DOMAINS IN A TWO DIMENSIONAL STEIN SPACE WITH QUOTIENT SINGULARITIES

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Abstract

Let X be a 2 - dimensional reduced Stein space such that every singular point of X is a quotient singular point. Let D be an open set of X. Let L be a positive dimensional commutative complex Lie group. Then $\mathrm{H}^1(D, \mathscr{A}_L)=0$ if and only if D is Stein and $\mathrm{H}^2(D, \pi_1(L))=0$.

0. Introduction

Let D be a domain in a complex manifold X. Assume that there exists a positive dimensional complex Lie group L such that $\mathrm{H}^1(D, \mathscr{A}_L)=0$, where $\mathscr{A}_L$ is the sheaf of groups over D of all germs of holomorphic maps into L. J. Kajiwara and H. Kazama [10] proved that D is a Stein manifold if X is a 2 - dimensional Stein manifold. J. Kajiwara and M. Takase [11] proved that D is a Stein manifold if X is a product manifold of two 1 - dimensional complex tori and L is a complex linear group whose Lie algebra contains a non-zero purely integral element. J. Kajiwara and K. Watanabe [12] proved that D is a Stein manifold if X is a 2 - dimensional projective space and $D \neq X$. Other related results are obtained in various situations (see for example [1], [8] and [9]).

In this paper we consider the case that X has singularities. We shall give a proof of the following fact. Let X be a purely 2 - dimensional reduced Stein space such that every singular point of X is a quotient singular point. Let D be an open set of X. If there exists a positive dimensional commutative complex Lie group L such that $\mathrm{H}^1(D, \mathscr{A}_L)=0$, then D is a Stein space. In section 1 we shall consider the Levi's problem for open sets of X. In section 2 we shall give a proof of the above fact using the result in section 1.

1. Levi's problem.

Let X be a reduced complex space. In this paper we always denote by $S(X)$ the singular locus of X. $p \in S(X)$ is called a quotient singular point if and only if there exist a neighbourhood U of p, a second countable connected complex manifold M and a finite subgroup G of $\mathrm{Aut}(M)$ such that U is biholomorphic to an open set of a quotient space M/G. Here we denote by $\mathrm{Aut}(M)$ the group of all holomorphic automorphisms of

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*一般科目（数学） 昭和63年9月1日受付

M. Every quotient singular point is a normal point (Kaup-Kaup [13], p. 312, 72.5). In this section we consider a variant of Levi's problem for open sets of a 2-dimensional Stein space every singular point of which is a quotient singular point. The following lemma is a special case of Lemma 2 of R. Narasimhan [15].

LEMMA 1. *Let M be a second countable connected complex manifold and G be a finite subgroup of* Aut(M). *Let π be a natural projection from M to a quotient space M/G. Let D be an open set of M/G. Then the following two conditions are equivalent.*

i) *$\pi^{-1}(D)$ is a Stein open set of M.*

ii) *D is a Stein open set of M/G.*

PROOF. i) → ii). M/G is a second countable reduced complex space and the map $\pi : M \to M/G$ is holomorphic, open, finite and surjective. Let $(p_\nu)_{\nu=1}^{\infty}$ be a sequence of pairwise disjoint points of D such that there is no subsequence converging in D. Let $(c_\nu)_{\nu=1}^{\infty}$ be an arbitrary sequence of complex numbers. D is Stein if we can find $F \in \mathcal{O}(D)$ such that $F(p_\nu) = c_\nu$ for every ν. Since $\bigcup_{\nu=1}^{\infty} \pi^{-1}(p_\nu)$ is a 0-dimensional analytic set of a Stein manifold $\widetilde{D} := \pi^{-1}(D)$, there exists $f \in \mathcal{O}(\widetilde{D})$ such that $f \equiv c_\nu$ on $\pi^{-1}(p_\nu)$ for every ν. $\tilde{f} := |G|^{-1} \cdot \sum_{g \in G} f \circ g$ is well-defined as a holomorphic function on $\widetilde{D}$, where $|G|$ denotes the order of G. Since $\tilde{f} \circ g = \tilde{f}$ for every $g \in G$, $F := \tilde{f} \circ \pi^{-1}$ is a single-valued function on D. Since $F \circ \pi = \tilde{f} \in \mathcal{O}(\widetilde{D})$, F is a holomorphic function on D by the definition of the complex structure of M/G. It holds that $F(p_\nu) = c_\nu$ for every ν. Therefore D is a Stein open set of M/G.

ii) → i). We set $\widetilde{D} = \pi^{-1}(D)$. Since $\pi|_{\widetilde{D}} : \widetilde{D} \to D$ is proper, D is holomorphically convex by Satz 2.2 of Stein [16]. Take an arbitrary $q \in \widetilde{D}$. There exist finitely many $f_1, f_2, \cdots, f_k \in \mathcal{O}(D)$ such that $\{x \in D \mid f_i(x) = 0,\ i = 1, 2, \cdots\cdots, k\} = \{\pi(q)\}$. Then q is an isolated point of $\{y \in \widetilde{D} \mid (f_i \circ \pi)(y) = 0,\ i = 1, 2, \cdots\cdots, k\} = \pi^{-1}(\pi(q))$. Therefore $\widetilde{D}$ is K-complete.

Let X be a reduced complex space and D be an open set of X. D is said to be locally Stein at $p \in \partial D$ if and only if there exists a neighbourhood U of p such that $U \cap D$ is a Stein open set of X. Let A be an analytic set of X. A point $p \in \partial D \cap A$ is said to be removable along A if and only if there exists a neighbourhood U of p such that $U - A \subset D$. The set $D^* = D \cup \{p \in \partial D \cap A \mid p$ is removable along $A\}$ is called the extension of D along A. To study the Levi's problem T. Ueda [17], [18] and K. Adachi [2] used the following lemma which is originally due to H. Grauert and R. Remmert [6].

LEMMA 2. *Let X be a complex manifold and A be a positive codimensional analytic set in X. Let D be an open set of X. Assume that D is locally Stein at every $p \in \partial D - A$*

and that D has no boundary point removable along A. Then D is locally Stein at every $p \in \partial D$.

LEMMA 3. *Let X be a reduced complex space. Assume that every $p \in S(X)$ is a quotient singular point. Let D be an open set of X. Assume that D is locally Stein at every $p \in \partial D - S(X)$ and that D has no boundary point removable along $S(X)$. Then D is locally Stein at every $p \in \partial D$.*

PROOF. Take an arbitrary $p \in \partial D \cap S(X)$. Since p is a quotient singular point, there exist a Stein neighbourhood V of p, a second countable connected complex manifold M and a finite subgroup G of $\mathrm{Aut}(M)$ such that V is biholomorphic to an open set of a quotient space M/G. $S(V) := S(X) \cap V$ is the singular locus of V. Choose a Stein neighbourhood U of p such that $U \Subset V$. U has no boundary point removable along $S(V)$ (see the proof of i) → iii) of Lemma 4). $W := U \cap D$ is an open set of V such that $W \Subset V$. We can prove that W is locally Stein at every $q \in \partial W - S(V)$ and has no boundary point removable along $S(V)$. We regard V as an open set of M/G. Let $\pi : M \to M/G$ be the natural projection. By Lemma 1 $\widetilde{V} := \pi^{-1}(W)$ is a Stein open set of M. $\widetilde{W} := \pi^{-1}(W)$ is an open set of $\widetilde{V}$ such that $\widetilde{W} \Subset \widetilde{V}$. $\widetilde{S} := \pi^{-1}(S(V))$ is a positive codimensional analytic set in $\widetilde{V}$. Take an arbitrary $r \in \partial \widetilde{W} - \widetilde{S}$. Since $\pi(r) \in \partial W - S(V)$ there exists a neighbourhood E of $\pi(r)$ in V such that $E \cap W$ is Stein. $\widetilde{E} := \pi^{-1}(E)$ is a neighbourhood of r in $\widetilde{V}$. $\widetilde{E} \cap \widetilde{W} = \pi^{-1}(E \cap W)$. By Lemma 1 $\widetilde{E} \cap \widetilde{W}$ is Stein. Hence $\widetilde{W}$ is locally Stein at every $r \in \partial \widetilde{W} - \widetilde{S}$. We can also prove that $\widetilde{W}$ has no boundary point removable along $\widetilde{S}$. Therefore $\widetilde{W}$ is locally Stein at every $r \in \partial \widetilde{W}$ by Lemma 2. By a theorem of Docquier - Grauert [5] $\widetilde{W}$ is a Stein open set of $\widetilde{V}$. By Lemma 1 $W = U \cap D$ is Stein. Thus it is proved that D is locally Stein at every $p \in \partial W \cap S(X)$.

Using the above lemma and a theorem due to A. Andreotti and R. Narasimhan [3] on Levi's problem for open sets in a Stein space with isolated singularities, we obtain the following lemma.

LEMMA 4. *Let X be a purely 2-dimensional reduced Stein space. Assume that every $p \in S(X)$ is a quotient singular point. Let D be an open set of X. Then the following three conditions are equivalent.*

i) *D is a Stein open set of X.*

ii) *D is locally Stein at every $p \in \partial D$.*

iii) *D is locally Stein at every $p \in \partial D - S(X)$ and has no boundary point removable along $S(X)$.*

PROOF. iii) → ii). Lemma 4.

ii) → i). Since X is a 2-dimensional normal complex space, $S(X)$ is a discrete closed set of X. By Corollary 1 to Theorem 4 of Andreotti - Narasimhan [3], D is a Stein open set.

i) → iii). Assume that there exists $p \in \partial D \cap S(X)$ removable along $S(X)$. Then there exists a neighbourhood U of p such that $U - S(X) \subset D$. There exists a sequence $(p_\nu)_{\nu=1}^{\infty} \subset U - S(X)$ such that $\lim_{\nu\to\infty} p_\nu = p$. There exists $f \in \mathcal{O}(D)$ such that $\lim_{\nu\to\infty} |f(p_\nu)| = +\infty$. Since U is normal, there exists $\tilde{f} \in \mathcal{O}(U)$ such that $\tilde{f} = f$ on $U - S(X)$. $|\tilde{f}(p)| = \lim_{\nu\to\infty} |\tilde{f}(p_\nu)| = +\infty$. It is a contradiction.

2. Domains with a vanishing cohomology group.

Let L be a positive dimensional commutative complex Lie group. The Lie algebra of L is isomorphic to $\mathbf{C}^m$, where m is the complex dimension of L. The exponential map exp $: \mathbf{C}^m \to L$ is holomorphic, surjective and homeomorphic. The kernel Γ of exp is a discrete subgroup of $\mathbf{C}^m$ and is isomorphic to the fundamental group of L. Let X be a 2-dimensional reduced Stein space and D be an open set of X. We denote by $\mathcal{A}_L$ the sheaf of commutative groups over D of all germs of holomorphic maps into L. We have an exact sequence $0 \to \Gamma \to \mathcal{O}^m \to \mathcal{A}_L \to 0$, where $\mathcal{O}$ is the structure sheaf of D. In this section we study the property of an open set D which satisfies the condition that $H^1(D, \mathcal{A}_L) = 0$.

LEMMA 5. Let X be a purely 2-dimensional normal Stein space. Let D be an open set of X such that $X - S(X) \subset D$. Assume that there exists a positive dimensional commutative complex Lie group L such that $H^1(D, \mathcal{A}_L) = 0$. Then $D = X$.

PROOF. Take an arbitrary point $p \in X$. Let V be a connected Stein neighbourhood of p. By Remmert's embedding theorem V is homeomorphic to an analytic set of $\mathbf{C}^N$. By Theorem 1 of Lojasiewicz [14] there exist a locally finite simplicial complex K and a homeomorphism $\tau : |K| \to V$ such that $\tau^{-1}(p) \in K$. Let U be the image by τ of the union of all $\sigma \in K$ such that $\tau^{-1}(p)$ is a 0-face of σ. Then U is a contractible neighbourhood of p and there exist finitely many contractible open sets $W_1, W_2, \cdots, W_k$ such that $U - \{p\} = \bigcup_{i=1}^{k} W_i$. Let Γ be the fundamental group of L. We have an exact sequence $0 \to H^1(\{W_i\}_{i=1}^{k}, \Gamma) \to H^1(U - \{p\}, \Gamma) \to \prod_{i=1}^{k} H^1(W_i, \Gamma)$. Since W_i is contractible, we have that $H^1(W_i, \Gamma) = 0$ for every i. Hence $H^1(U - \{p\}, \Gamma) \cong H^1(\{W_i\}_{i=1}^{k}, \Gamma)$. There exist a positive integer ℓ and a subgroup B of Γ^ℓ such that $H^1(U - \{p\}, \Gamma) \cong \Gamma^\ell / B$. In particular $H^1(U - \{p\}, \Gamma)$ consists of at most countably infinite elements. Assume that $D \subsetneqq X$. We set $P = X - D$. $\emptyset \neq P \subset S(X)$. Since X is normal, $S(X)$ is a finite or countably infinite discrete closed set of X. Let $(p_\nu)_{\nu=1}^{\infty}$ or $(p_\nu)_{\nu=1}^{n}$ be all the distinct points of P. By the above consideration there exists a neighbourhood U_ν of p_ν for every ν satisfying the following conditions. $U_\nu \cap S(X) = \{p_\nu\}$ and $U_\nu - \{p_\nu\}$ is a finite union of contractible open sets $(\nu = 1, 2, \cdots\cdots)$. $U_\mu \cap U_\nu = \emptyset$ if $\mu \neq \nu$. Then $\mathrm{H}^1(U_\nu - \{p_\nu\}, \Gamma)$ consists of at most countably infinite elements for every ν. We have a short exact sequence $0 \to \Gamma \to \mathcal{O}^m \to \mathcal{A}_L \to 0$, where $m = \dim L$. We obtain the exact sequence of cohomology groups $\mathrm{H}^1(D, \Gamma) \to \mathrm{H}^1(D, \mathcal{O}^m) \to \mathrm{H}^1(D, \mathcal{A}_L) = 0$. $U := \bigcup_\nu U_\nu$ is a neighbourhood of P. $U \cap D = U - P = \bigcup_\nu (U_\nu - \{p_\nu\})$. $\{U, D\}$ is an open covering of X. From the Meyer-Vietoris exact sequence $0 = \mathrm{H}^1(X, \mathcal{O}^m) \to \mathrm{H}^1(U, \mathcal{O}^m) \oplus \mathrm{H}^1(D, \mathcal{O}^m) \to \mathrm{H}^1(U - P, \mathcal{O}^m)$, we have an injection $\alpha : \mathrm{H}^1(D, \mathcal{O}^m) \to \mathrm{H}^1(U - P, \mathcal{O}^m)$. Since every holomorphic function on $U - P$ is extendable to U, $U - P$ is not Stein. By Theorem 2 of Ballico [4] it holds that $\mathrm{H}^1(U - P, \mathcal{O}) \neq 0$. Therefore $\mathrm{H}^1(U - P, \mathcal{O}^m) = \mathrm{H}^1(U - P, \mathcal{O})^m \neq 0$. In the same way $\mathrm{H}^1(D, \mathcal{O}^m) \neq 0$. Let $(\xi_\nu) : \mathrm{H}^1(U - P, \mathcal{O}^m) \to \prod_\nu \mathrm{H}^1(U_\nu - \{p_\nu\})$ be the natural isomorphism. There exists a number ν_0 such that $\xi_{\nu_0} \circ \alpha \neq 0$. We have a commutative diagram

$$\begin{tikzcd}
\mathrm{H}^1(D, \Gamma) \arrow[r] \arrow[d] & \mathrm{H}^1(D, \mathcal{O}^m) \arrow[r] \arrow[d, "\xi_{\nu_0} \circ \alpha"] & 0 \quad \text{exact} \\
\mathrm{H}^1(U_{\nu_0} - \{p_{\nu_0}\}, \Gamma) \arrow[r, "\rho_{\nu_0}"'] & \mathrm{H}^1(U_{\nu_0} - \{p_{\nu_0}\}, \mathcal{O}^m) &
\end{tikzcd}$$

We can prove that $\operatorname{im} \rho_{\nu_0} \supset \operatorname{im}(\xi_{\nu_0} \circ \alpha)$. Since $\operatorname{im}(\xi_{\nu_0} \circ \alpha)$ is a non-zero $\mathbf{C}$ - vector space, it consists of uncountably infinite elements. But $\operatorname{im} \rho_{\nu_0}$ consists of finite or countably infinite elements. It is a contradiction.

Using the method of J. Kajiwara and H. Kazama [10] we obtain the following lemma.

LEMMA 6. *Let X be a purely 2-dimensional reduced Stein space. Let D be an open set of X. Assume that there exists a positive dimensional commutative complex Lie group L such that* $\mathrm{H}^1(D, \mathcal{A}_L) = 0$. *Then D is locally Stein at every $p \in \partial D - S(X)$.*

PROOF. In the proof we use the method of the proof of Lemma 11 of Kajiwara - Kazama [10]. Suppose that there exists a point $p \in \partial D - S(X)$ such that D is not locally Stein at p. There exist a holomorphic map $\psi : X \to \mathbf{C}^2$ and a neighbourhood W of p such that $W \subset X - S(X)$, $\psi(W)$ is an open set of $\mathbf{C}^2$, $\psi(p) = (0, 0)$ and $\psi|_W : W \to \psi(W)$ is biholomorphic (Grauert-Remmert [7], p. 151, Consequence 2).

Choose a Stein open set V of $\mathbf{C}^2$ such that $(0, 0) \in V \Subset \psi(W)$. $V' := \psi^{-1}(V) \cap W$ is a neighbourhood of p and $\psi(V') = V$. Since D is not locally Stein at p, $\psi(D \cap V')$ is not a Stein open set of $\mathbf{C}^2$. By Lemma 1 of Kajiwara-Kazama [10] there exist H, P and φ satisfying the following properties. $H = \{ (w_1, w_2) \in \mathbf{C}^2 \mid |w_1| < 1, |w_2| < 1 \} \cup \{ (w_1, w_2) \in \mathbf{C}^2 \mid 1 - 2\varepsilon < |w_1| < 1 + 2\varepsilon, |w_2| < 1 + 2\varepsilon \}$, $P = \{ (w_1, w_2) \in \mathbf{C}^2 \mid |w_1| < 1 + 2\varepsilon, |w_2| < 1 + 2\varepsilon \}$, $0 < \varepsilon < 1/2$. $\varphi : \mathbf{C}^2 \to \mathbf{C}^2$ is a biholomorphic map. $\varphi(H) \subset \psi(D \cap V')$ and there exists (b_1, b_2) such that $|b_1| \leqq 1 - 2\varepsilon$, $|b_2| = 1$ and $\varphi(b_1, b_2) \in \partial(\psi(D \cap V'))$. Then $\varphi(H) \subset V$ implies $\varphi(P) \subset V$ since V is a Stein open set of $\mathbf{C}^2$. $\theta = (\theta_1, \theta_2) := \varphi^{-1} \circ \psi : X \to \mathbf{C}^2$ is a holomorphic map. We set $T = \{ x \in X \mid |\theta_1(x)| < 1 + 2\varepsilon \}$, $T_0 = \{ x \in T \mid |\theta_2(x)| < 1 + 2\varepsilon \} \cap V'$ and $T_1 = \{ x \in T \mid |\theta_2(x)| > 1 + \varepsilon \} \cup (T - V')$. T is a Stein open set of X and $\{ T_0, T_1 \}$ is an open covering of T. The function $(\theta_2 - b_2)^{-1}$ is holomorphic on $T_0 \cap T_1$. Since $\mathrm{H}^1(\{ T_0, T_1 \}, \mathcal{O}) = 0$, there exist $v_i \in \mathcal{O}(T_i)$ $(i = 0, 1)$ such that $(\theta_2 - b_2)^{-1} = v_1 - v_0$ on $T_0 \cap T_1$. We can define a holomorphic function v on $(T_0 \cap \{ x \in X \mid \theta_2(x) \neq b_2 \}) \cup T_1$ by the equations $v = v_0 + (\theta_2 - b_2)^{-1}$ on $T_0 \cap \{ x \in X \mid \theta_2(x) \neq b_2 \}$ and $v = v_1$ on T_1. We set $D_1 = D \cap \{ x \in X \mid \theta_1(x) \neq b_1 \}$ and $D_2 = D \cap (\{ x \in T \mid \theta_2(x) \neq b_2 \} \cup (T - V'))$. $\{ D_1, D_2 \}$ is an open covering of D. The function $(\theta_1 - b_1)^{-1} v$ is holomorphic on $D_1 \cap D_2$. Let $\exp : \mathbf{C}^m \to L$ be the exponential map, where $m = \dim L$. Let e be a non-zero element of $\mathbf{C}^m$. Since $\mathrm{H}^1(\{ D_1, D_2 \}, \mathscr{A}_L) = 0$ (Lemma 1 of Kajiwara [8]), there exist holomorphic maps $G_i : D_i \to L$ $(i = 1, 2)$ such that $\exp((\theta_1 - b_1)^{-1} v \cdot e) = G_2 \cdot G_1^{-1}$ on $D_1 \cap D_2$. The map $G_1' := \exp((\theta_1 - b_1)^{-1} v_0 \cdot e) \cdot G_1$ is holomorphic on $T_0 \cap D_1$ and it holds that $\exp((\theta_1 - b_1)^{-1}(\theta_2 - b_2)^{-1} \cdot e) = G_2 \cdot G_1'^{-1}$ on $T_0 \cap D_1 \cap D_2$. We set $H_i = \{ (w_1, w_2) \in H \mid w_i \neq b_i \}$ and $P_i = \{ (w_1, w_2) \in P \mid w_i \neq b_i \}$ $(i = 1, 2)$. Since $H \subset \theta(V')$ and $\theta^{-1}(H_i) \cap V' \subset T_0 \cap D_i$ $(i = 1, 2)$, the map $F_1 := G_1' \circ (\theta|_{V'})^{-1}$ is holomorphic on H_1 and the map $F_2 := G_2 \circ (\theta|_{V'})^{-1}$ is holomorphic on H_2. By Lemmas 3 and 4 of Kajiwara-Watanabe [12] there exist holomorphic maps $\widetilde{F}_i : P_i \to L$ such that $F_i = \widetilde{F}_i$ on H_i $(i = 1, 2)$. It holds that $\exp((w_1 - b_1)^{-1}(w_2 - b_2)^{-1} \cdot e) = \widetilde{F}_2(w_1, w_2) \cdot \widetilde{F}_1(w_1, w_2)^{-1}$ for every $(w_1, w_2) \in P_1 \cap P_2$. It contradicts a property of Thullen's domain (Lemma 5 of Kajiwara-Kazama [10]).

We now prove our main theorem.

THEOREM. *Let X be a purely 2-dimensional reduced Stein space such that every singular point of X is a quotient singular point. Let L be a positive dimensional commutative complex Lie group and $\Gamma = \pi_1(L)$ be the fundamental group of it. Let D be an open set of X. Then the following two conditions are equivalent.*

i) $\mathrm{H}^1(D, \mathscr{A}_L) = 0$.

ii) *D is a Stein open set of X and* $\mathrm{H}^2(D, \Gamma) = 0$.

PROOF. i)→ii). Let D^* be the extension of D along $S(X)$. By Lemma 6 D is locally Stein at every $p \in \partial D - S(X)$. Since $D - S(X) = D^* - S(X)$, D^* is locally Stein at every $p \in \partial D^* - S(X)$. D^* has no boundary point removable along $S(X)$. Therefore D^* is Stein by Lemma 4. Since $D^* - S(X) \subset D \subset D^*$, $D = D^*$ by Lemma 5. It follows that D is Stein. From the exact sequence $0 = \mathrm{H}^1(D, \mathscr{A}_L) \to \mathrm{H}^2(D, \Gamma) \to \mathrm{H}^2(D, \mathcal{O}^m) = 0$ we have that $\mathrm{H}^2(D, \Gamma) = 0$.

ii)→i). From the exact sequence $0 = \mathrm{H}^1(D, \mathcal{O}^m) \to \mathrm{H}^1(D, \mathscr{A}_L) \to \mathrm{H}^2(D, \Gamma) = 0$ we have that $\mathrm{H}^1(D, \mathscr{A}_L) = 0$.

References

[1] Y. ABE, *Steinness of a domain with a vanishing cohomology set in a product manifold of the one dimensional complex projective space and a one dimensional complex torus,* Mem. Fac. Sci. Kyushu Univ. **38** (1984), 17 – 22.

[2] K. ADACHI, *Le problème de Levi pour les fibrés grassmanniens et les variétés drapeaux,* Pacific J. Math. **116** (1985), 1 – 6.

[3] A. ANDREOTTI and R. NARASIMHAN, *Oka's Heftungslemma and the Levi problem for complex spaces,* Trans. Amer. Math. Soc. **111** (1964), 345 – 366.

[4] E. BALLICO, *A remark on a theorem by Fornaess and Narasimhan about the Levi problem,* Math. Ann. **253** (1980), 137 – 138.

[5] F. DOCQUIER and H. GRAUERT, *Levisches Problem und Rungescher Satz für Teilgebiete Steinscher Mannigfaltigkeiten,* Math. Ann. **140** (1960), 94 – 123.

[6] H. GRAUERT and R. REMMERT, *Konvexität in der komplexen Analysis. Nicht-holomorph-konvexe Holomorphiegebiete und Anwendungen auf die Abbildungstheorie,* Comm. Math. Helv. **31** (1956), 152 – 183.

[7] H. GRAUERT and R. REMMERT, *Theory of Stein spaces,* Grundl. 236, Springer-Verlag, Berlin Heidelberg New York, 1979.

[8] J. KAJIWARA, *Some extensions of Cartan-Behnke-Stein's theorem,* Publ. Res. Inst. Math. Sci. Kyoto Univ. **2** (1966), 135 – 156.

[9] J. KAJIWARA, *Steinness of a domain with a vanishing cohomology set in a product manifold of an open Riemann surface and a compact Riemann surface with genus* ≧ 1, Mem. Fac. Sci. Kyushu Univ. **32** (1978), 235 – 254.

[10] J. KAJIWARA, and H. KAZAMA, *Two dimensional complex manifold with vanishing cohomology set,* Math. Ann. **204** (1973), 1 – 12.

[11] J. KAJIWARA and M. TAKASE, *Steinitude des domaines à cohomologie nulle dans un produit de deux tores complexes de dimension un,* Mem. Fac. Sci. Kyushu Univ. **33** (1979), 159 – 171.

[12] J. KAJIWARA and K. WATANABE, *Domains with a vanishing cohomology set in two dimensional complex projective space,* Mem. Fac. Sci. Kyushu Univ. **32** (1978),

73 − 80.

[13] L. Kaup and B. Kaup, *Holomorphic functions of several variables,* Walter de Gruyter, Berlin New York, 1983.

[14] S. Lojasiewicz, *Triangulation of semi-analytic sets,* Ann. Scuola Norm. Sup. Pisa **19** (1965), 449 − 474.

[15] R. Narasimhan, *A note on Stein spaces and their normalizations,* Ann. Scuola Norm. Sup. Pisa **16** (1962), 327 − 333.

[16] K. Stein, *Überlagerungen holomorph-vollständigen komplexer Räume,* Arch. Math. **7** (1956), 354 − 361.

[17] T. Ueda, *Pseudoconvex domains over Grassmann manifolds,* J. Math. Kyoto Univ. **20** (1980), 391 − 394.

[18] T. Ueda, *Domains of holomorphy in Segre cones,* Publ. Res. Inst. Math. Sci. Kyoto Univ. **22** (1986), 561 − 569.