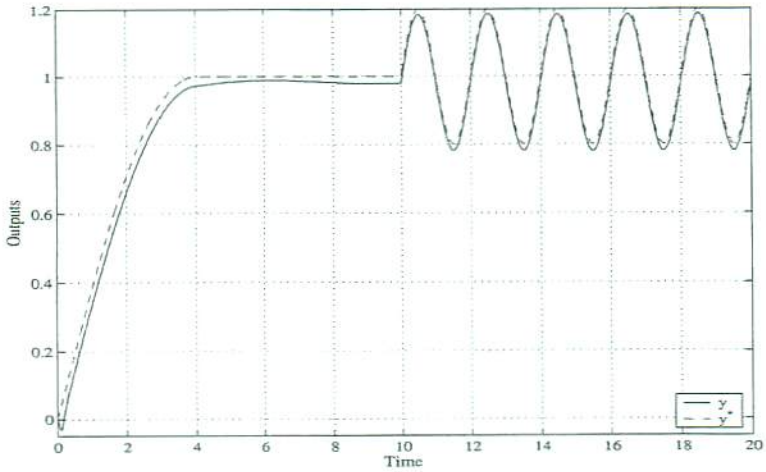


Figure 6.7: System output and reference signal: $y, y^*$

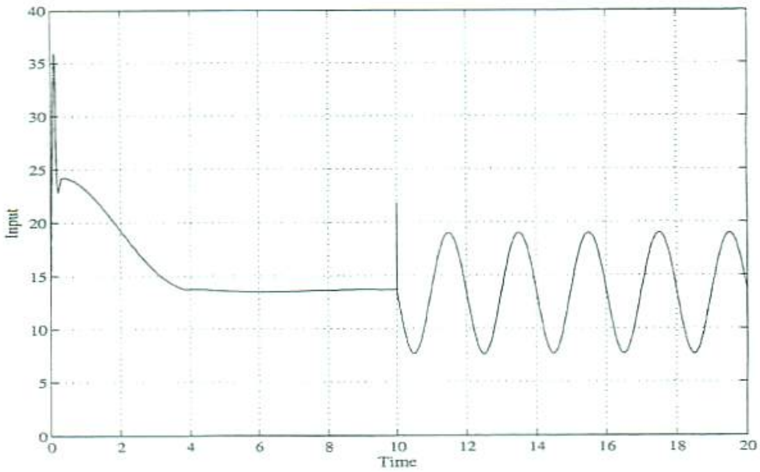


Figure 6.8: Control input: $u$

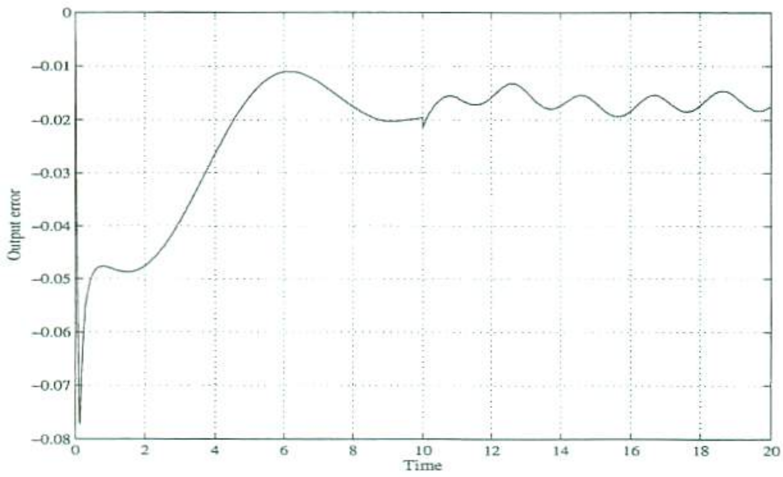


Figure 6.9: Output error: $\nu$

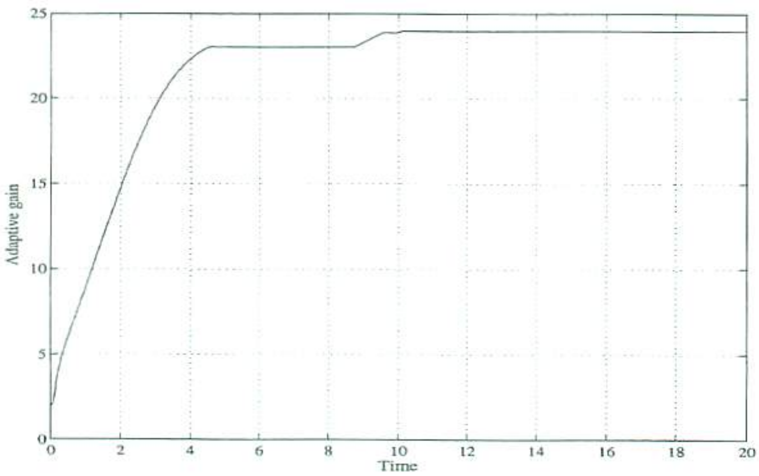


Figure 6.10: Adaptive feedback gain: $k$

# Summary


Adaptive control system based on high gain output feedback has a simple controller structure and high robustness for disturbances and unmodelled dynamics. Further this control system can be designed without the information about the upper bound of the controlled system's order. However, the application of the above adaptive control scheme for practical systems is very restricted, because it can only be applied to OFEP nonlinear systems and further most practical systems are not OFEP.

In this work, we proposed design methods for high gain adaptive control systems for non-OFEP nonlinear systems. The following is a summary of this work:

In chapter 2, we reviewed the definitions of *OFEP* and *relative degree*, and a basic controller design method for high gain adaptive output feedback control of OFEP nonlinear systems. Further, an alleviation method for OFEP restrictions, introducing a PFC, was also presented.

In chapter 3, a controller design method for high gain adaptive output feedback control system was proposed for non-OFEP nonlinear systems with a relative degree of 1 and non-Lipschitz nonlinearities. This control method can also be applied for nonlinear systems with an unknown and unbounded coefficient in the control input term.

In chapter 4, a high gain adaptive state feedback control system was designed for uncertain nonlinear systems with a higher order relative degree through backstepping strategy. The uncertainties dealt with in this chapter were nonparametric nonlinearities and the control input terms also have such nonparametric uncertainties. Thus we treated a wider class of nonlinear systems. The effectiveness of the proposed control system was confirmed through a numerical simulation of CSTR model, which is known as difficult to control.

In chapter 5, a high gain adaptive output feedback control system was designed for uncertain and time-varying nonlinear systems with non-Lipschitz nonlinearities and a higher order relative degree by introducing a virtual filter and applying backstepping strategy. Although the idea, introducing a virtual filter, has been proposed before, the coefficients in the control input terms are unknown constant or known functions in previous methods. The controlled system considered in this chapter has unknown and time-varying functions in the control input terms. Furthermore, since the proposed method is expanded one from the methods of chapter 3 and 4, it can be applied for uncertain nonlinear systems with non-Lipschitz nonlinearities and a higher order relative degree.

In chapter 6, we proposed a simple controller design method by introducing a PFC in parallel with the virtual filter proposed in chapter 5. The controller design method proposed in chapter 5 has a complex controller structure since the controller is designed through backstepping strategy in the virtual filter. In this chapter we designed a high gain adaptive output feedback control system by one-step backstepping, which is a new controller design method and allows us to design a controller by backstepping of only one step for controlled systems with a higher order relative degree.

In this work, design methods for a robust adaptive control system based on high gain feedback were proposed. None of the proposed methods require the information about the order of controlled system. Furthermore, in each chapter the effectiveness of each control methods were shown through several types of numerical simulations. To the best of my knowledge, none of the alleviation methods other than the introduction of a PFC have ever been proposed for OFEP conditions. The introduction of a PFC may cause a bias error. On the other hand the proposed methods do not cause the bias error even in a tracking control, so we believe that this work helps to expand the applicable class of high gain adaptive output feedback controls to practical systems.

# Acknowledgment

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# Appendix A

# The Proofs of the Proposition 6.2 and the Theorem 6.2.

## A.1 The Proof of the Proposition 6.2.

*Proof.* Since it follows from (6.67) that

$$\xi_2 = -b_z' u_{f_1} + z_2 + c_{\xi 1} z_1$$

we have

$$\dot{y} = -c_{\xi 1} y + \xi_2 + b_z' u_{f_1} + g_{c_1,1} \tag{A.1}$$

Further for $k = 2, \cdots, r-1$, $\xi_k$ is expressed from (6.67) by

$$\xi_k = -b_z' u_{f_{k-1}} + z_k + \sum_{j=1}^{k-1} c_{\xi j} z_{k-j}$$

the time derivative of $\xi_k$ yields that

$$\dot{\xi}_k = -b_z' u_{f_k} + z_{k+1} + g_{c_1,k} + \sum_{j=1}^{k-1} c_{\xi j}(z_{k-j+1} + g_{c_1,k-j}).$$

Since we have

$$\xi_{k+1} = -b_z' u_{f_k} + z_{k+1} + \sum_{j=1}^{k} c_{\xi j} z_{k-j+1}$$

from (6.67), the time derivative of $\xi_k$ is obtained by

$$\begin{aligned} \dot{\xi}_k &= (\xi_{k+1} - \sum_{j=1}^{k} c_{\xi j} z_{k-j+1}) + \sum_{j=1}^{k-1} c_{\xi j} z_{k-j+1} + g_{\xi k} \\ &= \xi_{k+1} - c_{\xi k} y + g_{\xi k}. \end{aligned} \tag{A.2}$$

Finally as for $\xi_r$, since

$$\xi_r = -b_z' u_{f_{r-1}} + z_r + \sum_{j=1}^{r-1} c_{\xi j} z_{r-j}$$

the time derivative of $\xi_r$ yields that

$$
\begin{aligned}
\dot{\xi}_r =& - bz'(-\sum_{i=1}^{r-1} \beta_i u_{f_i} + b_u u) - \sum_{i=1}^{r} a_{r-i} z_{r-i+1} + b_z u + c_z^T \eta + g_{c_1,r} \\
&+ \sum_{j=1}^{r-1} c_{\xi j}(z_{r-j+1} + g_{c_1,r-j}) \\
=& b_z' \sum_{i=1}^{r-1} \beta_i u_{f_i} - \sum_{i=1}^{r} a_{r-i} z_{r-i-1} + c_z^T \eta + \sum_{j=1}^{r-1} c_{\xi j} z_{r-j+1} + g_{\xi r}
\end{aligned}
$$

Here since

$$
c_{\xi k} = a_{r-k} - \theta_k
$$

the time derivative of $\xi_r$ is expressed by

$$
\begin{aligned}
\dot{\xi}_r =& b_z' \sum_{i=1}^{r-1} \beta_i u_{f_i} - \sum_{i=1}^{r} a_{r-i} z_{r-i+1} + c_z^T \eta + \sum_{j=1}^{r-1} (a_{r-j} - \theta_j) z_{r-j+1} + g_{\xi r} \\
=& b_z' \sum_{i=1}^{r-1} \beta_i u_{f_i} - a_0 z_1 + c_z^T \eta - \sum_{j=1}^{r-1} \theta_j z_{r-j+1} + g_{\xi r}
\end{aligned}
$$

and since we also have

$$
\begin{aligned}
u_{f_1} &= (-\xi_2 + z_2 + c_{\xi 1} z_1)/b_z' \\
u_{f_2} &= (-\xi_3 + z_3 + \sum_{j=1}^{2} c_{\xi j} x_{3-j})/b_z' \\
&\vdots \\
u_{f_{r-1}} &= (-\xi_r + z_r - \sum_{j=1}^{r-1} c_{\xi j} x_{r-j})/b_z'
\end{aligned}
$$

from (6.67). the time derivative $\xi_r$ can be represented by

$$
\begin{aligned}
\dot{\xi}_r =& \beta_1(-\xi_2 + z_2 + c_{\xi 1} z_1) + \beta_2(-\xi_3 + z_3 + \sum_{j=1}^{2} c_{\xi j} x_{3-j}) + \cdots \\
&+ \beta_{r-1}(-\xi_r + z_r + \sum_{j=1}^{r-1} c_{\xi j} x_{r-j}) - a_0 z_1 + c_z^T \eta + g_{\xi r} - \sum_{j=1}^{r-1} \theta_j z_{r-j+1} \\
=& - \sum_{j=1}^{r-1} \beta_j \xi_{j+1} + c_z^T \eta + g_{\xi r} \\
&+ (-a_0 + \beta_1 c_{\xi 1} + \beta_1 c_{\xi 2} + \cdots + \beta_{r-1} c_{\xi r-1}) z_1 \\
&+ (\beta_1 + \beta_2 c_{\xi 1} + \cdots + \beta_{r-1} c_{\xi r-2}) z_2 \\
&+ \cdots + (\beta_{r-2} + \beta_{r-1} c_{\xi 1}) z_{r-1} + \beta_{r-1} z_r - \sum_{j=1}^{r-1} \theta_j z_{r-j+1}
\end{aligned}
$$

Eventually, considering the facts that

$$\theta_1 = \beta_{r-1}$$
$$\theta_k = \beta_{r-k} + \sum_{j=1}^{k} \beta_{r-k+j} c_{\xi j}$$

the time derivative of $\xi_r$ is given by

$$\dot{\xi}_r = -\sum_{j=1}^{r-1} \beta_j \xi_{j+1} - c_{\xi r} y + c_z^T \eta + g_{\xi r}. \tag{A.3}$$

Thus we get the desired results. $\square$

## A.2 The Proof of the Theorem 6.2.

*Proof.* Consider the following positive and continuous function $V$:

$$V = \begin{cases} \frac{1}{2b_z'} \delta_\nu^2 + V_a, & \nu \in \Omega_{\nu_0} \\ \frac{1}{2b_z'} \nu^2 + V_a, & \nu \in \Omega_{\nu_1} \end{cases} \tag{A.4}$$

where

$$V_a = \begin{cases} \frac{1}{2}\delta_{y_f}^2 + \delta_{V_v}^2 + \frac{1}{2\gamma_I}\Delta k^2, & y_f \in \Omega_{y_{f_0}}, (\eta_\xi, \omega_1) \in \Omega_{v_0} \\ \frac{1}{2}\delta_{y_f}^2 + V_v + \frac{1}{2\gamma_I}\Delta k^2, & y_f \in \Omega_{y_{f_0}}, (\eta_\xi, \omega_1) \in \Omega_{v_1} \\ \frac{1}{2}y_f^2 + \delta_{V_v}^2 + \frac{1}{2\gamma_I}\Delta k^2, & y_f \in \Omega_{y_{f_1}}, (\eta_\xi, \omega_1) \in \Omega_{v_0} \\ \frac{1}{2}y_f^2 + V_v + \frac{1}{2\gamma_I}\Delta k^2, & y_f \in \Omega_{y_{f_1}}, (\eta_\xi, \omega_1) \in \Omega_{v_1} \end{cases}$$
$$V_v = \mu_0 \eta_\xi^T P_1 \eta_\xi + \frac{1}{2}\omega_1^2, \ \Delta k = k - k^*$$

and

$$\Omega_{v_0} = \{\eta_\xi \in R^{n-1}, \omega_1 \in R \mid V_v \le \delta_{V_v}^2\}$$
$$\Omega_{v_1} = \{\eta_\xi \in R^{n-1}, \omega_1 \in R \mid V_v > \delta_{V_v}^2\}$$

with a positive constant $\delta_{V_v}$. The positive constant $\delta_{V_v}$ is determined by

$$\delta_{V_v}^2 \ge \bar{R}/\bar{\alpha}_v \tag{A.5}$$

where

$$\bar{\alpha}_v = \min\left[\frac{\lambda_{min}[Q_1] - v_1'/\mu_0}{\lambda_{max}[P_1]}, 2(c_1 - \rho_5 - \frac{a_{f1}^2}{2\epsilon_3})\right]$$

for positive constants $\mu_0$ and $\rho_5$ that satisfy

$$\mu_0 \lambda_{min}[Q_1] - v_1' > 0, \ c_1 - \rho_5 - \frac{a_{f1}^2}{2\epsilon_3} > 0$$
$$v_1' = v_1 + \frac{3}{4\epsilon_1 l_1}$$

and

$$\begin{aligned}\bar{R} =& \frac{(a_y d_o + d_1 + m_0)^2}{4\rho_1 b_z^2} + \frac{\mu_0^2 \|P_1\|^2 (d_0\|b_1\| + m_\eta)^2}{\rho_4} \\ &+ \frac{(\mu_0\|b_1\|\|P_1\|\delta_\nu)^2}{\rho_3} + \frac{1}{4\epsilon_0}(|a_{a1}|^2 + \|a_{a2}\|^2) \\ &+ \frac{1}{4\epsilon_1}\left[\frac{3(a_y(d_0+\delta_\nu))^2}{l_1} + b_z'^2 + d_1^2 + \gamma_I^2\right] + \frac{\|a_{f_2}\|^2}{4\epsilon_2}\end{aligned}$$

Further we consider an ideal feedback gain $k^*$ such that the following inequality is satisfied

$$-(k^* - v_0')\delta_v^2 + R_2 \le -\gamma_\nu < 0 \tag{A.6}$$

for

$$\begin{aligned} v_0' &= v_0 + \frac{1}{4\rho_5} + \frac{3a_y^2}{4\epsilon_1 l_1} + \frac{1}{4a_{f_1}} + \frac{1}{2\epsilon_3} \\ R_2 &= \bar{R} + \frac{2\delta_{V_v}^2}{4\rho_5} \end{aligned}$$

where $\gamma_\nu$ is any positive constant.

Here we consider the time derivative of $V$ given in (A.4). First, we consider a case where $\nu \in \Omega_{\nu_0}$.

$(a-1)$ For the case $(\eta_\xi, \omega_1) \in \Omega_{v_0}, y_f \in \Omega_{y_{f_0}}$:

Since $V$ is given from (A.4) by

$$V = \frac{1}{2b_z'}\delta_\nu^2 + \frac{1}{2}\delta_{y_f}^2 + \frac{1}{2}\delta_{V_v}^2 + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.7}$$

we have $\dot{V} = 0$.

$(a-2)$ For the case $(\eta_\xi, \omega_1) \in \Omega_{v_1}, y_f \in \Omega_{y_{f_0}}$:

Since the $V$ is expressed from (A.4) by

$$V = \frac{1}{2b_z'}\delta_\nu^2 + \frac{1}{2}\delta_{y_f}^2 + V_v + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.8}$$

we have

$$\dot{V} = \dot{V}_v \le -\bar{\alpha}_v V_v + \bar{R}. \tag{A.9}$$

From (A.5), the time derivative of $V$ can be evaluated by $\dot{V} \le 0$.

$(a-3)$ For the case $(\eta_\xi, \omega_1) \in \Omega_{v_0}, y_f \in \Omega_{y_{f_1}}$:

Since from (A.4) we have

$$V = \frac{1}{2b_z'}\delta_\nu^2 + \frac{1}{2}y_f^2 + \frac{1}{2}\delta_{V_v}^2 + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.10}$$

the time derivative of $V$ can be evaluated by

$$\dot{V} \le -\gamma_f y_f^2 + \frac{\|a_{f_2}\|^2}{4\epsilon_2}. \tag{A.11}$$

Thus we have $\dot{V} \le 0$ from (6.74).

$(a-4)$ For the case $(\eta_\xi, \omega_1) \in \Omega_{v_1}, y_f \in \Omega_{y_{f_1}}$:

Since we have

$$V = \frac{1}{2b'_z}\delta_\nu^2 + \frac{1}{2}y_f^2 + V_v + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.12}$$

the time derivative of $V$ can be evaluated by

$$\dot{V} \le -\bar{\alpha}_v V_v - \gamma_f y_f^2 + \bar{R} < 0. \tag{A.13}$$

Thus we have $\dot{V} \le 0$ from (A.5).

Next, we consider the time derivative of $V$ for $\nu \in \Omega_{\nu_1}$.

$(b-1)$ For the case $(\eta_\xi, \omega_1) \in \Omega_{v0}, y_f \in \Omega_{y_{f_1}}$:

Since the $V$ is expressed from (A.4) by

$$V = \frac{1}{2b'_z}\nu^2 + \frac{1}{2}y_f^2 + \frac{1}{2}\delta_{V_v}^2 + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.14}$$

we have

$$\dot{V} \le -(k^* - v'_0)\nu^2 - \gamma_f y_f^2 + \frac{\|a_{f_2}\|^2}{4\epsilon_2} + R_2. \tag{A.15}$$

Therefore, the time derivative of $V$ can be evaluated by $\dot{V} \le -\gamma_\nu$ from (6.74) and (A.6).

$(b-2)$ For $(\eta_\xi, \omega_1) \in \Omega_{v_1}, y_f \in \Omega_{y_{f_1}}$:

Since we have

$$V = \frac{1}{2b'_z}\nu^2 + \frac{1}{2}y_f^2 + V_v^2 + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.16}$$

from (A.4), the time derivative of $V$ can be evaluated by

$$\dot{V} \le -(k^* - v'_0)\nu^2 - \bar{\alpha}_v V_v - \gamma_f y_f^2 + \bar{R}. \tag{A.17}$$

Thus we have $\dot{V} \le -\gamma_\nu$ from (A.5) and (A.6).

We can see from $(a-1)$ to $(a-4)$ and $(b-1)$, $(b-2)$ that the PFC output $y_f$ is bounded. Furthermore it follows from (6.14) that the PFC state $\eta_f$ is bounded. As a consequence, since the signal $y_f - \Psi_0$ is given by

$$\frac{d}{dt}(y_f - \Psi_0) = -a_{f_1}(y_f - \Psi_0) + a_{f_2}^T \eta_f \tag{A.18}$$

for $y_f \in \Omega_{y_{f_1}}$, $y_f - \Psi_0$ is also bounded. Thus there exists a positive constant $\Psi_{0M}$ such that

$$|y_f - \Psi_0| \le \Psi_{0M} \tag{A.19}$$

for the both regions $\Omega_{y_{f_0}}$ and $\Omega_{y_{f_1}}$. Here we consider the ideal feedback gain $k^*$ again. The ideal feedback gain is satisfied (A.6) and

$$-(k^* - v'_0)\delta_v^2 + \max(R_2, R_3) \le -\gamma_\nu < 0 \tag{A.20}$$

for

$$\begin{aligned} R_3 &= \frac{\sigma_0^2}{4\rho_1} \\ \sigma_0 &= \frac{|a_y|d_0}{b'_z} + \frac{\delta_{V_v}}{b'_z\sqrt{\mu_0\lambda_{min}[P_1]}} + \sqrt{2}\delta_{V_v} + \delta_{y_f} + \frac{d_1 + m_0}{b'_z}\Psi_{0M} \end{aligned}$$

$(b-3)$ For the case $(\eta_\xi, \omega_1) \in \Omega_{v0}, y_f \in \Omega_{y_{f_0}}$:

From (A.4) the function $V$ is represented by

$$V = \frac{1}{2b'_z}\nu^2 + \frac{1}{2}\delta_{y_f}^2 + \frac{1}{2}\delta_{V_v}^2 + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.21}$$

we have

$$\dot{V} \leq -(k^* - v_0')\nu^2 + R_3. \tag{A.22}$$

The time derivative of $V$ can be evaluated as $\dot{V} \leq -\gamma_\nu$ from (A.20).
$(b-4)$ For the case $(\eta_\xi, \omega_1) \in \Omega_{v_1}, y_f \in \Omega_{y_{f_0}}$:

Since the $V$ is expressed by

$$V = \frac{1}{2b'_z}\nu^2 + \frac{1}{2}\delta_{y_f}^2 + V_v + \frac{1}{2\gamma_I}\Delta k^2 \tag{A.23}$$

we have

$$\dot{V} \leq -(k^* - v_0')\nu^2 - \bar{\alpha}_v V_v + \bar{R} + R_3 \tag{A.24}$$

Thus we have $\dot{V} \leq -\gamma_\nu$ from considering (A.5) and (A.20).

From above analysis we have

$$\begin{aligned} &\dot{V} \leq 0, && \text{for } \nu \in \Omega_{\nu_0} \\ &\dot{V} \leq -\gamma_\nu < 0, && \text{for } \nu \in \Omega_{\nu_1} \end{aligned} \tag{A.25}$$

Thus it is easy to conclude that all the signals in the resulting closed-loop system are bounded because we have $\dot{V} \leq 0$ for $\forall t > 0$ from (A.25).

The poof of the convergence of the output error can be proved by the same way as the nonlinear case in theorem 6.1.

□

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**Chapter 3**

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