On the Warped Disk in NGC 4258

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We investigate evolution of the warped disk induced by torque due to radiation pressure from the central object. The warp is treated as a small perturbation on to a standard disk model around a supermassive black hole. We derive simultaneous differential equations which govern the development of the warp. Applying our procedure to NGC 4258, we specify the mass of the black hole $M = 3.9 \times 10^7 M_\odot$, the accretion rate $\dot{M} = 8.6 \times 10^{-4} M_\odot\ \mathrm{yr}^{-1}$ and the luminosity $L = 1.0 \times 10^{44}\ \mathrm{erg\ s}^{-1}$. The initial small tilt angle of the disk grows by a factor of 54 during 10 Gyr. It is found that the resulting warped disk can be well superposed on the observed map of water maser emitting clouds.

§1. Introduction

Strong maser emission of water has been detected[1] from many active galactic nuclei with luminosity $L_{H_2O} \sim 100 L_\odot$. The source is classified as a H_2O megamaser. High resolution interferometric images, obtained with Very Long Baseline Interferometry, reveals the distribution and motion of emitting clouds and provides a clue to the structure and dynamics of a masing disk. One of such examples is a weakly active Seyfert galaxy NGC 4258[2],[3] at a distance of 7.3 Mpc. The emission regions lie in a warped disk with radius $(4.3 - 8.6) \times 10^{17}$ cm. The rotation curve is accurately Keplerian, implying the central mass of $3.9 \times 10^7 M_\odot$. The disk is extremely thin and the thickness is less than 10^{15} cm.

The origin of the warped disk is still controversial. Several mechanisms have been proposed so far. The warp may be produced by a binary companion orbiting outside the disk.[4] Such a companion needs mass comparable to that of the disk. But it is much less than the mass of the disk which amounts to $\sim 10^4 M_\odot$ in NGC 4258. Another is the Bardeen-Petterson effect.[5] If we have a viscous disk around a misaligned spinning black hole, Lense-Thirring precession drives a warp in the disk. It becomes, however, significantly effective only for inner parts of the disk. A rough estimate shows that the Lense-Thirring effect dominates at $r \leq 10^{16}$ cm.

Alternatively, it is suggested that radiation from the central object acts pressure on the disk surface.[6],[7] Since the incident flux is radial, the associated force produces zero torque about the central object. If the disk is slightly warped, the surface is irradiated in a non-uniform manner. Provided that the disk is optically thick, it re-emits the absorbed radiation. The net re-emitted flux is normal to the local surface of the disk, which can produces torque on a given annulus of the disk. Such torque eventually changes the angular momentum of the annulus and amplifies the warp in the disk.

In the present paper we investigate the development of the warp induced by the radiation pressure in an accretion disk around a supermassive black hole. When we consider a geometrically thin, Keplerian disk, the tilt of the disk is treated as a small perturbation onto the standard disk model.[8] Then we linearlize a set of equations governing the development of the tilt.

§2. Basic equations

2.1. *Equation of motion*

We examine the evolution of a warped disk, which lies in an outer region of an accretion disk rotating around a black hole of mass M. The disk is considered to be thin and in a Keplerian motion. Self-gravity of the disk is neglected for simplicity. We use cylindrical coordinates (r, ϕ, z).

Let ρ be the gas density and H is the half-thickness of the disk, then the surface density is $\Sigma = 2\rho H$. The equation of continuity is

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\Sigma v_r) = 0, \tag{2.1}$$

where v_r is the radial velocity, which is negative for accreting gas.

The disk has a unit tilt vector $\boldsymbol{j}(r,t)$, normal to the disk plane. The angular momentum density $\boldsymbol{J}$ integrated with respect to z is

$$\boldsymbol{J} = \Sigma r^2 \Omega \boldsymbol{j}, \tag{2.2}$$

where Ω is the angular velocity. The equation for the angular momentum with including shear viscosity only is written as[6]

$$\frac{\partial}{\partial t}(\Sigma r^2 \Omega \boldsymbol{j}) + \frac{1}{r}\frac{\partial}{\partial r}(\Sigma v_r r^3 \Omega \boldsymbol{j}) = \frac{1}{r}\frac{\partial}{\partial r}(\nu \Sigma r^3 \Omega' \boldsymbol{j}) + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{2}\nu \Sigma r^3 \Omega \frac{\partial \boldsymbol{j}}{\partial r}\right). \tag{2.3}$$

Here the prime indicates differentiation with respect to r and ν is the kinematic viscocity coeffecient given by

$$\nu = \frac{2}{3}\alpha c_s H, \tag{2.4}$$

where α is the viscosity parameter and c_s is the sound velocity. When the disk is flat, $\partial \boldsymbol{j}/\partial r = 0$, Eq. (2.3) reduces to a familiar equation.

From Eqs. (2.1) and (2.3), we obtain for the standard disk

$$\frac{\partial \boldsymbol{j}}{\partial t} + \left[v_r - \frac{\nu\Omega'}{\Omega} - \frac{1}{2}\nu\frac{(\Sigma r^3\Omega)'}{\Sigma r^3 \Omega}\right]\frac{\partial \boldsymbol{j}}{\partial r} = \frac{\partial}{\partial r}\left(\frac{1}{2}\nu\frac{\partial \boldsymbol{j}}{\partial r}\right) - \frac{1}{\Sigma}\frac{\partial \Sigma}{\partial t}\boldsymbol{j}. \tag{2.5}$$

2.2. *Radiation torque*

The components of the tilt vector in an inertial frame XYZ are expressed in terms of the Euler angles as

$$\boldsymbol{j} = (\cos\gamma \sin\beta, \sin\gamma\sin\beta, \cos\beta), \tag{2.6}$$

where $\beta(r,t)$ is the local tilt angle of the disk with respect to the Z-axis and $\gamma(r,t)$ is the twist angle to the X-axis. The position $\boldsymbol{x}(r,\phi)$ of the disk surface is

$$\boldsymbol{x}(r,\phi) = r\hat{\boldsymbol{x}}(r,\phi), \tag{2.7}$$

where the radial unit vector $\hat{\boldsymbol{x}}$ is written by

$$\begin{aligned}\hat{\boldsymbol{x}} = (&\cos\phi\sin\gamma + \sin\phi\cos\gamma\cos\beta,\ \sin\phi\sin\gamma\cos\beta - \cos\phi\cos\gamma,\\ &-\sin\phi\sin\beta).\end{aligned} \tag{2.8}$$

The displacement vector is

$$\boldsymbol{dx} = \boldsymbol{s}_r dr + \boldsymbol{s}_\phi r d\phi, \tag{2.9}$$

where

$$\boldsymbol{s}_r = \frac{\partial \boldsymbol{x}}{\partial r}, \qquad \boldsymbol{s}_\phi = \frac{1}{r}\frac{\partial \boldsymbol{x}}{\partial \phi},$$

or explicitly

$$\boldsymbol{s}_r = \hat{\boldsymbol{x}} + \hat{\boldsymbol{x}}_\beta r\beta' + \hat{\boldsymbol{x}}_\gamma r\gamma', \tag{2.10}$$

$$\begin{aligned}\boldsymbol{s}_\phi &= \boldsymbol{j} \times \hat{\boldsymbol{x}}\\ &= (\cos\phi\cos\gamma\cos\beta - \sin\phi\sin\gamma,\ \cos\phi\sin\gamma\cos\beta + \sin\phi\cos\gamma,\\ &\qquad -\cos\phi\sin\beta),\end{aligned} \tag{2.11}$$

with

$$\begin{aligned}\hat{\boldsymbol{x}}_\beta &= \frac{\partial\hat{\boldsymbol{x}}}{\partial\beta}\\ &= -\sin\phi(\cos\gamma\sin\beta,\ \sin\gamma\sin\beta,\ \cos\beta),\end{aligned} \tag{2.12}$$

$$\begin{aligned}\hat{\boldsymbol{x}}_\gamma &= \frac{\partial\hat{\boldsymbol{x}}}{\partial\gamma}\\ &= (\cos\phi\cos\gamma - \sin\phi\sin\gamma\cos\beta,\ \sin\phi\cos\gamma\cos\beta + \cos\phi\sin\gamma,\ 0).\end{aligned} \tag{2.13}$$

The area vector of the surface element of the disk is

$$\boldsymbol{dS} = \boldsymbol{s}_r \times \boldsymbol{s}_\phi r dr d\phi. \tag{2.14}$$

Inserting Eqs. (2.10)–(2.11) into Eq. (2.14), we get

$$\boldsymbol{dS} = rdrd\phi[\boldsymbol{j} - r\hat{\boldsymbol{x}}(\gamma'\cos\phi\sin\beta - \beta'\sin\phi)]. \tag{2.15}$$

Since $\hat{\boldsymbol{x}}\cdot\boldsymbol{j} = 0$, we have

$$dS = rdrd\phi\left[1 + (r\gamma'\cos\phi\sin\beta - r\beta'\sin\phi)^2\right]^{1/2}. \tag{2.16}$$

If the tilt β is small, Eq. (2.16) reduces to its usual expression $dS = rdrd\phi$.

We assume a central object radiates in an isotropic manner and neglect shadowing by other inner parts of the warped disk. The radiation flux exerts force $\boldsymbol{dF}$ on the disk surface $\boldsymbol{dS}$:

$$\boldsymbol{dF} = \frac{L}{4\pi r^2}(\hat{\boldsymbol{x}} \cdot \boldsymbol{dS})\frac{2}{3c}\frac{\boldsymbol{dS}}{dS}, \tag{2.17}$$

where L is the luminosity of the central object and c is the light velocity. Hence the torque $\boldsymbol{dG}$ acting on the annulus of width dr at position $\boldsymbol{x}$ is

$$\boldsymbol{dG} = \oint \boldsymbol{x} \times \boldsymbol{dF}. \tag{2.18}$$

Using Eqs. (2.8), (2.16) and (2.17) for small β, and integrating Eq. (2.18) with respect to ϕ, we obtain

$$\boldsymbol{dG} = \frac{L}{6c}\left[r\beta\gamma'(\cos\gamma,\, \sin\gamma,\, 0) + r\beta'(\sin\gamma,\, -\cos\gamma,\, 0)\right] dr. \tag{2.19}$$

§3. Development of the tilt

We consider the small tilt, so that the disk can be approximated by the standard model. The disk rotates with the Keplerian angular velocity

$$\Omega = \sqrt{\frac{GM}{r^3}}, \tag{3.1}$$

where G is the gravitational constant. The steady inflow, $\partial\Sigma/\partial t = 0$, yields the constant mass accretion rate

$$\dot{M} = -2\pi r v_r \Sigma. \tag{3.2}$$

The radial velocity for an outer region of the disk is

$$v_r = -\frac{3\nu}{2r}. \tag{3.3}$$

The thickness and surface density are[8]

$$H = 8.8 \times 10^2 \alpha^{-1/10} m^{9/10} \dot{m}^{3/20} x^{9/8}\ \mathrm{cm}, \tag{3.4}$$

$$\Sigma = 4.0 \times 10^5 \alpha^{-4/5} m^{1/5} \dot{m}^{7/10} x^{-3/4}\ \mathrm{g\,cm^{-2}}. \tag{3.5}$$

Here $m = M/M_\odot$, $\dot{m} = \dot{M}/\dot{M}_{\mathrm{Edd}}$, $x = r/r_g$, where $\dot{M}_{\mathrm{Edd}} = 2.2 \times 10^{-9} m\ M_\odot\ \mathrm{yr}^{-1}$ is the Eddington accretion rate and $r_g = 2GM/c^2$ is the Schwarzschild radius of the black hole.

Using Eq. (3.3)–(3.5), Eq. (2.5) becomes

$$\frac{\partial \boldsymbol{j}}{\partial t} = \frac{1}{2}\nu\frac{\partial^2 \boldsymbol{j}}{\partial r^2} + \frac{1}{2}\left[\nu' + \nu\frac{(\Sigma r^3\Omega)'}{\Sigma r^3 \Omega}\right]\frac{\partial \boldsymbol{j}}{\partial r}. \tag{3.6}$$

From Eq. (2.4), we have

$$\nu = 2A r_g^2 x^{3/4} \tag{3.7}$$

with

$$A = 5.5 \times 10^{15} \alpha^{8/10} m^{18/10} \dot{m}^{3/10} r_g^{-3} \ \mathrm{s}^{-1}.$$

Therefore, we obtain

$$\frac{\partial \boldsymbol{j}}{\partial t} = A x^{3/4} \left[\frac{\partial^2 \boldsymbol{j}}{\partial x^2} + \frac{3}{2x} \frac{\partial \boldsymbol{j}}{\partial x} \right]. \tag{3.8}$$

When the radiation torque (2.19) is taken into account, Eq. (3.8) should be modified to be

$$\begin{aligned} \frac{\partial \boldsymbol{j}}{\partial t} &= A x^{3/4} \left[\frac{\partial^2 \boldsymbol{j}}{\partial x^2} + \frac{3}{2x} \frac{\partial \boldsymbol{j}}{\partial x} \right] + \frac{1}{2\pi \Sigma r^3 \Omega} \frac{d\boldsymbol{G}}{dr} \\ &= A x^{3/4} \left[\frac{\partial^2 \boldsymbol{j}}{\partial x^2} + \frac{3}{2x} \frac{\partial \boldsymbol{j}}{\partial x} \right] \\ &\quad + \frac{L x^{1/4}}{B} \left[\beta \frac{\partial \gamma}{\partial x} (\cos\gamma,\ \sin\gamma,\ 0) + \frac{\partial \beta}{\partial x} (\sin\gamma,\ -\cos\gamma,\ 0) \right], \end{aligned} \tag{3.9}$$

where

$$B = 9.6 \times 10^{27} \alpha^{-4/5} m^{1/5} \dot{m}^{7/10} r_g^2 \ \mathrm{erg}.$$

When the tilt is small, Eq. (2.6) becomes

$$\boldsymbol{j} = (P,\ Q,\ 1), \tag{3.10}$$

where

$$P(r,t) = \beta \cos\gamma, \qquad Q(r,t) = \beta \sin\gamma.$$

If we set

$$P = P_r(r) P_t(t), \qquad Q = Q_r(r) Q_t(t), \tag{3.11}$$

and assume that both P_t and Q_t have the same time-dependence, *i.e.*, $P_t = Q_t$, then we have

$$\frac{1}{P_t}\frac{dP_t}{dt} = \frac{1}{P_r} A x^{3/4} \left[\frac{d^2 P_r}{dx^2} + \frac{3}{2x}\frac{dP_r}{dx} \right] + \frac{L x^{1/4}}{B P_r}\frac{dQ_r}{dx} = \lambda,$$

$$\frac{1}{Q_t}\frac{dQ_t}{dt} = \frac{1}{Q_r} A x^{3/4} \left[\frac{d^2 Q_r}{dx^2} + \frac{3}{2x}\frac{dQ_r}{dx} \right] - \frac{L x^{1/4}}{B Q_r}\frac{dP_r}{dx} = \lambda,$$

where λ is a constant.

Finally, we obtain the time development

$$P_t = Q_t = e^{\lambda t} \tag{3.12}$$

and the simultaneous differential equations

$$\frac{d^2 P_r}{dx^2} + \frac{3}{2x}\frac{dP_r}{dx} + \frac{L}{A B x^{1/2}}\frac{dQ_r}{dx} = \frac{\lambda}{A x^{3/4}} P_r, \tag{3.13}$$

$$\frac{d^2 Q_r}{dx^2} + \frac{3}{2x}\frac{dQ_r}{dx} - \frac{L}{A B x^{1/2}}\frac{dP_r}{dx} = \frac{\lambda}{A x^{3/4}} Q_r. \tag{3.14}$$

These are linearized equations, so that if the disk is flat at some epoch, *i.e.*, $\beta = \beta' = 0$, which lead to $P_r = Q_r = P_r' = Q_r' = 0$, then it still remains flat forever. Therefore we need small perturbations to construct a warped disk.

§4. Results and discussion

When we consider the evolution of the warped disk in NGC 4258, it is necessary to specify values of the parameters of our disk model and the initial and boundary conditions for P_r and Q_r in Eqs. (3.13) and (3.14).

The mass of the central black hole is determined to be $M = 3.9 \times 10^7 M_\odot$ from the Keplerian rotation curve.[2)] A number of estimates for the accretion rate have been proposed:[9)] $\dot{M} = 8 \times 10^{-4} - 7 \times 10^{-3}\ M_\odot\ \mathrm{yr}^{-1}$. Then we set $\dot{m} = 0.01$, which corresponds to $\dot{M} = 8.6 \times 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$. The viscosity parameter is set to be $\alpha = 0.1$ as usual. The X-ray luminosity is 4×10^{40} erg s^{-1} in a energy range of 2 – 10 keV.[10)] Even if we take account that the X-ray luminosity is about 10 % of the bolometric one, we obtain only 4×10^{41} erg s^{-1}. It follows that NGC 4258 is one of low-luminosity active galactic nuclei. We have set $\dot{m} = 0.01$, which yields $L = 5 \times 10^{43}$ erg s^{-1}, so that we specify $L = 1.0 \times 10^{44}$ erg s^{-1} to examine the effects of radiation torque over the maximum.

The inner boundary of our numerical calculations is chosen to be $x_0 = 2 \times 10^4$, corresponding to $r_0 = 2.3 \times 10^{17}$ cm. The initial values at this point are $P_r = 10^{-3}$, $Q_r = 10^{-5}$ and $dP_r/dx = dQ_r/dx = 10^{-7}$, which indicate $\beta = 10^{-3}$, $\gamma = 10^{-2}$, $d\beta/dx = 10^{-7}$ and $d\gamma/dx = 10^{-4}$. The e-folding time in Eq. (3.12) is set to be $1/\lambda = 7.89 \times 10^{16}$ s.

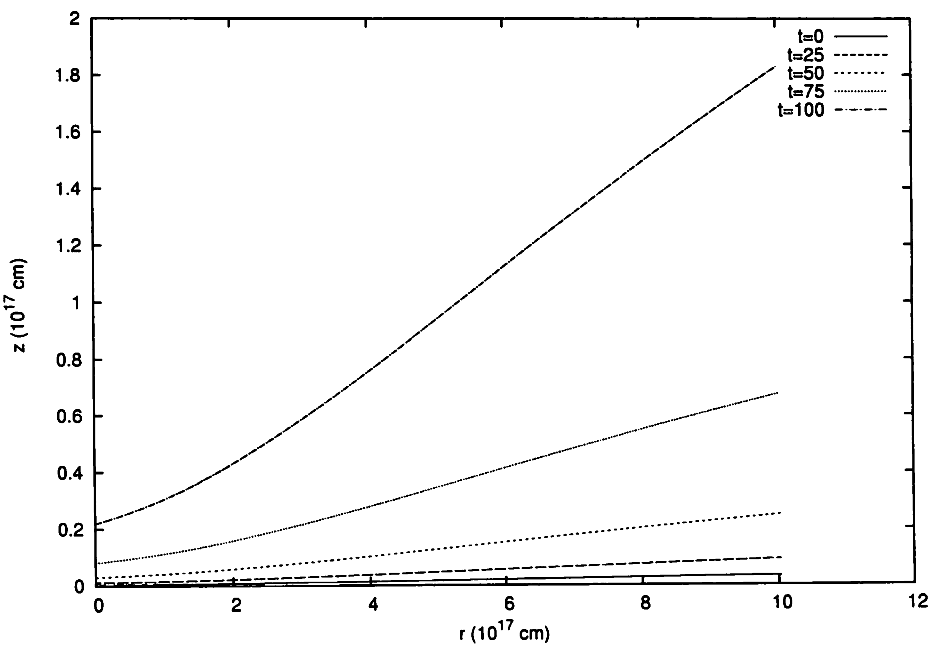

Fig. 1. Development of the warp along a spiral with a varying twist angle γ at t = 0, 25, 50, 75 and 100 $\times 10^8$ yr.

Figure 1 shows the evolution of the warp along a spiral with a varying γ at $t = 0$, 25, 50, 75 and 100 $\times 10^8$ yr. It should be noted that the tilt angle β increases

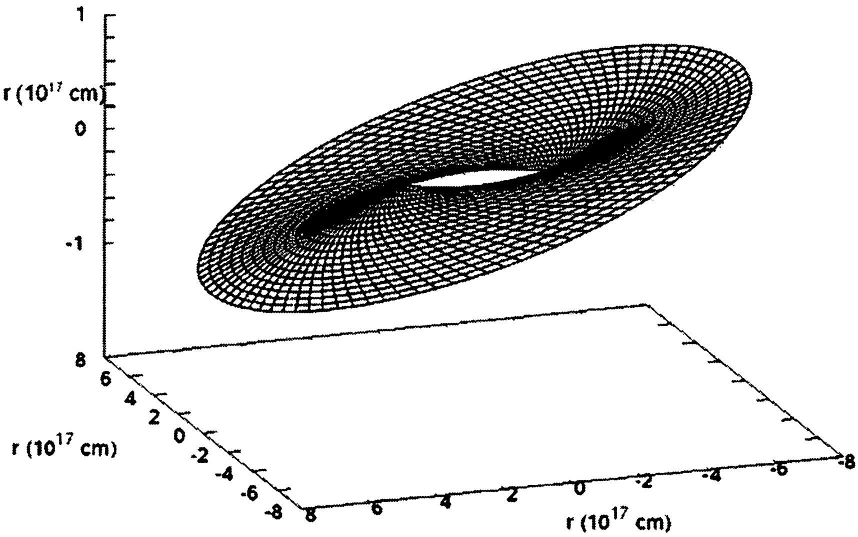

Fig. 2. Surface plot of the warped disk at 10 Gyr

gradually at a fixed epoch as one goes outward. Also β is amplified with $\exp(\lambda t)$ by a factor of 54 during 10 Gyr. Nevertheless, the assumption holds throughout our calculations that β remains small enough. It follows that the linearized equations (3.13) and (3.14) are still valid.

Figure 2 shows the surface plot of the warped disk at 10 Gyr. Note that the feature is 10 times extended along the vertical direction. It is found that the agreement is quite satisfactory with the observed disk.[11)]

Figure 3 shows the cross section of the warped disk at 10 Gyr. By comparing the observed map (Fig. 6) of water maser emmitting clouds,[12)] where the data are plotted in units of mas, and using 1 mas = 1.08×10^{17} cm for NGC 4258 at the distance of 7.3 Mpc, the resulting disk can be well superposed on to the data points.

§5. Concluding remarks

We have examined the evolution of the warped disk induced by the torque due to radiation pressure from the cetral object in NGC 4258. The warp is treated as a small perturbation to a standard disk model around a supermassive black hole of $M = 3.9 \times 10^7 M_\odot$. We have derived the simultaneous differential equations which govern the development of the warp. The numerical calculations have been carried out with suitable initial and boundary conditions for the accretion rate $\dot{M} = 8.6 \times 10^{-4} M_\odot$ yr^{-1} and the luminosity $L = 1.0 \times 10^{44}$ erg s^{-1}. The initial small tilt angle of the disk grows by a factor of 54 during 10 Gyr. It is found that the resulting warped disk can be well superposed on to the observed map of water maser emmitting clouds.

We have set the initial tilt angle at the inner boundary. These values could be originated from the Bardeen-Petterson effect which becomes significant at the more

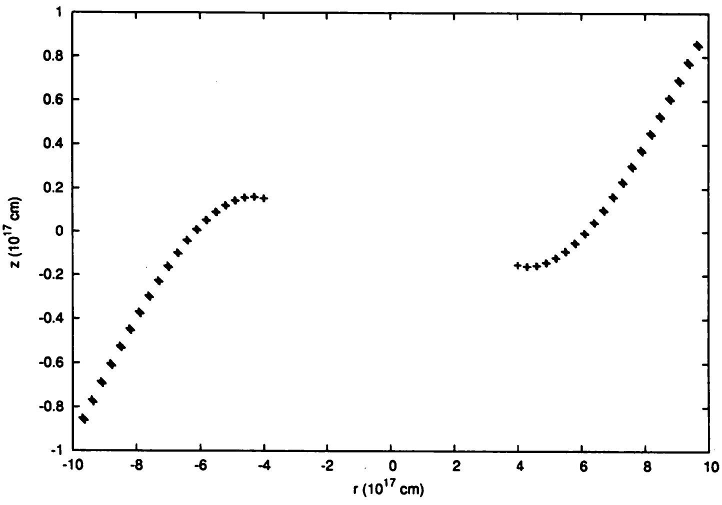

Fig. 3. Cross section of the warped, edge-on disk at 10 Gyr.

inner part of the disk. It is noted that the luminosity $L \simeq 10^{44}$ erg s^{-1} is rather high compared with the observed X-ray luminosity. The accretion time scale is evaluated to be $M/\dot{M} = 4.5 \times 10^{10}$ yr in our calculations, which is about 4 times longer than the age of the universe. Therefore NGC 4258 would be more active at the early stage of the evolution.

Physical quantities such as density, temperature etc. of the warped disk would be different from those of the standard model due to irradiation from the central object. Then it is worthwhile to examine formation of molecules[13] in the warped disk.

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