Observational Constraints on Brans-Dicke Theory with a Variable Cosmological Term

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We investigate Big-Bang nucleosynthesis in the Brans-Dicke model with a variable cosmological term ($BD\Lambda$) for the coupling constant $\omega = 10^4$. The model parameters are constrained from comparison between the resulting abundance of ^{4}He, D and ^{7}Li and the observed ones. Furthermore, we examine the magnitude redshift ($m - z$) relation for the $BD\Lambda$ with and without another constant cosmological term in a flat universe. Observational data of Type Ia Supernovae (SNIa) are used in the redshift range $0.01 < z < 2$. It is found that $BD\Lambda$ is inconsistent with the present accelerating universe but the model with a constant cosmological term with the value 0.7 can explain the SNIa data. The model parameters are insensitive to the $m - z$ relation.

§1. Introduction

For the last two decades we have heard many astronomical indications coming from observations like an age problem, large scale structure and Type Ia supernovae for the non-zero cosmological constant. The cosmological constant Λ is considered as one of the candidates of dark energy, though the acceptance of the conclusions from a theoretical point of view is limited. The constant has been discussed often as one of the fine tuning problems in cosmology called by "a cosmological constant problem".[1)] At the center of this, it lies a question why the observed value of the cosmological term in the very ealry universe exceeds the present value by some 120 orders of magnitude which is expected naturally from a theoretical point of view of elementory particle physics. It suggests that the cosmological term is not a true constant, but a variable quantity. To explain this kind of puzzle in cosmology new modified theories beyond the standard model are needed. Various functional forms have been proposed for the behavior of the cosmological term. The mechanism of the dynamical reduction of the cosmological term is formulated as a time dependent function[2)] and in terms of a scalar field.[1),3)] On the other hand, generalized scalar tensor theories have been investigated.[4),5)]

One of the approaches is the Brans-Dicke theory with a variable cosmological term as a function of scalar field ϕ.[5)] This model has been investigated for the Big Bang nucleosynthesis (BBN) in the early universe.[6)–8)] with the coupling constant $\omega \leq 500$. Current observations,[9)] however, suggest that ω exceeds 40,000. Therefore, it is worthwhile to examine the Brans-Dicke model with a variable cosmological term ($BD\Lambda$) for a new value of ω. Although $BD\Lambda$ has played a very important role in explaining the characteristics of the early universe,[6)–8)] we still need an answer to the question: "How does this model work at the present epoch?" We concentrate ourselves to the magnitude redshift $(m-z)$ relations of Type Ia Supernova (SNIa). This is because the cosmological term significantly affects the cosmic expansion rate of the universe at low redshifts. Observed data of SNIa imply the accelerating universe at the present epoch.[10)]

§2. Brans-Dicke model with a variable cosmological term

Expansion of $BD\Lambda$ is governed by[6)]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3\phi}(\rho_m + \rho_\gamma) - \frac{k}{a^2} + \frac{\Lambda}{3} + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}, \tag{2.1}$$

$$\dot{\phi} = \frac{1}{a^3}\left[\frac{8\pi\mu}{2\omega+3}\left(\rho_{m_0}t + \int_0^t (\rho_e - 3p_e)\,dt\right) + B\right], \tag{2.2}$$

where $a(t)$ is the scale factor, k is the curvature constant and B is an integral constant. We use the normalized value: $B^* = B/(10^{-24}\ \mathrm{g\ s\ cm^{-3}})$. ρ_i and p_i are the energy density and pressure of the constituent i, respectively. Energy density of matter varies as $\rho_m = \rho_{m0}a^{-3}$, while the radiation component is $\rho_\gamma = \rho_{\gamma_0}a^{-4}$ except $e^\pm$ epoch: $\rho_\gamma = \rho_{rad} + \rho_\nu + \rho_{e^\pm}$ at $t \leq 1$ s. The subscript "0" indicates the values at the present epoch.

Changes in the cosmological and gravitational terms in $BD\Lambda$ are written as

$$\Lambda = \frac{2\pi(\mu-1)}{\phi}\rho_{m_0}a^{-3}, \tag{2.3}$$

$$G = \frac{1}{2}\left(3 - \frac{2\omega+1}{2\omega+3}\mu\right)\frac{1}{\phi}, \tag{2.4}$$

where μ is a constant.

The original Brans-Dicke theory is deduced for $\mu = 1$ and is reduced to the Friedmann model when $\phi =$ constant and $\omega \gg 1$. To solve Eqs. (2.1) – (2.3), we specify the values at the present epoch: $G_0 = 6.6726 \times 10^{-8}\ \mathrm{cm^3\ g^{-1}\ s^{-2}}$, $H_0 = 71$ km s^{-1} Mpc^{-1}.[11)]

Figure 1 shows the evolution of the scalar field for several values of ω and B^* with $\mu = 0.6$ fixed. As shown in Eq. (2.2), the behavior of $\dot{\phi}$ is essentially determined from the value of B in the early universe. From Fig. 1 we can recognize that ϕ is an increasing function of time for positive B. Also ϕ becomes larger as ω increases. In contrast, if B is negative, ϕ decreases until $t \simeq 10$ s. Note that there is no considerable effect from B during the epoch $t > 10$ s.

Figure 2 shows the evolution of the scalar field for several values of ω and μ with $B^* = 0.5$ fixed. When μ is small, ϕ becomes large, because we have from Eq. (2.4)

$$\phi_0 \simeq \frac{2}{3-\mu}G_0$$

for large ω.

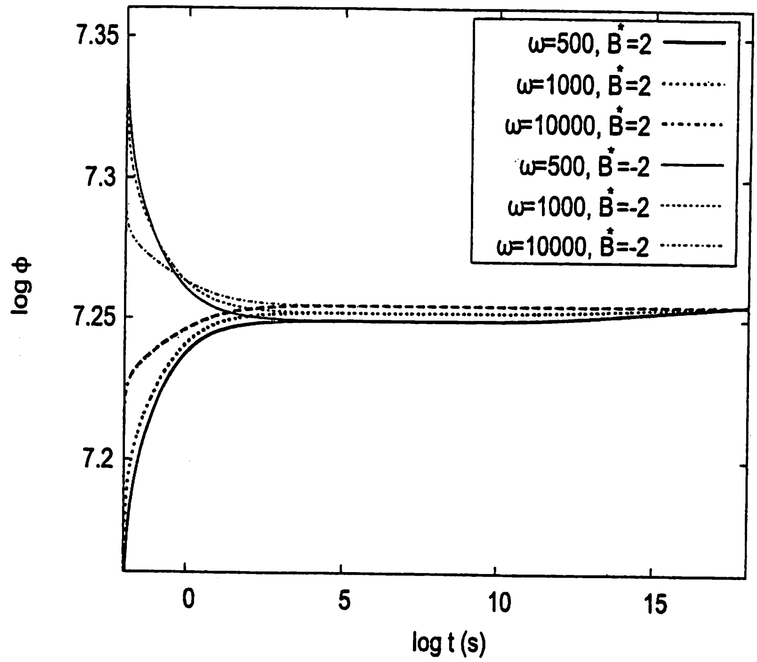

Fig. 1. Evolution of the scalar field in $BD\Lambda$ with $\mu = 0.6$ for several values of ω and B^*.

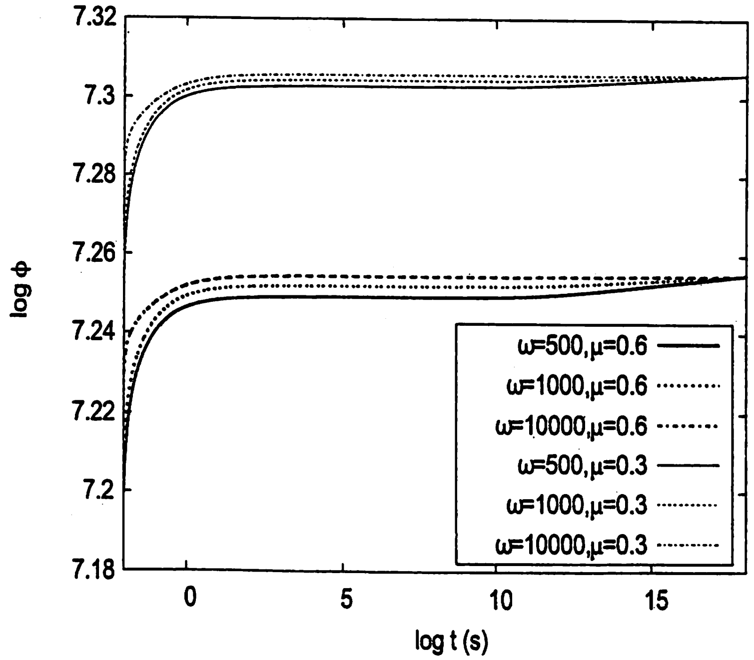

Fig. 2. Evolution of the scalar field in $BD\Lambda$ with $B^* = 0.5$ for several values of ω and μ.

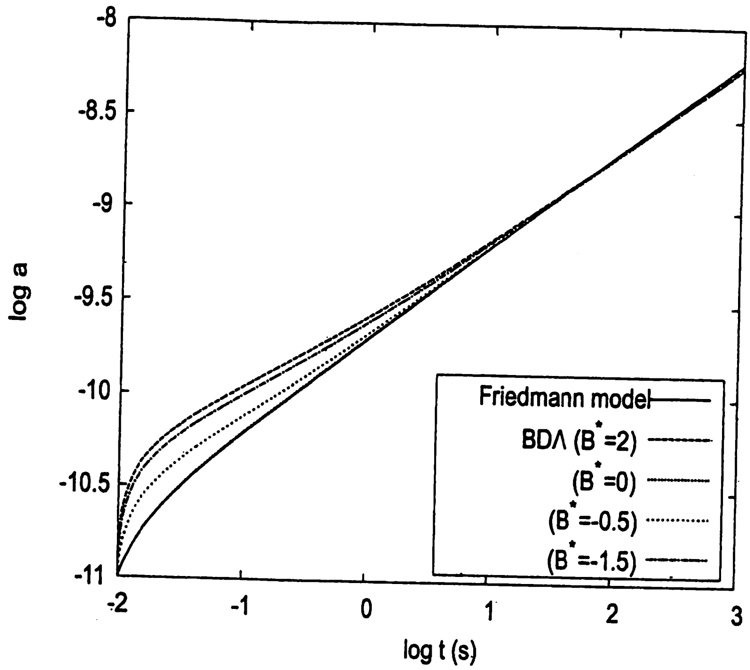

Fig. 3. Evolution of the scale factor in $BD\Lambda$ with $\mu = 0.6$ and $\omega = 10^4$ for several values of B^*. The solid line denotes the Friedmann model.

Figure 3 shows the evolution of the scale factor in $BD\Lambda$ for several values of B^* with $\mu = 0.6$ and $\omega = 10^4$. We recognize considerable deviations in $BD\Lambda$ from the Friedmann model at $t < 100$ s, which depends on the specified parameters. Therefore $BD\Lambda$ should be constrained from BBN.[6)–8)] Because of the deviation of the scalar field and the scale factor in $BD\Lambda$, it is worthwhile to investigate the plausible ranges in the parameters.

§3. Parameters constrained from the Big Bang nucleosynthesis

BBN provides powerful constraints on possible deviation from the standard cosmology.[12)] As shown in Fig. 3, the expansion rate of $BD\Lambda$ deviates significantly from that of the standard Friedmann model. Synthesis of ^{4}He is the most important consequence of BBN. Its abundance should be in principle used as the fundamental test of nonstandard BBN.

The abundance of light elements in $BD\Lambda$ has already been investigated,[6)–8)] where the parameters inherent in $BD\Lambda$ have been constrained for $\omega = 500$. But we consider the case $\omega = 10^4$ for convenience, because the Cassini measurements[9)] of the Shapiro time delay indicate $\omega \geq 4 \times 10^4$. The detailed method of nucleosynthesis is described in Ref. 8).

Figure 4 shows the calculated abundances of ^{4}He, D and ^{7}Li against the baryon-to-photon ratio η for $B^* = 2$ and $\mu = 0.6$. The $\pm 2\sigma$ uncertainties in nuclear reaction

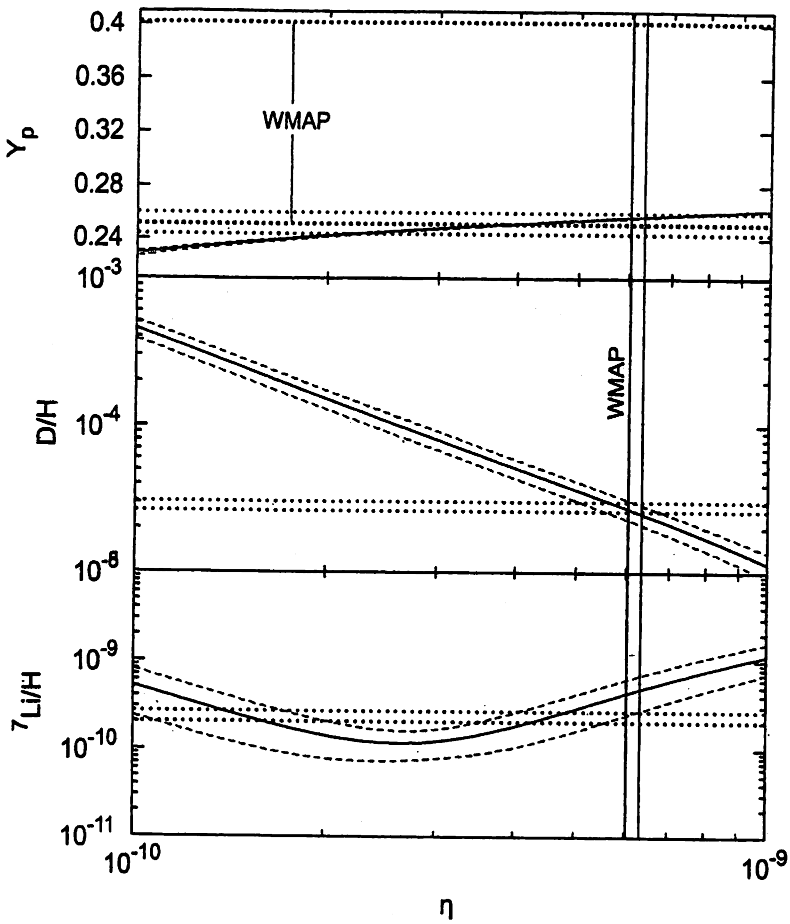

Fig. 4. Light element abundances of ^{4}He, D and ^{7}Li vs. the baryon-to-photon ratio η for $B^* = 2$, $\mu = 0.6$ and $\omega = 10^4$. The dashed lines indicate the $\pm 2\sigma$ uncertainties in nuclear reaction rates. The horizontal dotted lines denote the observed abundances. The two solid vertical lines indicate the range of η determined from WMAP.[15)]

rates are indicated by the dashed lines. The horizontal dotted lines indicate the observed values of ^{4}He, D/H and ^{7}Li/H.

The recent trend is that the mass fraction of Y_p has a larger value than thought before. It is found[13)] that $Y_p = 0.2516 \pm 0.0080$ with 2σ errors by reanalyzing the observed data.[14)] The results of the WMAP 7-year observations[15)] are $\eta = (6.19 \pm 0.15) \times 10^{-10}$ and $Y_p = 0.326 \pm 0.075$. We adopt the intersection of these values: $Y_p = 0.2510 - 0.2596$. Also we take $\mathrm{D/H} = (2.82 \pm 0.21) \times 10^{-5}$ from recent observations[16)] towards Q0913+07 and $^7\mathrm{Li/H} = (2.34 \pm 0.32) \times 10^{-10}$.[17)] It is found that the range of η derived from ^{4}He and D/H is tightly consistent with the value by WMAP, though the lower limit of ^{7}Li/H is barely consistent. These agreements lead us to obtain the parameter ranges $0.0 \leq \mu \leq 0.6$ and $-2 \leq B^* \leq 2$.

§4. The $m-z$ relation in BDA with and without a constant cosmological term

For the homogeneous and isotropic universe, the relation between the red shift z and the radial distance r_l is derived from the Robertson-Walker metric:

$$\int_0^z \frac{dz}{H} = \begin{cases} \sin^{-1} r_l & k=+1, \\ r_l & k=0, \\ \sinh^{-1} r_l & k=-1, \end{cases}$$

where $H = \dot{a}/a$ is the expansion rate written from Eq. (2.1) as

$$H = \left[\frac{1}{4}\left(\frac{\dot{\phi}}{\phi}\right)^2 - (1+z)^2 k + \frac{\Lambda}{3} + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{8\pi}{3}\frac{\rho}{\phi}\right]^{1/2} - \frac{1}{2}\frac{\dot{\phi}}{\phi}. \tag{4.1}$$

The WMAP results[18] tell that we live in a closely geometrically flat universe: $k=0$. Then we obtain from Eq. (2.1)

$$H_0^2 = \frac{1}{3}\left(\frac{8\pi\rho_{m_0}}{\phi_0} + \Lambda_0\right) + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)_0^2 - H_0\left(\frac{\dot{\phi}}{\phi}\right)_0, \tag{4.2}$$

$$\rho_{m_0} = 4\rho_c^{BDA}/(\mu+3), \quad \rho_c^{BDA} = 3\phi_0 H_0^2/8\pi, \tag{4.3}$$

where ρ_c^{BDA} is the critical density of BDA.

Using the analogy to the Lemaître model, Eq. (4.2) is written as

$$\Omega_{m_0} + \Omega_{\Lambda_0} + \Omega_{\phi_0} = 1, \tag{4.4}$$

where the energy density parameters are

$$\Omega_{m_0} = \frac{\rho_{m_0}}{\rho_c^{BDA}}, \tag{4.5}$$

$$\Omega_{\Lambda_0} = \frac{\Lambda_0}{3H_0^2} = \frac{(\mu-1)\rho_{m_0}}{4\rho_c^{BDA}}, \tag{4.6}$$

$$\Omega_{\phi_0} = \frac{\omega}{6H_0^2}\left(\frac{\dot{\phi}}{\phi}\right)_0^2 - \frac{1}{H_0}\left(\frac{\dot{\phi}}{\phi}\right)_0. \tag{4.7}$$

The value Ω_{ϕ_0} is found to be very small such as 7×10^{-5} for $\mu = 0.6$. If we consider the models in the parameter range $0.0 \le \mu \le 0.6$, the contribution from Ω_{ϕ_0} to Eq. (4.4) is always less than 10^{-5} and consequently can be neglected, as long as the present epoch is concerned.

The distance modulus μ_{th} of the source at the redshift z is

$$\mu_{th} = m - M = 5\log\left[(1+z)\, r_l\right] + 25, \tag{4.8}$$

where m and M are the apparent and absolute magnitudes, respectively, and r_l is measured in units of Mpc.

We use a united sample including three sets of recent data[19] on SNIa. They are the 5-year Supernovae Legacy Survey up to $z \simeq 1$, the Hubble Space Telescope with $z \leq 2$ and the Sloan Digital Sky Survey-II Supernovae Survey with $0.04 < z < 0.42$.

For the data sets we evaluate χ^2 by

$$\chi^2 = \sum_{i=1}^{n} \frac{(\mu_{th,i} - \mu_{obs,i})^2}{\sigma^2_{\mu_{obs,i}} + \sigma^2_{v,i}}, \tag{4.9}$$

where $\sigma_{\mu_{obs}}$ is the unceritinity in the individual distance moduli and σ_v is the dispersion in the redshift due to the peculiar velocity v written as

$$\sigma_v = \left(v \frac{d\mu_{th}}{dz} \right). \tag{4.10}$$

We adopt $v = 1.2 \times 10^{-3}$, in units of $c = 1$, by considering the mean value of peculiar velocities of the data sets. The total number of the united sample n is 557 for our analysis.

Figure 5 shows the $m - z$ relation of SNIa in $BD\Lambda$. Matter is dominant in this model. The energy density of the cosmological term is always less than 20% in the best-fit parameter region constrained in the previous section. We obtain negative Ω_{Λ_0} in the parameter range of our interest. The parameter B^* is not effective to change the values of both Ω_{m_0} and Ω_{Λ_0}. Because this model is matter dominant, it cannot be constrained from the SNIa observations.

The Friedmann model with the energy density parameters of $(\Omega_m, \Omega_\Lambda) = (1.0, 0.0)$ is merged into $BD\Lambda$ model with the reduced chi-square $\chi^2_r = \chi^2/N \simeq 4.117$, where $\chi^2 = 2293$ and N is the degree of freedom. This is inconsistent with the present accelerating universe, which should contain a sufficient amount of dark energy to accelerate the universe. To explain the present accelerating universe, we need some modification of the cosmological term.

As the next approach, $BD\Lambda$ is modified by adding another constant cosmological term Λ_{c0}. The expansion rate in this model is written by

$$H = \left[\frac{1}{4} \left(\frac{\dot{\phi}}{\phi} \right)^2 + \frac{\Lambda}{3} + \frac{\Lambda_{c0}}{3} + \frac{\omega}{6} \left(\frac{\dot{\phi}}{\phi} \right)^2 + \frac{8\pi}{3} \frac{\rho}{\phi} \right]^{1/2} - \frac{1}{2} \frac{\dot{\phi}}{\phi}. \tag{4.11}$$

The present matter density is

$$\rho_{m_0} = \frac{4\,(1 - \Lambda_{c0})\, \rho_c^{BD\Lambda}}{(\mu + 3)}. \tag{4.12}$$

Here the energy density parameter of the constant cosmological term Λ_{c0} is fixed to be 0.7.

We find that this model is consistent with the SNIa observations as seen in Fig. 5. The total cosmological term becomes large in this model and consistent with the present accelerating universe with reduced $\chi^2_r \simeq 0.98$ (where $\chi^2 = 546.92$). For $\mu = 0.5$, $BD\Lambda$ with Λ_{c0} predicts $\Omega_\Lambda = -4.29 \times 10^{-2}$ and $\Omega_m = 0.34$. Ω_Λ always gets

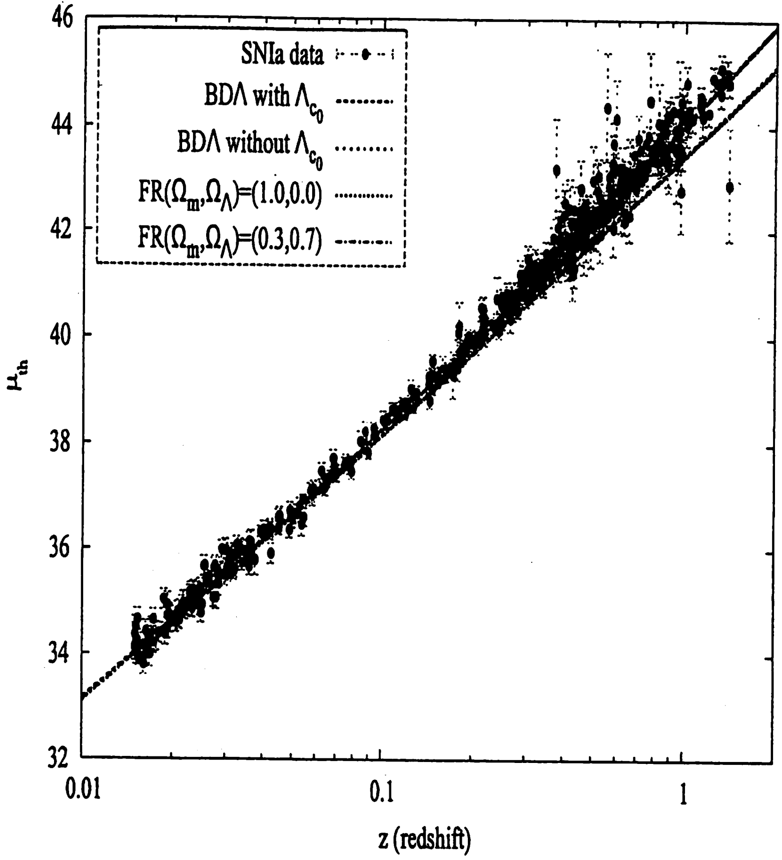

Fig. 5. Distance modulus vs. redshift for the flat universe in the Friedmann model and $BD\Lambda$ with and without a constant cosmological term constrained by SNIa observations.[19]

negative values in the parameter region of μ constrained in §3. If we consider the total value of energy densities, the contribution from $\Omega_{\Lambda_0} + \Omega_{\Lambda_{c_0}}$ to the total energy density is always between $60 - 67\%$. Therefore the cosmological term is dominant in the present epoch and can be constrained from the present SNIa observations. We conclude that $BD\Lambda$ with Λ_{c_0} has nearly the same energy density parameters as the Friedmann model with $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$. Although the cosmological term is not important at the early epoch, it plays a very important role at the present era. All the parameters inherent in $BD\Lambda$ become insufficient as far as the $m - z$ relation is concerned.

§5. Concluding remarks

We have derived the observational constrains on the Brans-Dicke theroy with a variable cosmological term. Previous BBN calculations[8] restricted the parameter range as $-0.5 \leq \mu \leq 0.8$ and $-10 \leq B^* \leq 10$ for $\omega = 500$. On the other hand, our large value of $\omega = 10^4$ leads to reduce the parameter range $-2 \leq B^* \leq -2$. It oppositely affects another parameter $0.0 \leq \mu \leq 0.6$. These model parameters are inefficient to explain the $m - z$ relations of SNIa.

In §4, the value of Ω_{ϕ_0} is found to be much smaller compared with the other terms in Eq. (4.4). Even though ω is increased until 10^4 the contribution from Ω_{ϕ_0} to Eq. (4.4) is always less than 1% in the particular parameter range. There is no considerable wrong effect from the assumption we made in §4 to neglect the value of Ω_{ϕ_0}. In the parameter range $0.0 \leq \mu \leq 0.6$, Λ has taken negative values according to Eq. (4.4). This may not conflict with theories, since the pressure of dark energy must be negative to reproduce the present accelerated expansion.[20]

It should be noted from Eq. (2.3) that $\Lambda \sim \rho_m/\phi$ and at the present epoch, Λ_0 is directly connected with ρ_{m_0}. Dark energy is written in terms of dark matter. However, dark energy and dark matter should be distinguishable to give rise to an accelerated expansion, since evolution of the scale factor seriously depends on the composition of individual energy density of the universe. Therefore, $BD\Lambda$ without a constant cosmological term is indistinguishable from the matter dominant Friedmann model with the parameters of $(\Omega_m, \Omega_\Lambda) = (1.0, 0.0)$. It should be noticed that the variable Λ term in $BD\Lambda$ plays a minor role to accelerate the universe at the present epoch. Consequently we have modified to add a constant cosmological term. It has no relation to the expansion rate of the universe at the early epoch. However, the energy stored in the constant cosmological term has played a major role to accelerate the universe at the present epoch as seen in Fig. 5. Since this model contains enough dark energy to accelerate the universe, it is constrained by the SNIa observations. In the present research, we have investigated $BD\Lambda$ at the early epoch to determine the intrinsic parameters and introduce new parameters at the present epoch for the $m - z$ relation.

Since we have succeeded in demonstrating a possibility of non-standard model which is compatible with the observations, it is worthwhile to examine more general functional forms to the cosmological term. Dark energy models with a cosmological constant and its variants are proposed to explain the acceleration of the universe. As an alternative to the dark energy model, modified gravitational models can explain the acceleration of the universe. It is worth to study the modified gravitational models to explain some puzzles in the universe.

References

1) S. Weinberg, Rev. Mod. Phys. **61** (1989), 1.
2) V. Silviera and I. Waga, Phys. Rev. D **56** (1997), 4625.
3) D. Huterer and M. S. Turner, Phys. Rev. D **60** (1999), 081301.
4) R. V. Wagoner, Phys. Rev. D **1** (1970), 3209.
   T. Fukui, K. Arai and M. Hashimoto, Class. Quant. Grav. **18** (2001), 2087.
5) M. Endo and T. Fukui, Gen. Rel. Grav. **8** (1977), 833.
6) K. Arai, M. Hashimoto and T. Fukui, Astron. Astrophys. **179** (1987), 17.
7) T. Etoh, M. Hashimoto, K. Arai and S. Fujimoto Astron. Astrophys. **325** (1997), 893.
8) R. Nakamura, M. Hashimoto, S. Gamow and K. Arai Astron. Astrophys. **448** (2006), 23.
9) E. Berti, A. Buonanno and C. M. Will Phys. Rev. D **71** (2005), 084025.
   B. Bertotti, L. Iess and P. Tortora, Nature **425** (2003), 374.
10) S. Perlmutter et al., Astrophys. J. **517** (1999), 565.
    A. G. Riess et al., Astrophys. J. **116** (1998), 1009.

A. G. Riess et al., Astrophys. J. **117** (1999), 707.
11) D. N. Spergel et al., Astrophys. J. Suppl. **148** (2003), 175.
12) R. A. Malaney and G. J. Mathews, Phys. Rep. **229** (1993), 145.
13) M. Fukugita and M. Kawasaki, Astrophys. J. **646** (2006), 691.
14) Y. I. Izotov and T. X. Thuan, Astrophys. J. **602** (2004), 200.
15) E. Komatsu et al., astro-ph/1001.4538v2 (2010).
16) M. Pettini et al., Mon. Not. Roy. Astron. Soc. **391** (2008), 1499.
17) J. Melendez and I. Ramirez, Astrophys. J. **615** (2004), L33. *Cosmology* (Oxford Unversity Press, New York 2008).
18) J. Dunckley et al., Astrophys. J. Suppl. **180** (2009), 306.
19) P. Astier et al., Astron. Astrophys. **447** (2006), 31.
A. G. Riess et al., Astrophys. J. **659** (2007), 98.
R. Kessler et al., Astrophys. J. Suppl. **185** (2009), 32.
20) S. M. Carroll and W. H. Press, Ann. Rev. Astron. Astrophys. **30** (1992), 499.