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ASPR-based Output Feedback Control with Virtual PFC for Output Tracking

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Abstract: Nowadays, the output feedback control method based on the Almost Strictly Positive Real (ASPR) property gets many attentions and has been researched widely. ASPR models can be stabilized by applying simple output feedback control; so the designed controllers have a simple structure. However, the systems have to satisfy quite strict conditions in order to obtain ASPR-ness, although almost all practical systems do not have the ASPR property. Therefore, for relaxing those conditions, the introduction of a Parallel Feedforward Compensator (PFC) has been proposed. This method can render the resulting augmented system ASPR. Up to now, several PFC design methods have been proposed, and one of them is an adaptive PFC design scheme. This technique has a feature that it can design a PFC automatically by utilizing online data. Furthermore, for the purpose of output regulation, the control design methods with an adaptive PFC have been proposed. Unfortunately, however, in almost all schemes, the discussion on the convergence of actual errors has not been conducted. Therefore, in this paper, introducing a virtual PFC model and an auxiliary input for ensuring ASPR-ness, a new ASPR-based output feedback control method is proposed, and the stability analysis and convergence of the actual error are discussed. Finally, the effectiveness of the proposed method is confirmed via numerical simulations.

Keywords: Adaptive control, Output feedback, Output regulation, Stability analysis, Convergence analysis

1. INTRODUCTION

Recently, with technology advances, controlled systems have become more complex and with higher orders, and controlling such complicated systems efficiently and safely is an important issue. Therefore, several control schemes such as model predictive control (Garcia et al., 1989; Yoon and Clarke, 1994) and model reference adaptive control (MRAC) (Monopoli, 1974; Nam and Araposthathis, 1988) have been proposed to tackle this challenge. However, in model predictive control, it requires more accurate controlled system models to achieve better control performance. On the other hand, in the MRAC method initiated by Monopoli (1974), it was only required to know some rough information about the controlled systems such as orders of plants to design controllers. Furthermore, in these two types of methods, since controllers should be designed according to plant orders, as the plant dimension becomes higher, controllers have a more complex structure, resulting in extra difficulties in the implementation of controllers.

Therefore, the control system design scheme based on Almost Strictly Positive Real (ASPR)-ness, with the advantage that controllers have a relatively simple structure, has been proposed and implemented (Bar-kana, 1987; Mizumoto and Iwai, 1996; Kaufman et al., 1997; Fradkov and Hill, 1998; Kim et al., 2016). The definition of an ASPR system is that the resulting close-loop system by applying the output feedback is Strictly Positive Real (SPR) (Bar-Kana, 1991). As one of the characteristics on an ASPR model, it is known that the control system can be stabilized by simple output feedback control. That is why the designed controllers can have the simple structure. For ensuring ASPR-ness, three conditions should be satisfied: 1) the relative degree of the system is 1 or 0; 2) the system is minimum-phase; 3) the high frequency gain of the system is positive. However, those three conditions, especially 1) and 2), are strict for practical systems; thus

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the applicable areas of this ASPR-based output feedback control had been limited in the past.

Taking this restriction into account, in order to relax the above three conditions, several alleviating methods have been proposed (Astrom, 1980; Bar-kana, 1987; Fradkov, 1996; Mizumoto and Iwai, 1996). One of them was to introduce a Parallel Feedforward Compensator (PFC) (Bar-kana, 1987; Mizumoto and Iwai, 1996). In this method, a PFC is applied to the controlled system in parallel as shown in Fig. 1 to render the augmented system ASPR. After applying the PFC, the augmented system applied with the output feedback control can be stabilized. However, any given PFC cannot render the augmented system ASPR; so it is important how to select PFC. For now, several PFC design methods have been proposed such as a model-based PFC (Mizumoto et al., 2010) and an adaptive PFC (Takagi et al., 2015). Among them, it is supposed that the adaptive PFC design method is more effective for uncertain systems since it does not need accurate information about the plant. After those methods had been developed, it turned out to be relatively easy to obtain the ASPR property; so more practical control design methods with a PFC have been proposed to solve different control tasks, including output tracking (Mizumoto et al., 2010; Mizumoto and Kawabe, 2017). In Mizumoto and Kawabe (2017), by introducing a feedforward input based on a radial basis function neural network (RBF NN), output tracking is achieved. In Mizumoto et al. (2010), a PID controller is applied with a PFC for the same purpose. In those methods, the stability of the designed control system is guaranteed based on the ASPR-ness, but the discussion on convergence of the actual error was neglected.

In this paper, considering the virtual PFC model and introducing an auxiliary input to guarantee the ASPR property, a new output feedback control method based on ASPR-ness is proposed for ensuring the convergence of the output tracking error. Furthermore, we compose a two-degree-of-freedom (TDOF) system with the feedforward based on RBF NN to achieve output tracking. In addition, we verify the stability of the resulting control system and the convergence of the actual error throughout the numerical analysis. Finally, numerical simulation is done to confirm the effectiveness of the proposed method.

2. PROBLEM STATEMENT

In this paper, the following n-th order LTI continuous time stable SISO system is considered:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + \boldsymbol{b}u(t)\\ y(t) &= \boldsymbol{c}^T\boldsymbol{x}(t),\end{aligned} \quad (1)$$

where, $\boldsymbol{x}(t) \in R^n$, $u(t)$ and $y(t) \in R$ are the state vector, input and output, respectively. $A \in R^{n\times n}$ is an unknown matrix, and $\boldsymbol{b}$, $\boldsymbol{c}$ are unknown vectors.

At this moment, regarding $G(s)$ as the transfer function of the control system in (1), the system output $y(t)$ will be also expressed as

$$y(t) = G(s)[u(t)]. \quad (2)$$

This notation in (2) means the output of the system $G(s)$ under the input $u(t)$.

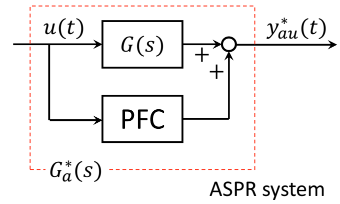

Fig. 1. The augmented system with a PFC

For ensuring the existence of an ideal feedforward input $v^*(t)$, the following assumption about reference signals is imposed.

Assumption 1. The reference model $r(t)$ should be generated by the following neutral stable exo-systems:

$$\begin{aligned}\dot{\boldsymbol{s}}(t) &= \boldsymbol{\mu}(\boldsymbol{s}(t))\\ r(t) &= \psi(\boldsymbol{s}(t))\\ \boldsymbol{\mu}(\boldsymbol{0}) &= \boldsymbol{0},\ \psi(\boldsymbol{0}) = 0,\end{aligned}$$

where $\boldsymbol{s}(t) \in R^r$ is an unknown vector, and this exo-system is neutral stable, which means all eigenvalues on the linear approximation $A_\mu = \left[\frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{s}}\right]_{\boldsymbol{s}=\boldsymbol{0}}$ of $\boldsymbol{\mu}(\boldsymbol{s}(t))$ are on the imaginary axis (Isidori, 1995).

Under this problem formulation, we design an output feedback control system based on ASPR-ness with a virtual PFC. In addition, we introduce the RBF NN-based feedforward input to achieve the output regulation.

3. THE DEFINITION OF THE IDEAL PFC

Here, the ideal PFC is defined (Mizumoto and Kawabe, 2017). First, designers give an n_a-th ASPR model for the controlled system (see Fig. 1). Then, it is supposed to be

$$G^*_a(s) = \frac{n^*_a(s)}{d^*_a(s)} = \frac{n_{an_a-1}s^{n_a-1} + \cdots + n_{a1}s + n_{a0}}{s^{n_a} + d_{an_a-1}s^{n_a-1} + \cdots + d_{a1}s + d_{a0}},$$

and the ideal ASPR model output $y^*_{au}(t)$ can be obtained as

$$y^*_{au}(t) = G^*_a(s)[u(t)] = y(t) + y^*_{fu}(t), \quad (3)$$

where $y^*_{fu}(t)$ is the ideal PFC output, and it is an available signal since $y(t)$ is measurable. Here, take the ideal PFC model $H^*(s)$ as

$$H^*(s) = \frac{n^*_H(s)}{d^*_H(s)} = \frac{b_{n_h-1}s^{n_h-1} + \cdots + b_1 s + b_0}{s^{n_h} + a_{n_h-1}s^{n_h-1} + \cdots + a_1 s + a_0};$$

$y^*_{fu}(t)$ can be also expressed by

$$y^*_{fu}(t) = H^*(s)[u(t)]. \quad (4)$$

Note that the ideal PFC model $H^*(s)$ is unknown.

Furthermore, introducing the following filter $f(s)$ to (4),

$$\frac{1}{f(s)} = \frac{1}{s^{n_h} + f_{n_h-1}s^{n_h-1} + \cdots + f_1 s + f_0}, \quad (5)$$

eventually, we can transform $y^*_{fu}(t)$ into the parametric representation as

$$y^*_{fu}(t) = \frac{z(s)}{f(s)}[y^*_{fu}(t)] + \frac{n^*_H(s)}{f(s)}[u(t)] = \boldsymbol{\rho}^{*T}_f \boldsymbol{z}^*_{fu}(t), \quad (6)$$

where,

$$\boldsymbol{\rho}_f^* = [z_{n_h-1}\, z_{n_h-2} \cdots z_0\;\, b_{n_h-1}\, b_{n_h-2} \cdots b_0]^T,$$
$$(z_i = f_i - a_i) \tag{7}$$
$$\boldsymbol{z}_{fu}^*(t) = \Big[\frac{s^{n_h-1}}{f(s)}[y_{fu}^*(t)] \cdots \frac{1}{f(s)}[y_{fu}^*(t)]$$
$$\frac{s^{n_h-1}}{f(s)}[u(t)] \cdots \frac{1}{f(s)}[u(t)]\Big]^T. \tag{8}$$

Here, we approximate the ideal PFC output $y_{fu}^*(t)$ with a smaller order n_f.

***Assumption** 2.* For the given order as $n_f < n_a + n$ and a compact set $\boldsymbol{\theta}(t) := [y_{fu}^*(t),\, u(t)]^T \in \Omega_{fl} \subset R^2$, the ideal weight vector $\boldsymbol{\rho}_{fl}^*$ is defined as

$$\boldsymbol{\rho}_{fl}^* = \arg\min_{\boldsymbol{\rho}_{fl} \in R^{2n_f}} \{ \sup_{\boldsymbol{\theta}(t) \in \Omega_{fl}} |y_{fu}^*(t) - \boldsymbol{\rho}_{fl}^T \boldsymbol{z}_{ful}^*(t)| \} \tag{9}$$
$$\text{subject to } \mathrm{b_{n_f-1} \neq n_{an_a-1}}$$

where $\boldsymbol{z}_{ful}^*(t)$ is the same as (8) substituting $n_h = n_f$, and then the ideal PFC output $y_{fu}^*(t)$ can be approximated as

$$y_{fu}^*(t) = \boldsymbol{\rho}_{fl}^{*T} \boldsymbol{z}_{ful}^*(t) + \varepsilon_{ful}(t), \tag{10}$$

satisfying

$$|\varepsilon_{ful}(t)| \le \varepsilon_{ful1}^*,\; |\dot{\varepsilon}_{ful}(t)| \le \varepsilon_{ful2}^*.$$

***Remark** 1.* Note that if the order of the filter $f(s)$ is given as $n_f \le n_a$, then we can make sure $n_f < n_a + n$.

4. THE IDEAL FEEDFORWARD INPUT

As mentioned before, under Assumption 1, there definitely exists an ideal feedforward input $v^*(t)$ (Isidori, 1995). It can be composed as a non-linear function of $\boldsymbol{s}(t)$ such as $v^*(t) = c(\boldsymbol{s}(t))$. Here, we try to approximate this function by using the following signal based on RBF NN:

$$\bar{v}(t) = \boldsymbol{\rho}_s^T \boldsymbol{\phi}_s(\boldsymbol{s}(t)), \tag{11}$$

where, $\boldsymbol{\phi}_s(\boldsymbol{s}(t))$ is a given l_s-th radial basis function and $\boldsymbol{\rho}_s$ is a weight vector. For sufficiently large node l_s and a compact set $S_s \in R^r$, the ideal weight vector $\boldsymbol{\rho}_s^*$ is defined as

$$\boldsymbol{\rho}_s^* = \arg\min_{\boldsymbol{\rho}_s \in R^{l_s}} \{ \sup_{\boldsymbol{s}(t) \in S_s} |v^*(t) - \boldsymbol{\rho}_s^T \boldsymbol{\phi}_s(\boldsymbol{s}(t))| \}, \tag{12}$$

and then, the approximation of $v^*(t)$ can be expressed as

$$v^*(t) = \boldsymbol{\rho}_s^{*T} \boldsymbol{\phi}_s(\boldsymbol{s}(t)) + \varepsilon_s(\boldsymbol{s}(t)),\; |\varepsilon_s(\boldsymbol{s}(t))| \le \varepsilon_s^*,$$

where $\varepsilon_s(\boldsymbol{s}(t))$ is an approximation error and should be bounded (Ge et al., 2002).

5. THE ADAPTIVE CONTROL SYSTEM DESIGN WITH AN AUXILIARY INPUT

In this section, considering the virtual PFC, an adaptive control system design scheme is explained, and Fig. 2 is the final obtained system.

Using (3), the actual output $y(t)$ can be expressed as

$$y(t) = G_a^*(s)[u(t)] - y_{fu}^*(t),$$

and, from Assumption 2, $y(t)$ can be represented as

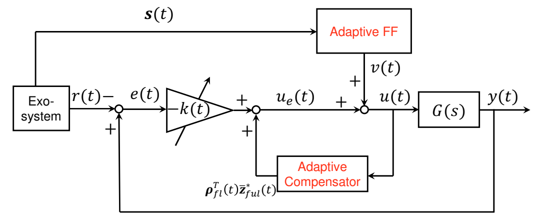

Fig. 2. The adaptive control system

$$y(t) = G_a^*(s)[u(t)] - \boldsymbol{\rho}_{fl}^{*T} \boldsymbol{z}_{ful}^*(t) - \varepsilon_{ful}(t).$$

Now, considering the following signal

$$\boldsymbol{\rho}_{fl}^{*T} \boldsymbol{z}_{ful}^*(t) = G_a^*(s)[\boldsymbol{\rho}_{fl}^{*T} \bar{\boldsymbol{z}}_{ful}^*(t)] \tag{13}$$
$$\bar{\boldsymbol{z}}_{ful}^*(t) = G_a^{*-1}(s)[\boldsymbol{z}_{ful}^*(t)] \tag{14}$$
$$\bar{\varepsilon}_{ful}(t) = G_a^{*-1}(s)[\varepsilon_{ful}(t)],\; |\bar{\varepsilon}_{ful}(t)| \le \varepsilon_{ful}^*, \tag{15}$$

we have

$$y(t) = G_a^*(s)[u(t) - \boldsymbol{\rho}_{fl}^{*T} \bar{\boldsymbol{z}}_{ful}^*(t) - \bar{\varepsilon}_{ful}(t)].$$

Finally, $y(t)$ can be also expressed as

$$y(t) = G_a^*(s)[u(t) - \boldsymbol{\rho}_{fl}^{*T} \bar{\boldsymbol{z}}_{ful}^*(t) - \bar{\varepsilon}_{ful}(t)] \pm G_a^*(s)[v^*(t)]$$
$$= G_a^*(s)[u(t) - v^*(t) - \boldsymbol{\rho}_{fl}^{*T} \bar{\boldsymbol{z}}_{ful}^*(t) - \bar{\varepsilon}_{ful}(t)]$$
$$+ G(s)[v^*(t)] + H^*(s)[v^*(t)],$$

and utilizing the following relationships

$$e(t) = y(t) - r(t),\; r(t) = G(s)[v^*(t)]$$
$$\varepsilon_{hv}(t) := G_a^{*-1}(s)[H^*(s)[v^*(t)]],\; |\varepsilon_{hv}(t)| \le \varepsilon_{hv}^*,$$

we have that the actual error $e(t)$ can be obtained as

$$e(t) = G_a^*(s)[u(t) - v^*(t) - \boldsymbol{\rho}_{fl}^{*T} \bar{\boldsymbol{z}}_{ful}^*(t) - \bar{\varepsilon}_{ful}(t) + \varepsilon_{hv}(t)].$$

Here, the control input is designed as

$$u(t) = u_e(t) + v(t) \tag{16}$$
$$u_e(t) = -k(t)e(t) + \boldsymbol{\rho}_{fl}^T(t) \bar{\boldsymbol{z}}_{ful}^*(t) \tag{17}$$
$$v(t) = \boldsymbol{\rho}_s^T(t) \boldsymbol{\phi}_s(\boldsymbol{s}(t)). \tag{18}$$

The first term in (17) plays a role as the output feedback, and the second term in (17) plays a role as the auxiliary input to ensure the ASPR-ness of the system. Moreover, (18) is the actual feedforward input.

Each parameter is estimated by adjusting laws as follows:

$$\dot{k}(t) = \gamma_k e^2(t) - \sigma_k k(t) \tag{19}$$
$$\dot{\boldsymbol{\rho}}_{fl}(t) = -\Gamma_{fl} \bar{\boldsymbol{z}}_{ful}^*(t) e(t) - \sigma_{fl} \boldsymbol{\rho}_{fl}(t) \tag{20}$$
$$\dot{\boldsymbol{\rho}}_s(t) = -\Gamma_s \boldsymbol{\phi}_s(\boldsymbol{s}(t)) e(t) - \sigma_s \boldsymbol{\rho}_s(\boldsymbol{s}(t)), \tag{21}$$

where $\Gamma_{fl}^T = \Gamma_{fl} > 0$, $\Gamma_s^T = \Gamma_s > 0$ and $\gamma_k, \sigma_k, \sigma_{fl}, \sigma_s > 0$.

Eventually, the actual error system can be represented as

$$e(t) = G_a^*(s)[-k(t)e(t) + \Delta\boldsymbol{\rho}_s^T(t) \boldsymbol{\phi}_s(\boldsymbol{s}(t))$$
$$+ \Delta\boldsymbol{\rho}_{fl}^T(t) \bar{\boldsymbol{z}}_{ful}^*(t) + \varepsilon_{hv}(t) - \bar{\varepsilon}_{ful}(t) - \varepsilon_s(t)]. \tag{22}$$

where, $\Delta\boldsymbol{\rho}_s(t) := \boldsymbol{\rho}_s(t) - \boldsymbol{\rho}_s^*$, $\Delta\boldsymbol{\rho}_{fl}(t) := \boldsymbol{\rho}_{fl}(t) - \boldsymbol{\rho}_{fl}^*$.

6. THE STABILITY ANALYSIS

Since $G_a^*(s)$ has a relative degree of 1 due to ASPR-ness, (22) can be transformed to the following canonical form:

$$\begin{aligned}\dot{e}(t) &= -(b_a k^* - a_a)e(t) + b_a\{-\Delta k(t)e(t)\\&+\Delta\boldsymbol{\rho}_s^T(t)\boldsymbol{\phi}_s(\boldsymbol{s}(t)) + \Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t)\\&+\varepsilon_{hv}(t) - \varepsilon_s(t) - \bar{\varepsilon}_{ful}(t)\} + \boldsymbol{c}_\eta^T\boldsymbol{\eta}_a(t)\\\dot{\boldsymbol{\eta}}_a(t) &= A_\eta\boldsymbol{\eta}_a(t) + \boldsymbol{b}_\eta e(t),\end{aligned}$$

with an appropriate constant a_a and a positive constant b_a and an appropriate vector $\boldsymbol{c}_\eta$. Where, $\Delta k(t) := k(t) - k^*$ with an ideal feedback gain k^*. Since $G_a^*(s)$ is minimum-phase, A_η should be a stable matrix, and then it is ensured the existence of symmetric positive definite matrices P_η and Q_η satisfying the following Lyapunov inequality:

$$A_\eta^T P_\eta + P_\eta A_\eta = -Q_\eta < 0.$$

Now, the following Lyapunov function is set:

$$\begin{aligned}V(t) &= e^2(t) + \boldsymbol{\eta}_a^T(t)P_\eta\boldsymbol{\eta}_a(t) + \frac{b_a}{\gamma_k}\Delta k^2(t)\\&+b_a\Delta\boldsymbol{\rho}_s^T(t)\Gamma_s^{-1}\Delta\boldsymbol{\rho}_s(t) + b_a\Delta\boldsymbol{\rho}_{fl}^T(t)\Gamma_{fl}^{-1}\Delta\boldsymbol{\rho}_{fl}(t).\end{aligned}$$

Eventually, the time derivative of $V(t)$ is evaluated as

$$\begin{aligned}\dot{V}(t) &\le -2\left\{b_a\left(k^* - \frac{1}{2}\delta_e\right) - a_a - \frac{1}{2\delta_1} - \frac{1}{2\delta_2}\right\}e^2(t)\\&-(\lambda_{min}[Q_\eta] - \delta_1\|\boldsymbol{c}_\eta\|^2 - \delta_2\|P_\eta\boldsymbol{b}_\eta\|^2)\|\boldsymbol{\eta}_a(t)\|^2\\&-\frac{b_a\sigma_k}{\gamma_k}\Delta k^2(t)\\&-b_a\sigma_{fl}(2\lambda_{min}[\Gamma_{fl}^{-1}] - \delta_3)\|\Delta\boldsymbol{\rho}_{fl}(t)\|^2\\&-b_a\sigma_s(2\lambda_{min}[\Gamma_s^{-1}] - \delta_4)\|\Delta\boldsymbol{\rho}_s(t)\|^2\\&+\frac{b_a\sigma_k}{\gamma_k}k^{*2} + \frac{b_a\sigma_{fl}}{\delta_3}\|\Gamma_{fl}^{-1}\|^2\|\boldsymbol{\rho}_{fl}^*\|^2\\&+\frac{b_a\sigma_s}{\delta_4}\|\Gamma_s^{-1}\|^2\|\boldsymbol{\rho}_s^*\|^2 + \frac{b_a}{\delta_e}\varepsilon^{*2}.\end{aligned}$$

with positive constants δ_1 to δ_4 and δ_e. Here, $|\varepsilon_{hv}(t) - \bar{\varepsilon}_{fl}(t) - \varepsilon_s(t)| \le \varepsilon^*$.

By considering a sufficiently large value of k^*, there exist δ_1 to δ_4 and δ_e such that all coefficients on $e(t)$, $\boldsymbol{\eta}_a(t)$, $\Delta k(t)$, $\Delta\boldsymbol{\rho}_{fl}(t)$, $\Delta\boldsymbol{\rho}_s(t)$ are positive, so these five signals must be bounded.

Next, the boundedness of the signal $\bar{\boldsymbol{z}}_{ful}^*(t)$ is discussed. Before that, we consider $\boldsymbol{z}_{ful}^*(t)$ as follows:

$$\boldsymbol{z}_{ful}^*(t) = \left[\frac{s^{n_f-1}}{f(s)}[y_{fu}^*(t)]\cdots\frac{d_a^*(s)}{f(s)n_a^*(s)}\cdot\frac{n_a^*(s)}{d_a^*(s)}[u(t)]\right]^T.$$

We apply $G_a^{*-1}(s)G_a^*(s)$ to the element related to $u(t)$. Here, $y_{au}^*(t)$ can be obtained as

$$\begin{aligned}y_{au}^*(t) &= G_a^*(s)[-k(t)e(t) + \boldsymbol{\rho}_s^T(t)\boldsymbol{\phi}_s(\boldsymbol{s}(t))\\&+\Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t)] + \boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t)\\&= y_1(t) + \boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t),\end{aligned}$$

where

$$y_1(t) = G_a^*(s)[-k(t)e(t) + \boldsymbol{\rho}_s^T(t)\boldsymbol{\phi}_s(\boldsymbol{s}(t)) + \Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t)].$$

Furthermore, from (22), it can be obtained that

$$\begin{aligned}e(t) &= G_a^*(s)[-k(t)e(t) + \boldsymbol{\rho}_s^T(t)\boldsymbol{\phi}_s(\boldsymbol{s}(t)) + \Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t)]\\&+G_a^*(s)[-\boldsymbol{\rho}_s^{*T}\boldsymbol{\phi}_s(\boldsymbol{s}(t)) + \varepsilon_{hv}(t) - \bar{\varepsilon}_{ful}(t) - \varepsilon_s(t)]\end{aligned}$$

Since $e(t)$ and $-\boldsymbol{\rho}_s^{*T}\boldsymbol{\phi}_s(\boldsymbol{s}(t)) + \varepsilon_{hv}(t) - \bar{\varepsilon}_{ful}(t) - \varepsilon_s(t)$ are bounded and $n_a^*(s)$ is stable, $y_1(t)$ is bounded.

Then, given that $y_{fu}^*(t) = \boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) + \varepsilon_{ful}(t)$ and $y_1(t)$, $\boldsymbol{z}_{ful}^*(t)$ can be also expressed by

$$\begin{aligned}\boldsymbol{z}_{ful}^*(t) &= \begin{bmatrix}C_{fa1} & O\\ O & C_{fa2}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_{fu}^*(t)\\ \boldsymbol{x}_{fh}^*(t)\end{bmatrix} + \begin{bmatrix}\boldsymbol{0}\\ d_{fh}\\ \boldsymbol{0}\end{bmatrix}u(t)\\&= C_{fa}\boldsymbol{x}_{fa}^*(t) + \boldsymbol{d}_{fa}(y_1(t) + \boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t)), \qquad (23)\\\dot{\boldsymbol{x}}_{fa}^*(t) &= \begin{bmatrix}A_f & O\\ O & A_{fh}\end{bmatrix}\boldsymbol{x}_{fa}^*(t) + \boldsymbol{b}_{fa1}(\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) + \varepsilon_{ful}(t))\\&+\boldsymbol{b}_{fa2}(y_1(t) + \boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t))\\&= A_{fa}\boldsymbol{x}_{fa}^*(t) + \boldsymbol{b}_{fa3}\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t)\\&+\boldsymbol{b}_{fa1}\varepsilon_{ful}(t) + \boldsymbol{b}_{fa2}y_1(t), \qquad (24)\end{aligned}$$

with appropriate matrices C_{fa1}, C_{fa2} A_f, A_{fh} and vectors $\boldsymbol{b}_{fa1}$, $\boldsymbol{b}_{fa2}$, $\boldsymbol{b}_{fa3} = \boldsymbol{b}_{fa1} + \boldsymbol{b}_{fa2}$ and scalar $d_{fh} = 1/n_{an_a-1}$; and $\boldsymbol{x}_{fa}^*(t) := [\boldsymbol{x}_{fu}^{*T}(t), \boldsymbol{x}_{fh}^{*T}(t)]^T$.

Since $f(s)$ and $n_a^*(s)$ are given as stable polynomials, A_{fa} should be a stable matrix. Therefore, there exist symmetric positive definite matrices P_{fa} and Q_{fa}, which satisfy the following Lyapunov inequality:

$$A_{fa}^T P_{fa} + P_{fa}A_{fa} = -Q_{fa} < 0.$$

From (23), it can be obtained that

$$\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) = \boldsymbol{\rho}_{fl}^{*T}C_{fa}\boldsymbol{x}_{fa}^*(t) + \frac{b_{n_f-1}}{n_{an_a-1}}(y_1(t) + \boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t))$$

where $\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{d}_{fa} = b_{n_f-1}/n_{an_a-1}$.

Under Assumption 2, it can be obtained that

$$\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) = \frac{n_{an_a-1}}{n_{ab}}\boldsymbol{\rho}_{fl}^{*T}C_{fa}\boldsymbol{x}_{fa}^*(t) + \frac{b_{n_f-1}}{n_{ab}}y_1(t) \qquad (25)$$

where $n_{ab} = n_{an_a-1} - b_{n_f-1}$.

Setting $V_{fa}(t) = \boldsymbol{x}_{fa}^{*T}(t)P_{fa}\boldsymbol{x}_{fa}^*(t)$, $\dot{V}_{fa}(t)$ is

$$\begin{aligned}\dot{V}_{fa}(t) &= -\boldsymbol{x}_{fa}^{*T}(t)Q_{fa}\boldsymbol{x}_{fa}^*(t)\\&+\frac{2n_{an_a-1}}{n_{ab}}\boldsymbol{x}_{fa}^*(t)P_{fa}\boldsymbol{b}_{fa3}\boldsymbol{\rho}_{fl}^{*T}C_{fa}\boldsymbol{x}_{fa}^*(t)\\&+\frac{2b_{n_f-1}}{n_{ab}}\boldsymbol{x}_{fa}^*(t)P_{fa}\boldsymbol{b}_{fa3}y_1(t)\\&+2\boldsymbol{x}_{fa}^{*T}(t)P_{fu}\boldsymbol{b}_{fu1}\varepsilon_{ful}(t) + 2\boldsymbol{x}_{fa}^{*T}(t)P_{fu}\boldsymbol{b}_{fu2}y_1(t)\end{aligned}$$

Here, the following assumption related to $\boldsymbol{\rho}_{fl}^*$ is imposed.

Assumption 3. There exists $\boldsymbol{\rho}_{fl}^*$ which satisfies either of the following three cases:
Case 1: $n_{ab} > 0$, and all eigenvalues of $P_{fa}\boldsymbol{b}_{fa3}\boldsymbol{\rho}_{fl}^{*T}C_{fa}$ is negative or 0.

Case 2: $n_{ab} < 0$, and all eigenvalues of $P_{fa}\boldsymbol{b}_{fa3}\boldsymbol{\rho}_{fl}^{*T}C_{fa}$ is positive or 0.
Case 3: $\|P_{fa}\boldsymbol{b}_{fa3}\boldsymbol{\rho}_{fl}^{*T}C_{fa}\| \ll \lambda_{min}[Q_{fa}]$.

Under Assumption 3, it can be evaluated that

$$\begin{aligned}\dot{V}_{fa}(t) \leq &-(\lambda_{min}[Q_{fa}] - \delta_{fa1}\|P_{fa}\boldsymbol{b}_{fa3}\|^2 - \delta_{fa2}\|P_{fa}\boldsymbol{b}_{fa2}\|^2 \\ &-\delta_{fa3}\|P_{fa}\boldsymbol{b}_{fa1}\|^2])\|\boldsymbol{x}_{fa}^*(t)\|^2 \\ &+\left(\frac{b_{n_f-1}^2}{\delta_{fa1}n_{ab}^2} + \frac{1}{\delta_{fa2}}\right)y_1^2(t) + \frac{1}{\delta_{fa3}}\varepsilon_{ful}^2(t)\end{aligned}$$

with some positive constants δ_{fa1} and δ_{fa3}. By setting $0 < \|Q_{fa}\| < 1$, the coefficient on $\boldsymbol{x}_{fa}^*(t)$ should be positive, so the boundedness of $\boldsymbol{x}_{fa}^*(t)$ can be ensured.

Moreover, under Assumption 2 and from (23), we can obtain that

$$\begin{aligned}(I - \boldsymbol{d}_{fa}\boldsymbol{\rho}_{fl}^{*T})\boldsymbol{z}_{ful}^*(t) &= C_{fa}\boldsymbol{x}_{fa}^*(t) + \boldsymbol{d}_{fa}y_1(t) \\ \boldsymbol{z}_{ful}^*(t) &= E^{-1}C_{fa}\boldsymbol{x}_{fa}^*(t) + E^{-1}\boldsymbol{d}_{fa}y_1(t) \quad (26)\end{aligned}$$

with $E = I - \boldsymbol{d}_{fa}\boldsymbol{\rho}_{fl}^*$. It can be evaluated that

$$\|\boldsymbol{z}_{ful}^*(t)\| \leq c_1\|\boldsymbol{x}_{fa}^*(t)\| + c_2|y_1(t)| + c_3 < +\infty.$$

with positive constants c_1 to c_3.

Finally, the boundedness of $\bar{\boldsymbol{z}}_{ful}^*(t)$ is discussed based on the preliminary. Here, it can be obtained from (14) that

$$\begin{aligned}\bar{\boldsymbol{z}}_{ful}^*(t) &= \frac{s^{n_a} + d_{an_a-1}s^{n_a-1} + \cdots + d_{a1}s + d_{a0}}{n_{an_a-1}s^{n_a-1} + \cdots + n_{a1}s + n_{a0}}[\boldsymbol{z}_{ful}^*(t)] \\ &= \frac{1}{n_{an_a-1}}\dot{\boldsymbol{z}}_{ful}^*(t) + \boldsymbol{y}_{f1}(t) + \boldsymbol{y}_{f2}(t), \quad (27)\end{aligned}$$

where m is an appropriate constant and $m(s)$ is an appropriate polynomial, and since $\boldsymbol{z}_{ful}^*(t)$ is bounded and $n_a^*(s)$ is a stable polynomial, $\boldsymbol{y}_{f1}(t) = m\boldsymbol{z}_{ful}^*(t)$ and $\boldsymbol{y}_{f2}(t) = \dfrac{m(s)}{n_a^*(s)}[\boldsymbol{z}_{ful}^*(t)]$ are bounded.

Here, $\dot{\boldsymbol{z}}_{ful}^*(t)$ can be obtained as

$$\begin{aligned}\dot{\boldsymbol{z}}_{ful}^*(t) &= E^{-1}C_{fa}\dot{\boldsymbol{x}}_{fa}^*(t) + E^{-1}\boldsymbol{d}_{fa}\dot{y}_1(t) \\ &= E^{-1}C_{fa}(A_{fa}\boldsymbol{x}_{fa}^*(t) + \boldsymbol{b}_{fa3}\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) + \boldsymbol{b}_{fa2}y_1(t) \\ &\quad +\boldsymbol{b}_{fa1}\varepsilon_{ful}(t)) + E^{-1}\boldsymbol{d}_{fa}\dot{y}_1(t).\end{aligned}$$

Since $G_a^*(s)$ is an ASPR model, $y_1(t)$ can be obtained as

$$\begin{aligned}\dot{y}_1(t) &= a_a y_1(t) + b_a(-k(t)e(t) + \boldsymbol{\rho}_s^T(t)\boldsymbol{\phi}_s(\boldsymbol{s}(t)) \\ &\quad +\Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t)) + \boldsymbol{c}_\eta^T\boldsymbol{\eta}_1(t) \\ \dot{\boldsymbol{\eta}}_1(t) &= A_\eta\boldsymbol{\eta}_1(t) + \boldsymbol{b}_\eta y_1(t).\end{aligned}$$

Then, $\dot{\boldsymbol{z}}_{ful}^*(t)$ can be also represented as

$$\begin{aligned}\dot{\boldsymbol{z}}_{ful}^*(t) &= E^{-1}C_{fa}(A_{fu}\boldsymbol{x}_{fu}^*(t) + \boldsymbol{b}_{fu3}\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) + \boldsymbol{b}_{fu2}y_1(t) \\ &\quad +\boldsymbol{b}_{fu1}\varepsilon_{ful}(t)) + E^{-1}\boldsymbol{d}_{fa}\{a_a y_1(t) + b_a(-k(t)e(t) \\ &\quad +\boldsymbol{\rho}_s^T(t)\boldsymbol{\phi}_s(\boldsymbol{s}(t)) + \Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t)) + \boldsymbol{c}_\eta^T\boldsymbol{\eta}_1(t)\}.\end{aligned}$$

Setting $V_\eta(t) = \boldsymbol{\eta}_1^T(t)P_\eta\boldsymbol{\eta}_1(t)$, $\dot{V}_\eta(t)$ can be obtained as

$$\dot{V}_\eta(t) \leq -(\lambda_{min}[Q_\eta] - \delta_\eta\|P_\eta\boldsymbol{b}_\eta\|^2)\|\boldsymbol{\eta}_1(t)\|^2 + \frac{1}{\delta_\eta}y_1^2(t).$$

Since $y_1(t)$ is bounded, there exists δ_η that the coefficient on $\boldsymbol{\eta}_1(t)$ is positive, which leads to the fact that $\boldsymbol{\eta}_1(t)$ is bounded.

Defining

$$\begin{aligned}\boldsymbol{z}_r(t) := &E^{-1}C_{fa}(A_{fu}\boldsymbol{x}_{fu}^*(t) + \boldsymbol{b}_{fu1}\varepsilon_{ful}(t) + \boldsymbol{b}_{fu2}y_1(t)) \\ &+E^{-1}\boldsymbol{d}_{fa}\{a_a y_1(t) + b_a(-k(t)e(t) + \boldsymbol{\rho}_s^T(t)\boldsymbol{\phi}_s(\boldsymbol{s}(t))) \\ &+\boldsymbol{c}_\eta^T\boldsymbol{\eta}_1(t)\},\end{aligned}$$

we have that it is a bounded signal. Therefore, it can be obtained that

$$\begin{aligned}\dot{\boldsymbol{z}}_{ful}^*(t) = &E^{-1}C_{fa}\boldsymbol{b}_{fu3}\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) + \boldsymbol{z}_r(t) \\ &+b_a E^{-1}\boldsymbol{d}_{fa}\Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t).\end{aligned}$$

Then, from (27), $\bar{\boldsymbol{z}}_{ful}^*(t)$ can be expressed by

$$\begin{aligned}\bar{\boldsymbol{z}}_{ful}^*(t) = &\frac{1}{n_{an_a-1}}(E^{-1}C_{fa}\boldsymbol{b}_{fu3}\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) + \boldsymbol{z}_r(t) \\ &+b_a E^{-1}\boldsymbol{d}_{fa}\Delta\boldsymbol{\rho}_{fl}^T(t)\bar{\boldsymbol{z}}_{ful}^*(t)) + \boldsymbol{y}_{f1}(t) + \boldsymbol{y}_{f2}(t).\end{aligned}$$

From $b_a = n_{an_a-1}$, setting $F(t) = I = E^{-1}\boldsymbol{d}_{fa}\Delta\boldsymbol{\rho}_{fl}^T(t)$, we can obtain that

$$\begin{aligned}F(t)\bar{\boldsymbol{z}}_{ful}^*(t) = &\frac{1}{n_{an_a-1}}\left\{E^{-1}C_{fa}\boldsymbol{b}_{fu3}\boldsymbol{\rho}_{fl}^{*T}\boldsymbol{z}_{ful}^*(t) + \boldsymbol{z}_r(t)\right\} \\ &+\boldsymbol{y}_{f1}(t) + \boldsymbol{y}_{f2}(t). \quad (28)\end{aligned}$$

Here, the following assumption for the designed PFC parameter is imposed.

Assumption 4. For all operating time, the PFC parameter $b_{n_f-1}(t)$ is designed which satisfies $b_{n_f-1}(t) \neq n_{an_a-1}$.

Remark 2. Under Assumption 2, by setting Γ_{fl}, σ_{fl} appropriately, Assumption 4 can be satisfied.

Under Assumption 4, the inverse matrix $F^{-1}(t)$ exists. Therefore, from (28), we can obtain that

$$\begin{aligned}\|\bar{\boldsymbol{z}}_{ful}^*(t)\| \leq &c_{fa1}\|\boldsymbol{z}_{ful}^*(t)\| + c_{fa2}\|\boldsymbol{y}_{f1}(t)\| \\ &+c_{fa3}\|\boldsymbol{y}_{fa2}(t)\| + c_{fa4}\|\boldsymbol{z}_r(t)\| + c_{fa5}\end{aligned}$$

with positive constants c_{fa1} to c_{fa5}. Since all the signals in the right-hand side are bounded, $\bar{\boldsymbol{z}}_{ful}^*(t)$ is bounded. Hence the following result is obtained.

Theorem 1. All the signals in the control system composed of (16) to (22) are bounded provided Assumptions 2-4 hold.

7. THE CONVERGENCE OF THE ACTUAL ERROR

By setting $\delta_3 = \lambda_{min}[\Gamma_{fl}^{-1}]$, $\delta_4 = \lambda_{min}[\Gamma_s^{-1}]$, $\delta_e = k^*$, R^* can be represented as

$$\begin{aligned}R^* = &\frac{b_a\sigma_k}{\gamma_k}k^{*2} + \frac{b_a}{k^*}\varepsilon^{*2} \\ &+\frac{b_a\sigma_{fl}\|\Gamma_{fl}^{-1}\|^2}{\lambda_{min}[\Gamma_{fl}^{-1}]}\|\boldsymbol{\rho}_{fl}^*\|^2 + \frac{b_a\sigma_s\|\Gamma_s^{-1}\|^2}{\lambda_{min}[\Gamma_s^{-1}]}\|\boldsymbol{\rho}_s^*\|^2.\end{aligned}$$

Then, it can be obtained that

$$\dot{V}(t) \leq -\alpha_1 e^2(t) + R^*. \quad (29)$$

Integrating and taking limit of (29), we can eventually evaluate

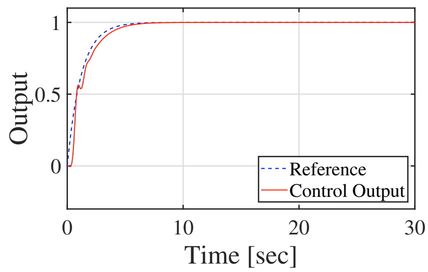

Fig. 3. The reference signal and output

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T e^2(t)dt \le \frac{R^*}{\alpha_1}.$$

Applying the Cauchy-Schwarz inequality, we have that

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T |e(t)|dt \le \sqrt{\frac{R^*}{\alpha_1}}.$$

Here, we set some parameters as follows:

$$\frac{\sigma_k}{\gamma_k} \le k^{*-2},\ \frac{\sigma_{fl}\|\Gamma_{fl}^{-1}\|^2}{\lambda_{min}[\Gamma_{fl}^{-1}]} \le 1,\ \frac{\sigma_s\|\Gamma_s^{-1}\|^2}{\lambda_{min}[\Gamma_s^{-1}]} \le 1, \qquad (30)$$

and then, we can evaluate that

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T |e(t)|dt \le \sqrt{\frac{R^*}{\alpha_1}} \in o(k^{*-1/2}). \qquad (31)$$

This leads to the following result.

***Theorem** 2.* Under the conditions in (30), the mean absolute value of the actual error $e(t)$ satisfies the convergence condition given in (31).

8. THE NUMERICAL SIMULATION

At this time, we set the following 3rd-order non-minimum system having a relative degree of 2:

$$G(s) = \frac{-100s + 1200}{s^3 + 41s^2 + 500s + 2500}. \qquad (32)$$

This model in (32) is definitely a non-ASPR model. Thus we give $G_a^*(s)$ and $f(s)$ as follows:

$$G_a^*(s) = \frac{s + 110}{s^2 + 50s + 100},\ \frac{1}{f(s)} = \frac{1}{s^2 + 24s + 144}.$$

The parameters are set as

$$\gamma_k = 50,\ \sigma_k = 10^{-5},\ \Gamma_s = 10,\ \sigma_s = 10^{-4}$$
$$\Gamma_{fl} = \text{diag}[10,\ 10,\ 1,\ 10],\ \sigma_{fl} = 10^{-6}.$$

The initial values of the parameters are given by

$$k(0) = 0, \rho_s(0) = 0, \boldsymbol{\rho}_{fl}(0) = [1, 1, 0.5, 1]^T.$$

The result is shown in Fig. 3, which shows the reference signal and output. The system output tracks the step signal quite accurately.

9. CONCLUSION

In this paper, a new ASPR-based output feedback control method was proposed with a virtual PFC model. In addition, introducing an RBF NN-based feedforward input, we obtained output tracking with respect to the actual output. Finally, the stability of the obtained control system and the convergence of the actual error were ensured with some assumptions. In the future, we would analyze how to set design parameters for Assumption 4.

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