Entangled States, EPR Problem -Teleportation and Quantum Communication-

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Up to now there seem to be no phenomenon which contradicts quantum theory. Moreover, recently, new applications of quantum mechanics are expected in an area known as quantum communication, cryptography and computation. In the quantum information processing of these applications, the EPR correlation arisen from the maximally entangled states among constituent particles acts an essential and indispensable role. Therefore, thorough understanding of these maximally entangled states is necessary both for the establishment of the foundation of quantum mechanics and for these applications. In this talk, we want to report our recent investigations on the EPR problem, teleportation and quantum communication with their experimental implementations.

§ 1. Two Spin 1/2 Particles (EPR Problem)

Singlet, Bell State, Entanglement

$$|\Psi^{(-)}\rangle_{12} = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\Big), \tag{1}$$

Ket, Neumann Projection

⇓

State of 2 Changes after the measurement in 1

⇓

EPR paradox, Incomplete Measurement

Statistical Operator :

Apparatus,
Composite System and its Subsystem,
Measuring Process

$$\hat{\rho}_{12} = \frac{1}{2}\Big(|\uparrow\rangle_1\langle\uparrow| \otimes |\downarrow\rangle_2\langle\downarrow| - |\uparrow\rangle_1\langle\downarrow| \otimes |\downarrow\rangle_2\langle\uparrow| - |\downarrow\rangle_1\langle\uparrow| \otimes |\uparrow\rangle_2\langle\downarrow| + |\downarrow\rangle_1\langle\downarrow| \otimes |\uparrow\rangle_2\langle\uparrow|\Big). \tag{2}$$

Decoherence

$$\hat{\rho}_{12} \to \hat{\rho}_{12,\infty} = \frac{1}{2}\Big(|\uparrow\rangle_1\langle\uparrow| \otimes |\downarrow\rangle_2\langle\downarrow| + |\downarrow\rangle_1\langle\downarrow| \otimes |\uparrow\rangle_2\langle\uparrow|\Big). \tag{3}$$

Expectation Value

$$\langle \hat{S}_z^{(1)} \otimes \hat{\mathbf{1}}^{(2)} \rangle = \mathrm{Tr}^{(12)}(\hat{\rho}_{12} \cdot \hat{S}_z^{(1)} \otimes \hat{\mathbf{1}}^{(2)}) = \mathrm{Tr}^{(1)}\{\mathrm{Tr}^{(2)}(\hat{\rho}_{12}) \cdot \hat{S}_z^{(1)}\} = \mathrm{Tr}^{(1)}(\hat{\rho}_1 \cdot \hat{S}_z^{(1)}), \tag{4}$$

States of Subsystems before Measurement

$$\hat{\rho}_1 = \mathrm{Tr}^{(2)}(\hat{\rho}_{12}) = \frac{1}{2}(|\uparrow\rangle_1\langle\uparrow| + |\downarrow\rangle_1\langle\downarrow|), \ \hat{\rho}_2 = \mathrm{Tr}^{(1)}(\hat{\rho}_{12}) = \frac{1}{2}(|\downarrow\rangle_2\langle\downarrow| + |\uparrow\rangle_2\langle\uparrow|). \tag{5}$$

State of 2 does not Change before and after the measurement in 1

$$\begin{cases} \hat{\rho}_1 \to \hat{\rho}_{1,\infty} = \mathrm{Tr}^{(2)}(\hat{\rho}_{12,\infty}) = \hat{\rho}_1, \\ \hat{\rho}_2 \to \hat{\rho}_{2,\infty} = \mathrm{Tr}^{(1)}(\hat{\rho}_{12,\infty}) = \hat{\rho}_2. \end{cases} \tag{6}$$

Probabilistic Exclusive Event

$$\hat{\rho}_{1,\infty} \longrightarrow |\uparrow\rangle_1\langle\uparrow| \quad or \quad |\downarrow\rangle_1\langle\downarrow| \tag{7}$$

Law of Conservation of the Spin Angular Momentum

Statistical Operator of the Subsystem 2 for Successive Measurement

$$\begin{cases} \hat{\rho}_{2,\infty}(1, up) = \mathrm{Tr}^{(1)}(|\uparrow\rangle_1\langle\uparrow|\hat{\rho}_{12,\infty}) = \frac{1}{2}|\downarrow\rangle_2\langle\downarrow|, \\ \hat{\rho}_{2,\infty}(1, down) = \mathrm{Tr}^{(1)}(|\downarrow\rangle_1\langle\downarrow|\hat{\rho}_{12,\infty}) = \frac{1}{2}|\uparrow\rangle_2\langle\uparrow|. \end{cases} \tag{8}$$

$|\Psi^{(\pm)}\rangle_{12}$:two combinations $\uparrow_1\downarrow_2$, $\downarrow_1\uparrow_2$

$$|\Psi^{(+)}\rangle_{12} = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2\Big), \; |\Phi^{(\pm)}\rangle_{12} = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_1|\uparrow\rangle_2 \pm |\downarrow\rangle_1|\downarrow\rangle_2\Big). \tag{9}$$

$|\Phi^{(\pm)}\rangle_{12}$: two combinations $\uparrow_1\uparrow_2$, $\downarrow_1\downarrow_2$

Superposition of the Maximally Entangled Bell States

$a|\Phi^{(\pm)}\rangle_{12} \pm b|\Psi^{(\pm)}\rangle_{12}$

$$|\;\rangle_{12} = a|\Phi^{(+)}\rangle_{12} + b|\Psi^{(+)}\rangle_{12}, \quad |a|^2 + |b|^2 = 1, \tag{10}$$

$$\begin{aligned} |\;\rangle_{12} &= \frac{1}{\sqrt{2}}\{|\uparrow\rangle_1(a|\uparrow\rangle_2 + b|\downarrow\rangle_2) + |\downarrow\rangle_1(b|\uparrow\rangle_2 + a|\downarrow\rangle_2)\} \\ &= \frac{1}{\sqrt{2}}\{(a|\uparrow\rangle_1 + b|\downarrow\rangle_1)|\uparrow\rangle_2 + (b|\uparrow\rangle_1 + a|\downarrow\rangle_1)|\downarrow\rangle_2\}, \end{aligned} \tag{11}$$

$a = b = 1/\sqrt{2}$: factorizable

$\hat{\rho}_{12} = |\;\rangle_{12}\langle\;|$

$$\hat{\rho}_i = \mathrm{Tr}^{(j)}(\hat{\rho}_{12}) = \frac{1}{2}\{|\uparrow\rangle_i\langle\uparrow| + (ab^* + a^*b)(|\uparrow\rangle_i\langle\downarrow| + |\downarrow\rangle_i\langle\uparrow|) + |\downarrow\rangle_i\langle\downarrow|\}, \tag{12}$$

$i, j = 1, 2$ and $i \neq j$

Measurement of $S_z^{(1)}$

$$\begin{cases} \hat{\rho}_1 \to \hat{\rho}_{1,\infty} = \mathrm{Tr}^{(2)}(\hat{\rho}_{12,\infty}) = \frac{1}{2}(|\uparrow\rangle_1\langle\uparrow| + |\downarrow\rangle_1\langle\downarrow|), \\ \hat{\rho}_2 \to \hat{\rho}_{2,\infty} = \mathrm{Tr}^{(1)}(\hat{\rho}_{12,\infty}) = \hat{\rho}_2, \end{cases} \tag{13}$$

$$\begin{aligned} \hat{\rho}_{2,\infty}(1, up) &= \mathrm{Tr}^{(1)}(|\uparrow\rangle_1\langle\uparrow|\hat{\rho}_{12,\infty}) \\ &= \frac{1}{2}\{|a|^2|\uparrow\rangle_2\langle\uparrow| + ab^*|\uparrow\rangle_2\langle\downarrow| + a^*b|\downarrow\rangle_2\langle\uparrow| + |b|^2|\downarrow\rangle_2\langle\downarrow|\}, \end{aligned} \tag{14}$$

for (1, *up*)

Measurement of $S_z^{(2)}$

$$\hat{\rho}_{2,\infty}(1,up)_{,\infty} = \frac{1}{2}(|a|^2|\uparrow\rangle_2\langle\uparrow| + |b|^2|\downarrow\rangle_2\langle\downarrow|). \tag{15}$$

Four combinations: $\uparrow_1\uparrow_2$, $\uparrow_1\downarrow_2$, $\downarrow_1\uparrow_2$, $\downarrow_1\downarrow_2$

Probabilities: $|a|^2/4$, $|b|^2/4$, $|b|^2/4$, $|a|^2/4$

A new Method for Quantum Communication by two two-State Systems

§ 2. Three Spin 1/2 Particles (Teleportation)

Bennett's Method

$$\begin{aligned}|\Psi\rangle_{123} &= |\phi\rangle_1|\Psi^{(-)}\rangle_{23} \\ &= \frac{1}{2}\Big\{-|\Psi^{(-)}\rangle_{12}|\phi\rangle_3 + |\Psi^{(+)}\rangle_{12}\big(-a|\uparrow\rangle_3 + b|\downarrow\rangle_3\big) \\ &\quad + |\Phi^{(-)}\rangle_{12}\big(a|\downarrow\rangle_3 + b|\uparrow\rangle_3\big) + |\Phi^{(+)}\rangle_{12}\big(a|\downarrow\rangle_3 - b|\uparrow\rangle_3\big)\Big\},\end{aligned} \tag{16}$$

Qubit(unknown state), Bell States

$$\begin{cases} |\phi\rangle = a|\uparrow\rangle + b|\downarrow\rangle,\ |a|^2+|b|^2 = 1\ , \\ |\Psi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle|\downarrow\rangle \pm |\downarrow\rangle|\uparrow\rangle\big)\ , \\ |\Phi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle|\uparrow\rangle \pm |\downarrow\rangle|\downarrow\rangle\big) \equiv \frac{1}{\sqrt{2}}\big(|1,1\rangle \pm |1,-1\rangle\big)\ , \end{cases} \tag{17}$$

Motoyoshi's Method

The Most General State of This System

$$\begin{aligned}|\Psi\rangle_{123} &= |\uparrow\rangle_1|\uparrow\rangle_2\big(c_1|\uparrow\rangle_3 + c_2|\downarrow\rangle_3\big) + |\uparrow\rangle_1|\downarrow\rangle_2\big(c_3|\uparrow\rangle_3 + c_4|\downarrow\rangle_3\big) \\ &\quad + |\downarrow\rangle_1|\uparrow\rangle_2\big(c_5|\uparrow\rangle_3 + c_6|\downarrow\rangle_3\big) + |\downarrow\rangle_1|\downarrow\rangle_2\big(c_7|\uparrow\rangle_3 + c_8|\downarrow\rangle_3\big) \\ &= \big(c_1|\uparrow\rangle_1 + c_5|\downarrow\rangle_1\big)|\uparrow\rangle_2|\uparrow\rangle_3 + \big(c_2|\uparrow\rangle_1 + c_6|\downarrow\rangle_1\big)|\uparrow\rangle_2|\downarrow\rangle_3 \\ &\quad + \big(c_3|\uparrow\rangle_1 + c_7|\downarrow\rangle_1\big)|\downarrow\rangle_2|\uparrow\rangle_3 + \big(c_4|\uparrow\rangle_1 + c_8|\downarrow\rangle_1\big)|\downarrow\rangle_2|\downarrow\rangle_3\ ,\end{aligned} \tag{18}$$

Swapping

$$\begin{aligned}|\Psi\rangle_{123} &= \big(c_1|\uparrow\rangle_1 + c_2|\downarrow\rangle_1\big)|1,1\rangle_{23} + \sqrt{2}|\phi'\rangle_1|\Psi^{(+)}\rangle_{23} + \big(c_7|\uparrow\rangle_1 + c_8|\downarrow\rangle_1\big)|1,-1\rangle_{23} \\ &= |1,1\rangle_{12}\big(c_1|\uparrow\rangle_3 + c_2|\downarrow\rangle_3\big) + \sqrt{2}|\Psi^{(+)}\rangle_{12}|\phi'\rangle_3 + |1,-1\rangle_{12}\big(c_7|\uparrow\rangle_3 + c_8|\downarrow\rangle_3\big),\end{aligned} \tag{19}$$

$$\begin{cases} |\phi'\rangle = c_2|\uparrow\rangle + c_7|\downarrow\rangle, \\ |\Psi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle|\downarrow\rangle \pm |\downarrow\rangle|\uparrow\rangle\big)\ , \\ |\uparrow\rangle|\uparrow\rangle = |1,1\rangle, |\downarrow\rangle|\downarrow\rangle = |1,-1\rangle. \end{cases} \tag{20}$$

Othogonality

$$c_1 = -e^{2i\alpha}c_7{}^*\ ,\ c_8 = -e^{2i\beta}c_2{}^*\ , \tag{21}$$

$$
\begin{aligned}
|\Psi\rangle_{123} &= \frac{1}{2}\Big\{|\psi'\rangle_1\Big(e^{2i\alpha}|1,1\rangle_{23} - e^{2i\beta}|1,-1\rangle_{23}\Big) + \sqrt{2}|\phi'\rangle_1|\Psi^{(+)}\rangle_{23}\Big\} \\
&= \frac{1}{2}\Big\{\Big(e^{2i\alpha}|1,1\rangle_{12} - e^{2i\beta}|1,-1\rangle_{12}\Big)|\psi'\rangle_3 + \sqrt{2}|\Psi^{(+)}\rangle_{12}|\phi'\rangle_3\Big\} . \qquad (22)
\end{aligned}
$$

Mixture

$$
\hat{\rho}_{1,23} = \frac{1}{4}\Big\{|\psi\rangle_1\langle\psi| \otimes |1,1\rangle_{23}\langle 1,1| + |\psi\rangle_1\langle\psi| \otimes |1,-1\rangle_{23}\langle 1,-1| + 2|\phi\rangle_1\langle\phi| \otimes |\Psi^{(+)}\rangle_{23}\langle\Psi^{(+)}|\Big\} . \qquad (23)
$$

Qubit and its Othogonal State

$$
|\phi\rangle = a|\uparrow\rangle + b|\downarrow\rangle \quad , \quad |\psi\rangle = -b^*|\uparrow\rangle + a^*|\downarrow\rangle \qquad (24)
$$

States of Subsystems

$$
\begin{cases}
\hat{\rho}_1 = \mathrm{Tr}^{(23)}(\hat{\rho}_{1,23}) = \frac{1}{2}\Big\{|\psi\rangle_1\langle\psi| + |\phi\rangle_1\langle\phi|\Big\}, \\
\hat{\rho}_{23} = \mathrm{Tr}^{(1)}(\hat{\rho}_{1,23}) = \frac{1}{4}\Big\{|1,1\rangle_{23}\langle 1,1| + |1,-1\rangle_{23}\langle 1,-1| + 2|\Psi^{(+)}\rangle_{23}\langle\Psi^{(+)}|\Big\} .
\end{cases} \qquad (25)
$$

Recomposition of the Subsystems 1,23 $\longrightarrow$ 12,3

$$
\begin{cases}
\hat{\rho}_3 = \mathrm{Tr}^{(12)}(\hat{\rho}_{12,3}) = \frac{1}{2}\Big\{|\psi\rangle_3\langle\psi| + |\phi\rangle_3\langle\phi|\Big\}, \\
\hat{\rho}_{12} = \mathrm{Tr}^{(3)}(\hat{\rho}_{12,3}) = \frac{1}{4}\Big\{|1,1\rangle_{12}\langle 1,1| + |1,-1\rangle_{12}\langle 1,-1| \\
\qquad + |\Psi^{(+)}\rangle_{12}\langle\Psi^{(+)}| + |\Psi^{(-)}\rangle_{12}\langle\Psi^{(-)}|\Big\}.
\end{cases} \qquad (26)
$$

Measurement in the Subsystem 12 of 12,3

$$
\hat{\rho}_{12,3} \rightarrow \hat{\rho}_{12,3,\infty} \qquad (27)
$$

$$
\begin{cases}
\hat{\rho}_{3,\infty} = \mathrm{Tr}^{(12)}(\hat{\rho}_{12,3,\infty}) = \hat{\rho}_3 , \\
\hat{\rho}_{12,\infty} = \mathrm{Tr}^{(3)}(\hat{\rho}_{12,3,\infty}) = \hat{\rho}_{12} .
\end{cases} \qquad (28)
$$

State of 3 does not Change $\longrightarrow$ Teleportation (Law of Conservation)

§ 3. Three Spin 1/2 Particles (Quantum Communication)

The Polarization of Photons

$$
\hat{\rho}_{1,23} = \frac{1}{4}\begin{bmatrix}
\begin{pmatrix} |b|^2 & -a^*b \\ -ab^* & |a|^2 \end{pmatrix} & 0 & 0 \\
0 & \begin{pmatrix} |a|^2 & a^*b & |a|^2 & a^*b \\ ab^* & |b|^2 & ab^* & |b|^2 \\ |a|^2 & a^*b & |a|^2 & a^*b \\ ab^* & |b|^2 & ab^* & |b|^2 \end{pmatrix} & 0 \\
0 & 0 & \begin{pmatrix} |b|^2 & -a^*b \\ -ab^* & |a|^2 \end{pmatrix}
\end{bmatrix} . \qquad (29)
$$

Recomposition of the Subsystems 1,23 $\longrightarrow$ 12,3

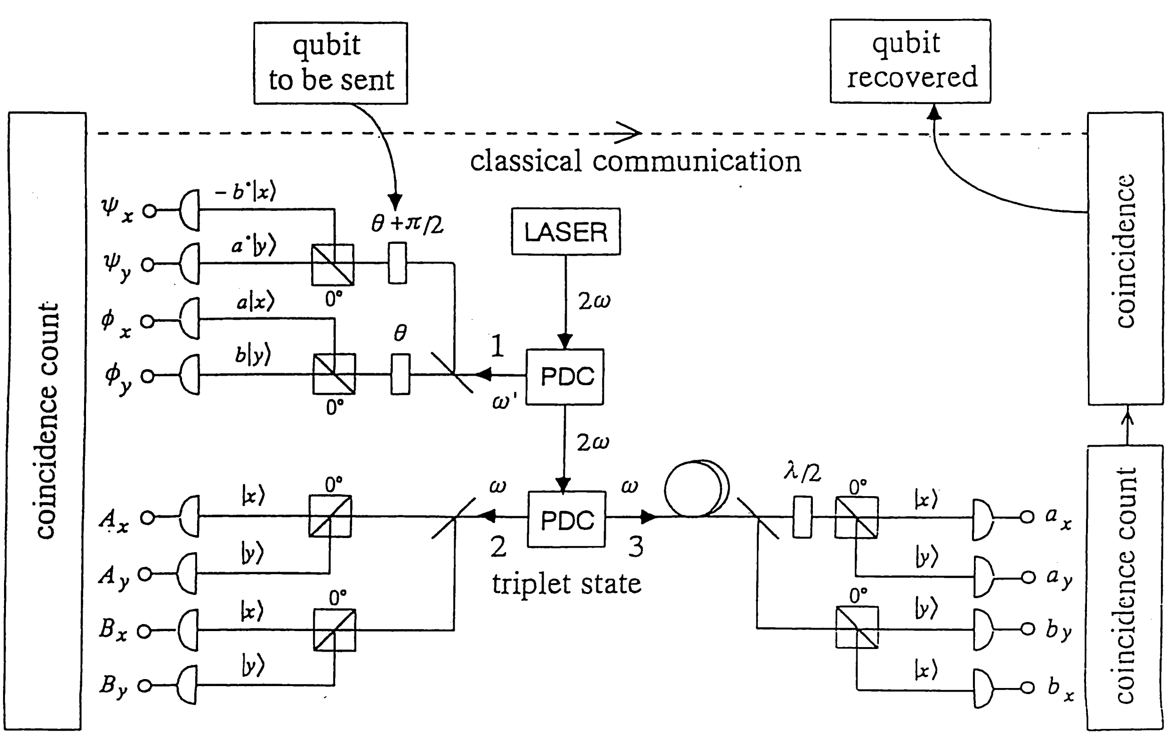
$$\Downarrow$$

$$\hat{\rho}_{12,3} = \frac{1}{4}\begin{bmatrix} |b|^2 & 0 & 0 & -a^*b & 0 & 0 \\ 0 & \begin{pmatrix} |a|^2 & |a|^2 \\ |a|^2 & |a|^2 \end{pmatrix} & 0 & 0 & \begin{pmatrix} a^*b & a^*b \\ a^*b & a^*b \end{pmatrix} & 0 \\ 0 & 0 & |b|^2 & 0 & 0 & -a^*b \\ -ab^* & 0 & 0 & |a|^2 & 0 & 0 \\ 0 & \begin{pmatrix} ab^* & ab^* \\ ab^* & ab^* \end{pmatrix} & 0 & 0 & \begin{pmatrix} |b|^2 & |b|^2 \\ |b|^2 & |b|^2 \end{pmatrix} & 0 \\ 0 & 0 & -ab^* & 0 & 0 & |a|^2 \end{bmatrix}. \quad (30)$$

Coincidence Counting of 12

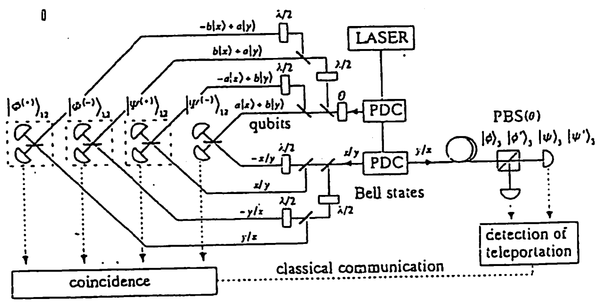
$\Downarrow$

Information of a, b will be appeared in 3



§ 4. Symmetric Teleportation in Time Reversal

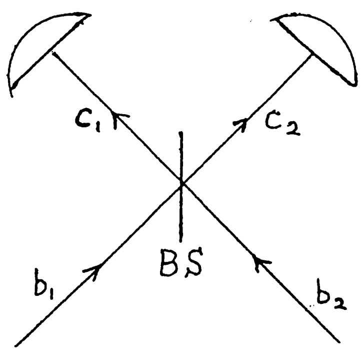
$$-\frac{1}{2}\Big\{|\phi\rangle_1|\Psi^{(-)}\rangle_{23}+\sigma_z|\phi\rangle_1|\Psi^{(+)}\rangle_{23}-\sigma_x|\phi\rangle_1|\Phi^{(-)}\rangle_{23}+i\sigma_y|\phi\rangle_1|\Phi^{(+)}\rangle_{23}\Big\} \longrightarrow |\Psi^{(-)}\rangle_{12}|\phi\rangle_3, \quad (31)$$



§ 5. Method for the Photon Counting

Incoming Single-Photon Modes: $\hat{b}_1^\dagger$, $\hat{b}_2^\dagger$
50:50 beam splitter (BS)
Outgoing Modes: $\hat{c}_1^\dagger$, $\hat{c}_2^\dagger$

$$\hat{c}_{1j}^\dagger = \frac{1}{\sqrt{2}}\left(\hat{b}_{1j}^\dagger + \hat{b}_{2j}^\dagger\right), \quad \hat{c}_{2j}^\dagger = \frac{1}{\sqrt{2}}\left(\hat{b}_{1j}^\dagger - \hat{b}_{2j}^\dagger\right), \quad (32)$$



$$\hat{c}_{1j}^\dagger\hat{c}_{2k}^\dagger = \frac{1}{2}(\hat{b}_{1j}^\dagger\hat{b}_{1k}^\dagger - \hat{b}_{1j}^\dagger\hat{b}_{2k}^\dagger + \hat{b}_{2j}^\dagger\hat{b}_{1k}^\dagger - \hat{b}_{2j}^\dagger\hat{b}_{2k}^\dagger) \quad (33)$$

$$\hat{c}_{1j}^\dagger\hat{c}_{2k}^\dagger|0\rangle = \begin{cases} \pm\frac{1}{\sqrt{2}}|\Psi^{(-)}\rangle_{12}, & (j \neq k) \\ 0, & (j = k) \end{cases} \quad (34)$$

$$\hat{c}_{1j}^\dagger\hat{c}_{1k}^\dagger|0\rangle \;\; or \;\; \hat{c}_{2j}^\dagger\hat{c}_{2k}^\dagger|0\rangle = \begin{cases} \frac{1}{\sqrt{2}}|\Psi^{(+)}\rangle_{12} & (j \neq k) \\ \frac{1}{\sqrt{2}}(|\Phi^{(+)}\rangle_{12} + |\Phi^{(-)}\rangle_{12}) & (j = k = \uparrow) \\ \frac{1}{\sqrt{2}}(|\Phi^{(+)}\rangle_{12} - |\Phi^{(-)}\rangle_{12}) & (j = k = \downarrow) \end{cases} \quad (35)$$

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