The Influence of the Viscosity Prescription on Self-Gravitating Accretion Disk

Takashi KAI[1], Tamon BABA[1], Kenzo ARAI[1] and Shin-ichiro FUJIMOTO[2]

[1] *Department of Physics, Kumamoto University, Kumamoto 860-8555*
[2] *Kumamoto National College of Technology, Kumamoto 861-1102*

(Received September 30, 2010)

We investigate the difference between the accretion disk models based on two kinds of α-prescription of viscosity. The models are constructed with self-gravity in the vertical direction of the disk around a supermassive black hole. It is found that the structures are completely different in the self-gravity dominant region, though they are almost the same for the standard disk model. We should be careful in treating viscosity, if self-gravity is taken into account for a disk model of an emitting region of H_2O masers in an active galactic nucleus (AGN).

§1. Introduction

Luminous H_2O masers have been observed in many active galactic nuclei (AGN). The maser spots in NGC 4258 reveal[1] that they rotate with Keplerian velocity around a central black hole of mass $3.9 \times 10^7 M_{\odot}$, they lie in the range 0.14 – 0.28 pc from the center, which is an outer region of the disk, and the thickness of the disk is less than 0.003 pc. These facts indicate the existence of a thin, accretion disk around the black hole. Although the standard accretion disk model[2] is of great success in explaining many observed properties of the disk, self-gravity of the disk could be crucial in constructing a disk model of the emitting region of H_2O masers.[3]

A key ingredient of the disk model is viscosity. Due to our ignorance on the viscosity coefficient, we usually adopt α-prescription. There have been proposed two approaches so far. One is the kinematic viscosity coeficient given by

$$\nu = \alpha c_s H \qquad (0 < \alpha \leq 1), \tag{1.1}$$

where c_s is the sound speed, H is the half thickness of the disk and α is the viscosity parameter. The other is the $r\phi$-component of the shear stress tensor given by

$$t_{r\phi} = -\alpha P, \tag{1.2}$$

where P is the gas pressure.

These two kinds of prescription yield the same consequences for the standard disk model. It is, however, not clear whether they are the same for a self-gravitating disk. As previously stated, the maser spots are in Keplerian motion. Then the self-gravity should be taken into account only in the vertical direction of the disk.

In the present paper we investigate the influence of the two kinds of α-prescription (1.1) and (1.2) on the structure of the self-gravity dominant region of the accretion disk around a supermassive black hole.

§2. Accretion disk model

2.1. *Basic Equations*

We construct an accretion disk model for the emitting region of H_2O masers in AGN. The disk is assumed to be steady and axisymmetric. We adopt cylindrical coordinates (r, ϕ, z).

The continuity equation integrated along the z-direction is

$$\dot{M} = 4\pi r \rho H |v_r|, \tag{2.1}$$

where $\dot{M}$ is the mass accretion rate, ρ the gas density and v_r the radial velocity which is negative for accretion.

The maser spots rotate around a central black hole of mass M with the Keplerian angular velocity given by

$$\Omega = \Omega_{\mathrm{K}} = \sqrt{\frac{GM}{r^3}}, \tag{2.2}$$

where G is the gravitational constant.

The angular momentum transfer is written, in the outer region of the disk, as

$$\nu \rho H = \frac{1}{6\pi}\dot{M}. \tag{2.3}$$

When the self-gravity is incorporated with a one-zone model, hydrostatic equilibrium in the z-direction leads to

$$H = \frac{c_s}{\sqrt{\Omega_{\mathrm{K}}^2 + 4\pi G\rho}}, \tag{2.4}$$

where $c_s = \sqrt{P/\rho}$.

In the outer region of the disk, the equation of state is given by

$$P = \frac{\rho k_{\mathrm{B}} T}{\mu m_{\mathrm{H}}} \tag{2.5}$$

where k_{B} is the Boltzmann constant, T the temperature, μ the mean molecular weight and m_{H} the mass of a hydrogen atom.

Energy generated through the viscous process is released as blackbody radiation at the disk surface. Thus the energy balance is written as

$$Q_{\mathrm{vis}}^{+} = Q_{\mathrm{rad}}^{-}. \tag{2.6}$$

Here

$$Q_{\mathrm{vis}}^{+} = \frac{9}{2}\nu\rho H \Omega_{\mathrm{K}}^2, \tag{2.7}$$

$$Q_{\mathrm{rad}}^{-} = \frac{8acT^4}{3\kappa\rho H}, \tag{2.8}$$

where a is the radiation energy constant and c is the speed of light. We adopt the opacity κ evaluated[4] through transitions in molecules and dust grains at low temperatures. It should be noted that this is not the case where the Kramers' opacity is usually extrapolated in the standard disk model.[2]

2.2. *Viscosity*

It is believed that turbulent viscosity is effective for accretion of material in a disk. The kinematic viscosity is given by the product of the size $l_{\rm turb}$ and turnover velocity $v_{\rm turb}$ of the largest turbulent eddies:

$$\nu_{\rm turb} \sim l_{\rm turb} v_{\rm turb}. \tag{2.9}$$

When we put

$$l_{\rm turb} \le H, \quad v_{\rm turb} \le c_{\rm s} \tag{2.10}$$

then we obtain

$$\nu = \alpha c_{\rm s} H, \qquad (0 < \alpha \le 1)$$

which is the first prescription (1.1).

Using Eq. (2.2), the $r\phi$-component of the shear stress tensor is written as

$$t_{r\phi} = \rho\nu r \frac{d\Omega}{dr} = -\frac{3}{2}\rho\nu\Omega_{\rm K}. \tag{2.11}$$

It follows from the second prescription (1.2) that

$$\nu = \frac{2}{3}\alpha\frac{c_{\rm s}^2}{\Omega_{\rm K}}. \tag{2.12}$$

If we neglect the self-gravity, as in the standard disk model, from Eq. (2.4) we have $c_{\rm s} = \Omega_{\rm K} H$. Therefore, we obtain

$$\nu = \frac{2}{3}\alpha c_{\rm s} H, \tag{2.13}$$

which is identical to Eq. (1.1) within a factor 2/3.

The above results do not hold, however, in a self-gravitating disk. To distinguish these expressions in our disk model, we introduce two kinds of viscosity:

$$\nu_1 = \alpha c_{\rm s} H, \tag{2.14}$$

$$\nu_2 = \frac{2}{3}\alpha\frac{c_{\rm s}^2}{\Omega_{\rm K}}. \tag{2.15}$$

§3. Results and discussion

Our disk model is specified by three parameters M, $\dot{M}$ and α. We set $M = 3.9 \times 10^7 M_\odot$, which is the mass of the central black hole in NGC 4258.[1] We introduce $\dot{m} = \dot{M}/\dot{M}_{\rm Edd}$, where $\dot{M}_{\rm Edd} = 1.4 \times 10^{17} m$ g s^{-1} is the Eddington accretion rate. Also we set $\dot{m} = 10^{-3}$ and $\alpha = 0.1$. We numerically solve Eqs. (2.1) – (2.8) with (2.14) or (2.15) to obtain ρ, T, v_r and H for the outer region $r = 10^{16} - 10^{18}$ cm. In this range we adopt $\mu = 2.3$ because gas is composed mainly of H_2 molecules.

Figure 1 shows the self-gravity of the disk and the z-component of the gravity of the central black hole. The solid line indicates the self-gravity with ν_1 and the dotted one the self-gravity with ν_2. The dashed line is the central gravity with ν_1

and the dot-dashed one the central gravity with ν_2. Two vertical lines denote the inner and outer boundaries of the emitting region, $r = (4.3 - 8.6) \times 10^{17}$ cm, of H_2O masers. It can be seen that the self-gravity becomes dominant over the central gravity at $r \geq 4 \times 10^{16}$ cm for both kinds of α-prescription and the influence is more prominent for the viscosity ν_1.

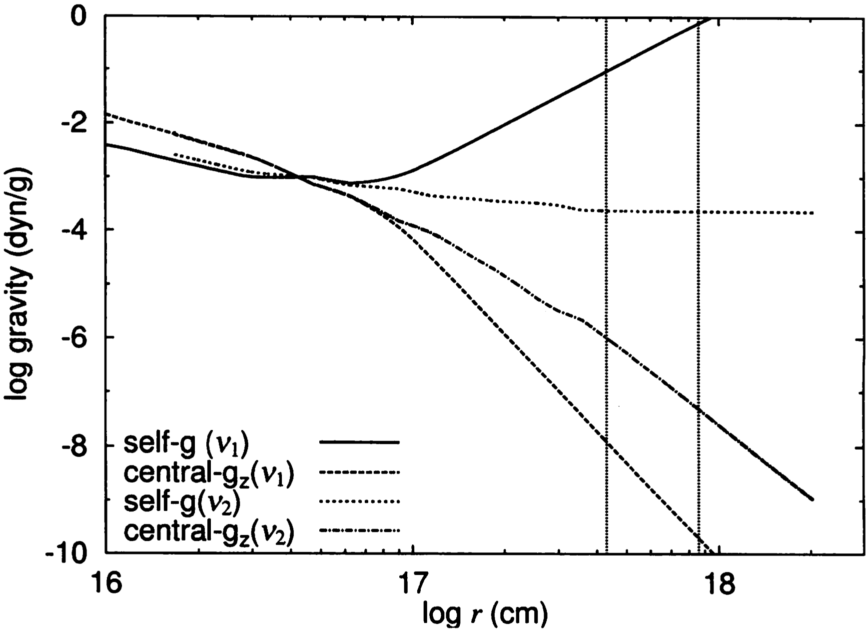

Fig. 1. Self-gravity of the disk and the z-component of the gravity of the central black hole. Two vertical lines denote the inner and outer boundaries of the maser emitting region.

We examine the disk structure of the following cases.

Case A: self-gravitating disk with ν_1.

Case B: self-gravitating disk with ν_2.

Case C: disk with neglecting self-gravity.

Case D: standard disk.

It should be noted that Case C contains the opacity through molecules and dust grains but Case D extrapolates the Kramers' opacity unreasonably to low temperatures. As seen from Fig. 1, the self-gravity cannot be neglected in the outer region of the disk. Then Cases C and D are not valid in the maser emitting region. These are the cases only for comparison.

Figure 2 shows the density profile in the disk. In Case A, denoted by the solid line, density increases drastically outwards such that $\rho \sim r^6$ in the region of our interest. This is caused by the strong self-gravity but weak viscosity. Gas accumulates in outer parts of the disk. The disk becomes gravitationally unstable to be broken up into many clumps. On the other hand, density in Case B, indicated by the dashed line, is nearly constant and slightly increases outwards. Small undulations of the curve are originated from the weak temperature dependence of the opacity around 300 K.

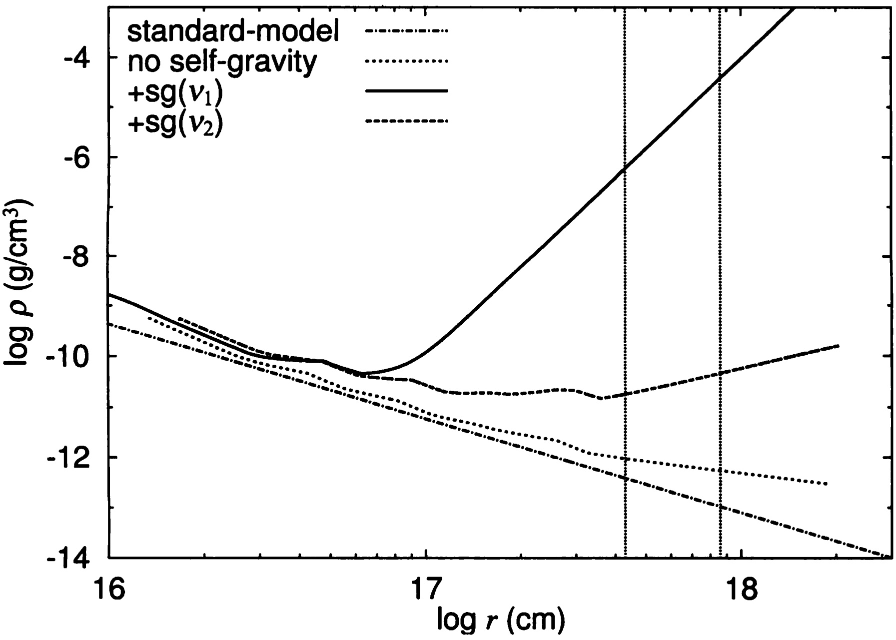

Fig. 2. Density profile of the disk in Case A (solid line), Case B (dashed line), Case C (dotted line) and Case D (dot-dashed line). Two vertical lines denote the inner and outer boundaries of the maser emitting region.

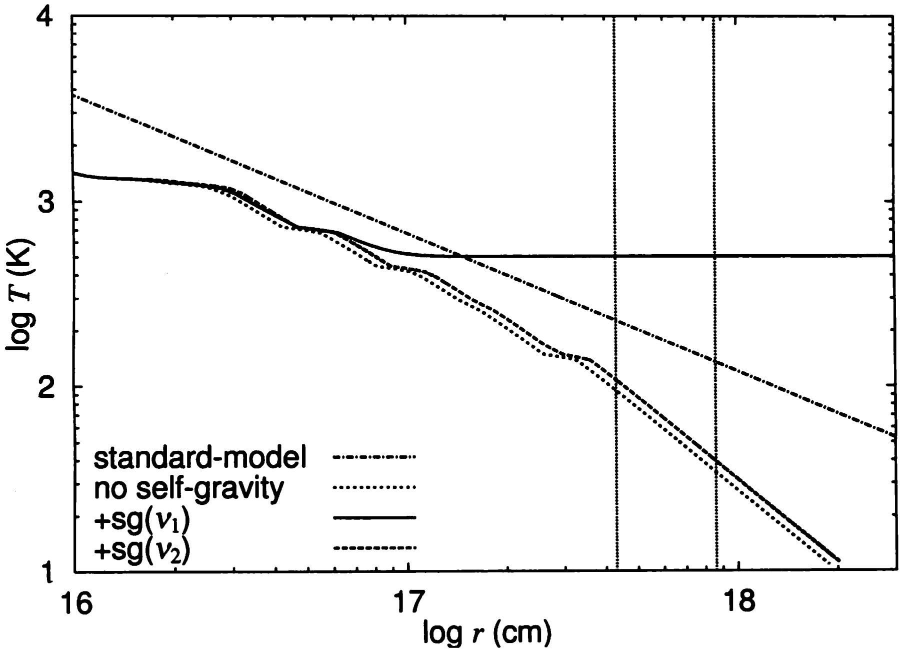

Fig. 3. Temperature in the disk. Notations are the same as in Fig. 2.

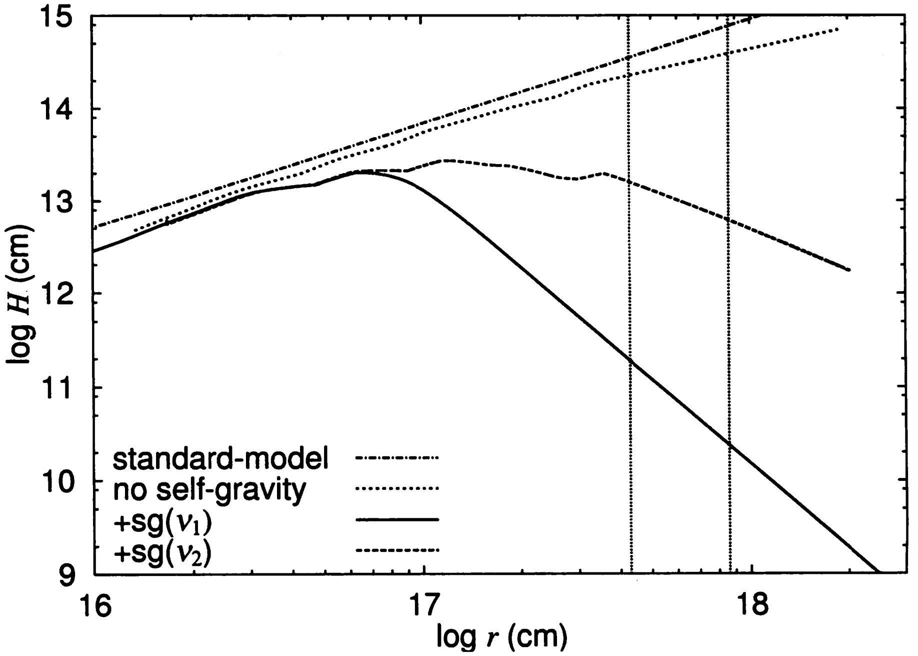

Fig. 4. Half thickness of the disk. Notations are the same as in Fig. 2.

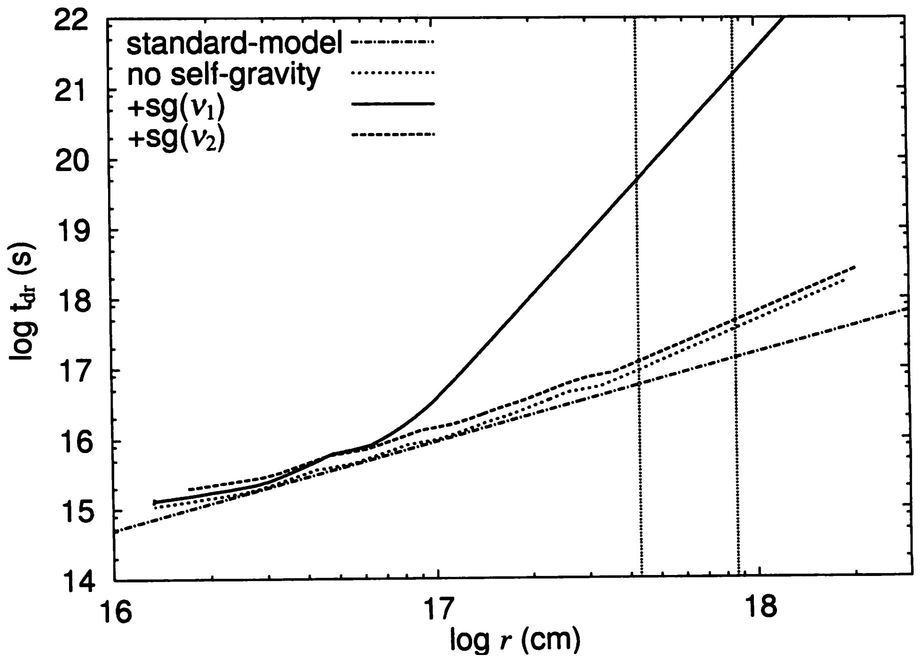

Fig. 5. Drift time in the disk. Notations are the same as in Fig. 2.

Figure 3 shows the temperature distribution in the disk. We obtain a nearly isothermal disk in Case A. The radiative cooling is rather inefficient in this high density model. Note that temperatures in Case B are similar to those in Case C where the self-gravity is neglected. This is because both Eqs. (2.7) and (2.8) contain the product ρH and the increase in the density due to self-gravity compensates the decrease in the thickness of the disk.

The thickness of the disk is shown in Fig. 4. In Case A we obtain $H \sim r^{-3}$ in the self-gravity dominant region. As previously stated, the disk shrinks in the z-direction due to strong self-gravity.

Gas drifts inward with time scale given by

$$t_{\rm dr} = \frac{r}{|v_r|}. \tag{3.1}$$

Figure 5 shows the drift time. In Case A, from Eq. (2.1) we get $|v_r| \sim (\rho H r)^{-1} \sim r^{-4}$ in the self-gravity dominant region. Then the drift time increases outwards such that $t_{\rm dr} \sim r^5$ and exceeds extremely the age of the universe 4.3×10^{17} s. It is a trouble that gas hardly accretes through the weak viscosity. The drift time in Case B is the same order as in Case C because the product ρH is the same.

We have examined the disk structure with $\dot{m} = 10^{-3}$ so far. If $\dot{m}$ is reduced by a factor 100, gas density in the disk decreases and the critical radius at which the self-gravity is dominant over the central gravity shifts to around 10^{17} cm. The turning points of ρ, T, H and $t_{\rm dr}$ in Figs. 2 – 5 also shift outwards with keeping their features.

§4. concluding remarks

We have investigated the influence of the viscosity prescription on the accretion disk. The models have been constructed with self-gravity in the z-direction of the disk around a supermassive black hole based on two kinds of α-prescription Eqs. (1.1) and (1.2), though the structures are the same for the standard disk. It is found that the model with Eq. (1.1) has weaker viscosity, higher density, higher temperature and longer drift time in the self-gravity domonant region than that with Eq. (1.2). The drastic increase in the density would lead to the gravitational instability of the disk. It is worthwhile to decide which is better prescription.

We have examined the disk structure in NGC 4258 with the accretion rate $\dot{M} = 5.5 \times 10^{21}$ g s^{-1}. It is found that the self-gravity dominates over the central gravity of the black hole in the maser emitting region. The disk would be gravitationally unstable to be broken up into many clumps in this region. We should be careful in treating viscosity, if self-gravity is taken into account for a disk model in AGN.

References

1) M. Miyoshi et al., Nature **373** (1995), 127.
J. R. Herrnstein et al., Nature **400** (1999), 539.
2) N. I. Shakura and R. A. Sunyaev, Astron. Astrophys. **24** (1973), 337.
S. Kato, J. Fukue and S. Mineshige, *Black Hole Accretion Disks*, (Kyoto Univ. Press, Kyoto, 2008)

J. Frank, A. King, and D. Raine, *Accretion Power in Astrophysics*, (Cambridge Univ. Press, Cambridge, 2002)

3) J. -M. Huré, S. Collin-Souffrin, J. Le Bourlot and G. Pineau des Forets, Astron. Astrophys. **290** (1994), 19.

4) D. Semenov, Th. Henning, Ch. Helling, M. Ilgner and E. Sedlmayr, Astron. Astrophys. **410** (2003), 611.