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論 説

Geological Hazard Risk Assessment by Using Fuzzy Sets Theory*

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Abstract: In this study, a low-cost, rapid and qualitative evaluation procedure using a fuzzy set analysis for assessment and prediction of liquefaction potential of saturated sandy grounds is presented. Eight items affecting liquefaction resistance of ground including geology, geomorphology, seismic (relative site amplification and intensity increments), and geotechnical items (sandy layers thickness, water table level, thickness of surface layers, and type of soils) are considered to express the basic characteristics of liquefaction potential of the ground. These items are chosen and established from a review of the various literatures, engineering judgment, available statistical data, and previous observations of liquefaction in the world. A set of evaluation criteria was established or selected for each item and a total of eight factors is used in the proposed evaluation system. In the proposed evaluation system, liquefaction potential of ground is assessed and expressed in linguistic terms based on the considered criteria. Then the linguistic data is analysed by using fuzzy sets. The liquefaction index is defined for assessment of liquefaction potential of soils.

An example of application of the method is presented to liquefaction potential analysis of saturated soft ground in northwestern Iran (Gilan plain). The studied area was suffered catastrophic earthquake in June 1990, and the properties of the earthquake were widespread liquefaction, several huge landslides and more than tens other slope failures.

This study revealed that, the proposed method is able to predict the liquefaction potential of the ground for preparation of hazard potential maps and zoning, which is useful for general hazard assessment and delineation of areas.

Key Words: Fuzzy sets, Hazard criteria, Hazard rate, hazard evaluation, Liquefaction

1 Introduction

There are some uncertainties in geotechnical engineering problems, which may be associated with ambiguity, qualitative, vagueness, and imprecision of the events. These uncertainties can result from factors of complexity of natural events, in which the knowledge is imprecise and / or imcomplete; use of the natural languages which can be meaningful but not clearly defined; inexact or ill-defined figures, pictures, and scenes due to the lack of informations.

Some concepts encountered in hazard assessment, such as considered criteria and hazard rate which are always described by natural language, such as very high, high, medium, low, and very low, are linguistic descriptions used to define hazard conditions. They inherently possess vagueness and ambiguity.

On the other hand, the evaluation system using these terms as a qualitative assessment enables the preparation of hazard potential maps at low-cost for purposes such as land-use planning or regional risk analysis.

Also, the accuracy of the risk analysis system is related to evaluation hazard criteria or effective factors in hazard occurrence. Furthermore, the most desired analysis system is the one which can evaluate hazard potential on the

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basis of the all hazard criteria with minimum uncertainty.

Then, the hazard risk assessed and recorded by linguistic terms, must be expressed by some numerical grades as a hazard potential rate to be used in engineering practice.

In order to quantify these linguistic descriptions, many methods have been proposed. Almost all of which use discrete numbers on the real axis to express different levels of hazards. For removing these problems, Zadeh(1965), proposed an extensive axiomatic system that attemts to recognize, capture, and exploit nonstatistical uncertainties, which may arise in mathematical models.

The basic structure in this system is the "fuzzy set". In particular fuzzy models are especially well suited for many problems arising in ill-defined or complex systems. Fuzzy sets is a set of numbers that describe the "degree of belonging" to each level of rating.

The fuzzy sets theory is a methodology for the formulation and solution of problems which are too complex or ill-defined to be susceptible to analysis by conventional techniquies (Zadeh, 1980).

The applicability of this multicriteria technique to geological hazard assessment was examined in northwestern Iran (Gilan area). For this, the liquefaction potential of the area was analyzed using this method. The evaluated locations is depicted in Figure 1.

2 Fuzzy sets theory

2.1 Principle of fuzzy sets theory

In any set, A, the belongness of an element to the universe, whether the element is a member of A or not, is described by a function. Such a function is called the characteristic function or membership function of A, and is defined by:

$$Charc_A(x) = \begin{cases} 0 & \text{if } x \text{ is not in the set A} \\ 1 & \text{if } x \text{ is in the set A} \end{cases} \quad (1)$$

Since every real number either is a member of A or not, we can associate "belongs to A" with the number 1 and "not in A" with the number of 0. This association is called a mapping from U_R to $\{0,1\}$. This mapping is rep-

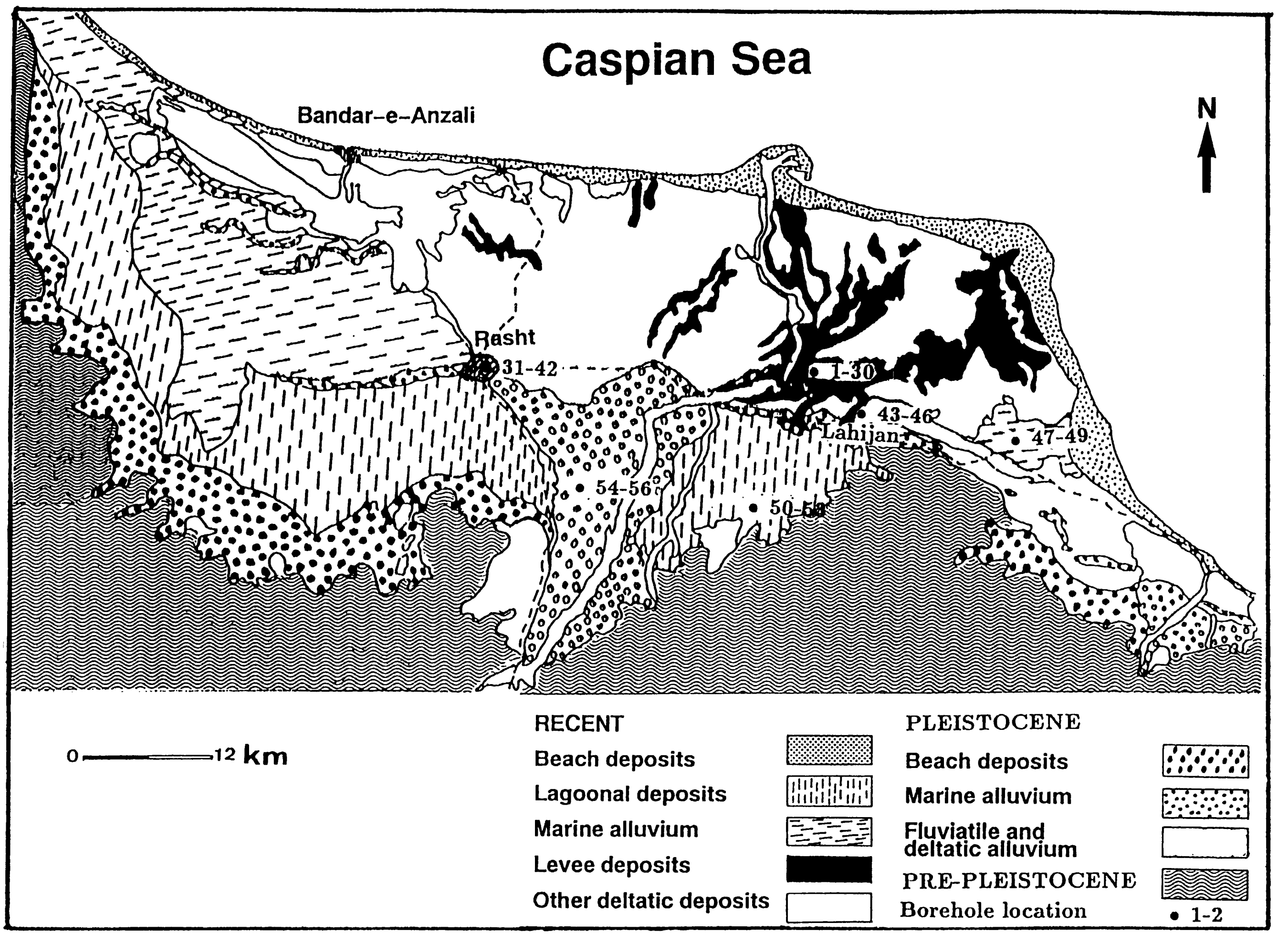

Fig. 1 Geological map of the Gilan plain area (northwestern Iran)

resented symbolically by μ_A, and can be shown by:

$$\mu_A : U_R \rightarrow \{0, 1\} \tag{2}$$

Then in a set theory, an element is completely in a set or completely out of a set, but what if an element was not completely in a set and was not also completely out of a set ? This notion of the plausibility of a set membership, leads to the generalization of the degree of membership in a set, and from this generalization comes a variant of the set theory which is called fuzzy set theory (Schmucker, 1984).

A set is fuzzy when it has no realization as subsets. To define a fuzzy subset of some universe U_R which is a collection of objects from U (the set part) with each object which is associated with a degree of membership (the fuzzy part), the range of membership functions from the two point set $\{0, 1\}$ is extented to the unit interval [0,1]. Then a fuzzy set of A in U is a set of oredered pairs:

$$A = \{(x, \mu_A(x)) \mid x \in A \text{ and } A \subset U\} \tag{3}$$

In Equation(3) x is the value assumed by the linguistic variable and μ_A is respective grades of membership and, $\mid$ is a delimeter. If x is any set, then μ_A is fuzzy subset of x which:

$$\mu_A : X \rightarrow [0, 1] \tag{4}$$

and can be defined:

$$(\mu_A \text{ is a fuzzy subset of } x) \leftrightarrow (\mu_A \text{ is a function mapping } x) \rightarrow [0, 1] \tag{5}$$

For $x \in X$, the value $\mu_A(x)$ is called the degree of membership of x in A. The $\mu_A(x)$ measures the extent to which x possesses the imprecisely defined object peroperties which characterize A or measures the degree of similarity of things to A.

In fuzzy environment, the data are fuzzy numbers, *i.e.*, fuzzy variables defined on the real line. Then fuzzy numbers can be processed in a manner similar to the nonfuzzy case, and the results of the analysis are given in terms of fuzzy numbers.

2.2 Extension principle

Let A and B be two fuzzy subsets of U and $a(x)$ be the degree of membership of x in A and $b(x)$ be the degree of membership of x in B. According to the extension principle (Schmucker, 1984), the following relationships are derived.

$$A \cup B = \{\max(a(x), b(x)) / x \mid x \text{ is an element of } U\} \tag{6}$$

and

$$A \cap B = \{\min(a(x), b(x)) / x \mid x \text{ is an element of } U\} \tag{7}$$

Another set operation that is useful for risk analysis is complement of set. On the proposed definition by Zadeh(1978) the complement of a fuzzy set A is:

$$A' = \{(1 - a(x)) / x \mid x \text{ is in } U\} \tag{8}$$

These set operations are given graphicaly in Figure 2.
By generalization of given definition by Zadeh if $A_1, A_2, \ldots, A_n$ are fuzzy sets defined on universe $X_1, X_2, \ldots, X_n$, $*$ is fuzzy arithmetic operations, and f is a function which maps $X_1 * X_2 * \ldots * X_n$ to the universe Y, the fuzzy image B of $A_1, A_2, \ldots, A_n$ through f has the membership function, μ_B(Y) which is:

$$\mu_B(Y) = \max\{\min(\mu_{A1}(x_1), \mu_{A2}(x_2), \ldots, \mu_{An}(x_n) / x_1, x_2, \ldots, x_n \mid x_1, x_2, \ldots, x_n \text{ are elements of universe } Y\} \tag{9}$$

which

$$Y = f(x_1, x_2, \ldots, x_n) \tag{10}$$

Then the extension principle is an extremely powerful tool because any functional relationship between nonfuzzy (crisp) elements can be extended to fuzzy entities. Algebric operations used extensively in risk and decision analysis, are special case and hence, governed by the same principle.

2.3 Natural language

The notions of linguistic variables and of fuzzy sets are not one and the same but rather have the relationship of goal and tool: having precisely manipulatable natural language expressions is the goal, and fuzzy set theory is a tool to achieve the goal.

A linguistic variable is a variable whose values are natural language expressions referring to some quantity of interest.

"A linguistic variable differs from a numerical variable in that its values are not numbers but words or sentences in a natural or artificial language. Since words in general are less precise than numbers, the concept of a linguistic variable serves the purpose of providing a means of approximate characterization of phenomena which are too

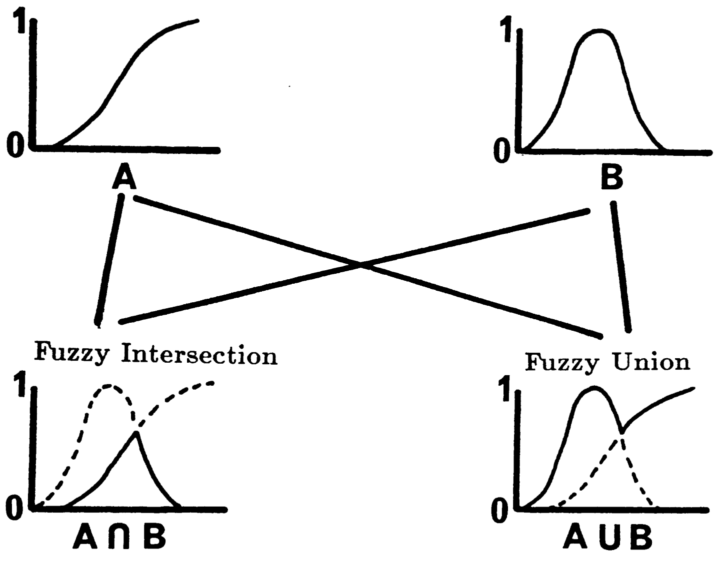

Fig. 2 Fuzzy extention principle

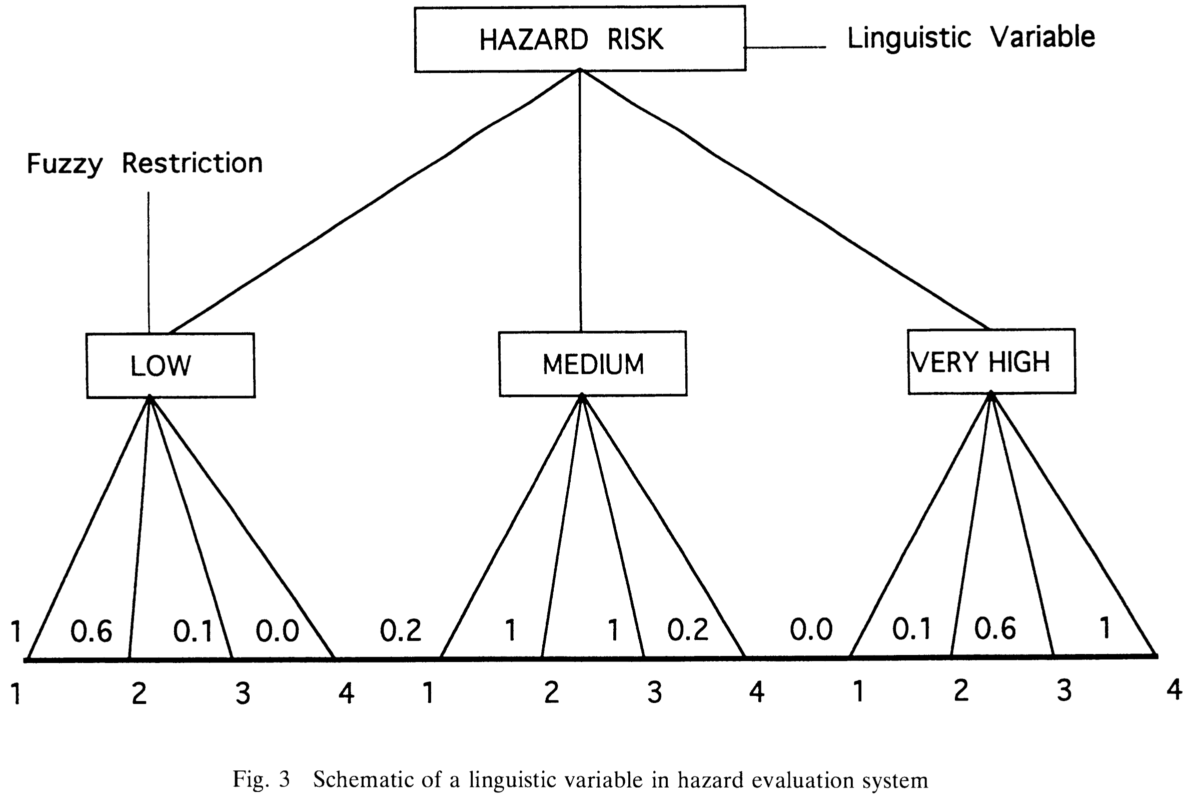

Fig. 3 Schematic of a linguistic variable in hazard evaluation system

complex or too ill-defined to be a meanable to description in conventional quantitative terms. More specifically, the fuzzy sets which represent the restriction associated with the values of a linguistic variable may be viewed as summaries of various subclasses of elements in a univeres of discourse. This of course, is analogous to the role played by words and sentences in a natural language" (Zadeh, 1975).

For example Figure 3 shows the case where the quantity of interest is the risk of geological hazards, which is described by linguistic words such as very low to very high. Then we can construct a fuzzy set meaning for a wide range of natural language expressions, and we have alluded to the fact that we will use these fuzzy sets in a series of yet unspecified computations to determine the overall risk of a system being analyzed.

2.4 Fuzzy weighted average operation

The possibility and severity of geological hazrads(liquefaction and slope failure) and the reliability for each branch of a given evaluation system are all taken to account in order to calculate the total hazard rate for the system. The calculation is done by a generalization of the normal weighted average, which is called the fuzzy weighted average operation. For example, if R_i is a sequence of integers, and W_i a sequence of integer weights, then the weighted average of the R_i's by ordinary set operations is defined as:

$$\overline{R} = \frac{\sum_{i=1}^{n}(R_i * W_i)}{\sum_{i=1}^{n} W_i} \qquad (11)$$

On the descriptions of Dubois *et al.*(1976b), this definition can be extended to provide a similar computation where both the entities being weighted and their weights are fuzzy quantities. To do this, we must extend the arithmetic operations used in the computation of the mean: addition, multiplication, and division, from operators defined on the reals to operators defined for fuzzy sets described earlier in Zadeh's extention principle.

Because the results of fuzzy algebric operations will be given the set over the set of many integers, Clements (1977) gave the method, in which the set over the reals is reduced to one over the integers by deleting any element not over an integer base. This reduction from a fuzzy set over the reals to one over the integers is done using Equation (9).

Using this definition for fuzzy algebric operations(Clements method), the fuzzy weighted average is defined. To evaluate the overall risk of the system and reduce the number of alternatives, the fuzzy weighted average operation *FWA* is used.

If R_i and W_i are a sequence of fuzzy sets, and all the fuzzy arithmetic operations show by symbol $*$, then the fuzzy weighted average of the R_i's using the W_i's as weights, is defined as follows:

$$R = \frac{\sum_{i=1}^{n}(R_i * W_i)}{\sum_{i=1}^{n} W_i} \qquad (12)$$

where R is the fuzzy set representing the overall rating of an alternative, R_i is the fuzzy set that represents rating of the alternative based on a particular criterion; and W_i is the fuzzy set representing the weight or relative importance assigned to that particular criterion.

The fuzzy arithmetic operations, summation, multiplication and division, which are used in equation (12) are defined as follows(Schmucker, 1984):

if

$$X = \{x(i)/i; 1 \leq i \leq n\} \qquad (13)$$

$$Y = \{y(j)/j; 1 \leq j \leq n\} \qquad (14)$$

where *i, j,* and *n* are integers; $x(i)$ and $y(j)$ are membership functions that characterize the fuzzy sets X and Y, respectively. Then the fuzzy addition is defined as:

$$X + Y = \{\min[x(i), y(j)]/(i+j); 1 \leq i, j \leq n\} \qquad (15)$$

The fuzzy summation is a repeated process of the fuzzy addition. The fuzzy multiplication is defined as:

$$X * Y = \{\min[x(i), y(j)]/(i*j); 1 \leq i, j \leq n\} \qquad (16)$$

and the fuzzy division is defined as:

$$X / Y = \{\min[x(i), y(j)]/(i/j); 1 \leq i, j \leq n\} \qquad (17)$$

Another concern in the implementation of equation(12) is whether the fuzzy normalization operation should be conducted after each fuzzy operation. The fuzzy normalization *NOR* is defined as:

If

$$Z = NOR[X] \qquad (18)$$

then

$$Z = \{z(i)/i; 1 \leq i \leq n\} \qquad (19)$$

where

$$z(i) = \{x(i)/\max(x(i)); 1 \leq i \leq n\} \qquad (20)$$

The overall hazard evaluation of system which is obtained by the *FWA* operation,can be presented as value based on a ranking index.

There are several procedures for ranking of fuzzy subsets such as methods proposed by: Yager (1981), Juang (1988), Adamo (1980), Baas *et al.* (1977), Jain (1977), Dubois *et al.* (1983), Baldwin *et al.* (1979). A simple model for the ranking index developed by Juang (1988) can be defined as follows:

$$HPI = \frac{(A_L - A_R + C)}{2C} \qquad (21)$$

where:

$0 \leq HPI \leq 1$, is the utility

A_L is area enclosed by the universe and to the left of the membership function of the final fuzzy set obtained

A_R is the area enclosed by the universe and to the right of the membership function of the final fuzzy set obtained.

C is a constant which is the area enclosed by the universe.

We can demonstrate this procedure by means of a small example. Let $R_1 = \{1/0, 0.6/1, 0.2/2, 0/3\}$, $R_2 = \{0/0, 0.2/1, 0.6/2, 1/3\}$, $W_1 = \{1/0, 0.5/1, 0/2\}$, and $W_2 = \{0/1, 0.4/2, 0.8/3\}$. Then:

$$R = \frac{\sum_{i=1}^{2}(R_i * W_i)}{\sum_{i=1}^{2} W_i} = \frac{(W_1 * R_1) + (W_2 * R_2)}{W_1 + W_2} \tag{22}$$

We calculate:

$W_1 + W_2 = \{\min(1, 0) / [0+1], \min(1, 0.4) / [0+2], \min(1, 0.8) / [0+3], \min(0.5, 0) / [1+1], \min(0.5, 0.4) / [1+2], \min(1, 0.8) / [1+3], \min(0, 0) / [2+3], \min(0, 0.4) / [2+2], \min(0, 0.8) / [2+3]\}$

$= \{0/1, 0.4/2, 0/2, 0.8/3, 0.4/3, 0/3, 0.8/4, 0/4, 0/5\}$

after normalization:

$= \{0/1, 0.5/2, 1/3, 1/4, 0/5\}$

In the same manner:

$(W_1 * R_1) = \{\min(1, 1) / [0 * 0], \min(1, 6) / [1 * 0], \min(1, 0.2) / [0 * 2], \min(1, 0) / [3 * 0], \min(0.5, 1) / [0 * 1], \min(0.5, 0.6) / [1 * 1], \min(0.5, 0.2) / [1 * 2], \min(0.5, 0) / [1 * 3], \min(0, 1) / [0 * 2], \min(0, 0.6) / [1 * 2], \min(0, 0.2) / [2 * 2], \min(0, 0) / [2 * 3]\}$

$= \{1/0, 0.6/0, 0.2/0, 0/0, 0.5/0, 0.5/1, 0.2/2, 0/2, 0/3, 0/4, 0/6\}$

$= \{1/0, 0.5/1, 0.2/2, 0/3, 0/4, 0/6\}$

$(W_2 * R_2) = \{\min(0, 0) / [1 * 0], \min(0, 0.2) / [1 * 1], \min(0, 0.6) / [1 * 2], \min(0, 1) / [1 * 3], \min(0.4, 0) / [2 * 0], \min(0.4, 0.2) / [2 * 1], \min(0.4, 0.6) / [2 * 2], \min(0.4, 1) / [2 * 3], \min(0.8, 0) / [3 * 0], \min(0.8, 0.2) / [3 * 1], \min(0.8, 0.6) / [3 * 2], \min(0.8, 1) / [3 * 3]\}$

$= \{0/0, 0/1, 0/2, 0.2/2, 0/3, 0.2/3, 0.4/4, 0.4/6, 0.6/6, 0.8/9\}$

after normalization:

$= \{0/0, 0/1, 0.25/2, 0.25/3, 0.5/4, 0.75/6, 1/9\}$

and

$(W_1 * R_1) + (W_2 * R_2) = \{1/0, 0.5/1, 0.2/2, 0/3, 0/4, 0/6\} + \{0/0, 0/1, 0.25/2, 0.25/3, 0.5/4, 0.75/6, 1/9\}$

$= \{0/0, 0/1, 0.25/2, 0.25/3, 0.5/4, 0.75/6, 1/9, 0/1, 0/2, 0.25/3, 0.25/4, 0.5/5, 0.5/7, 0.5/10, 0/2, 0/3, 0.2/4, 0.2/5, 0.2/6, 0.2/8, 0.2/11, 0/3, 0/4, 0/5, 0/6, 0/7, 0/9, 0/12, 0/4, 0/5, 0/6, 0/7, 0/8, 0/10, 0/13, 0/6, 0/7, 0/8, 0/9, 0/10, 0/12, 0/15\}$

$= \{0/0, 0/1, 0.25/2, 0.25/3, 0.5/4, 0.5/5, 0.75/6, 0.5/7, 0.2/8, 1/9, 0.5/10, 0.2/11, 0/12, 0/13, 0/15\}$

The R is calculated by division of $((W_1 * R_1) + (W_2 * R_2))$ by $(W_1 + W_2)$, which is:

$R = \max\{0/0, 0/1, 0/2, 0.25/1, 0/3, 0.25/1, 0/4, 0.5/2, 0.5/1, 0/5, 0/1, 0/6, 0.5/3, 0.75/2, 0/7, 0/8, 0.2/4, 0.2/2, 0/9, 1/3, 0/10, 0.5/5, 0/2, 0/11, 0/12, 0/6, 0/4, 0/3, 0/13, 0/15, 0/5, 0/13\}$

$= \{0/0, 0.5/1, 0.75/2, 1/3, 0.2/4, 0.5/5, 0/6, 0/7, 0/8, 0/9, 0/10, 0/11, 0/12, 0/13, 0/15\}$

In order to reduction of number elements of the set to the integers base, the elements over the integer base of the set are deleted using Clements method. Thus the final fuzzy set is:

$R = \{0/0, 0.5/1, 0.75/2, 1/3\}$

The normalized fuzzy set is ranked for presentation of the set as a value. The ranking of depicted final fuzzy set in Figure 4 is:

$$HPI = \frac{(A_L - A_R + C)}{2C} = \frac{1.12 + 3}{6} = 0.708 \tag{23}$$

Then using fuzzy set theory, extention principle, *FWA*, and ranking of fuzzy set, the overall risk of system is evaluated on the basis of considered criteria.

3 Liquefaction evaluation criteria

3.1 Influence items on liquefaction potential of soils

Based on a comprehensive review of literatures on the subject, past observations, and field study of the northwestern part of Iran, where more than ten slope failures and liquefaction phenomena occurred as a result of the 1990 Manjil-Iran earthquake, heavy rainfalls, and human activities, the most important factors in liquefaction phenomena are geological, geomorphological, seismic and geotechnical characteristics of the ground. Evaluation criteria for liquefaction hazard will be presented here. Because the criteria were established on the basic characteristics of the hazards and the most significant properties of past observation in the world were considered in their selection, those will, no doubt, find global application.

As the liquefaction potential of any soil deposits is affected by the soil properties, environmental factors, and

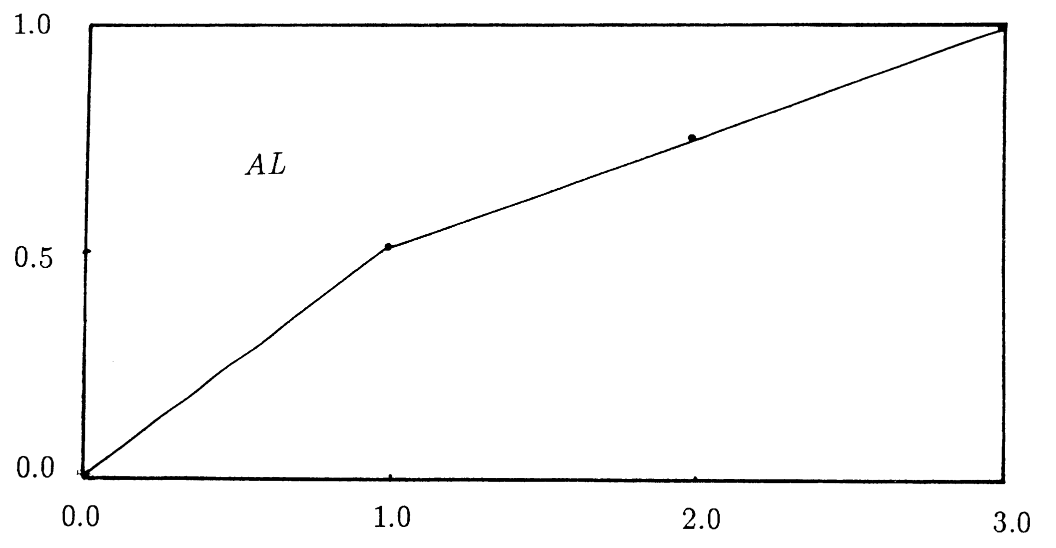

Fig. 4 Membership function of final fuzzy set of given example

characteristics of the earthquake(Seed *et al.*, 1982), among many factors which may have some influence on liquefaction potential of soils, the following eight items are selected from a comprehensive review of the literature on the subject, past expereince, engineering judgment, *etc.*

1. Geological characteristics of site
2. Geomorphological characteristics of site
3. Relative site amplification
4. Intensity increments
5. Sandy layer thickness
6. Water table level
7. Surface layer thickness
8. Type of soils

Each item has 5 to 29 factors. A total 8 factors were adopted for evaluation of liquefaction potential in each mesh or point by using geological and topographical maps; seismic characteristics by considering geological aspects of area; and geotechnical features of the area.

The considered criteria are defined for the earthquakes with a minimum magnitude of 5.2 which had induced liquefaction in the world, on the basis of comprehensive review of published papers of reportd earthquakes in the various countries.

3.2 Geological and geomorphological criteria

The geological and geomorphological factors directly or indirectly influence geotechnical properties that control the liquefaction susceptibility of sediments. Thus, the correlation between past liquefaction occurrence with geological and geomorphological parameters is the best way to clarifying the reliability of these factors in prediction of liquefaction.

The geological criteria were established on the susceptibility of geological units to liquefaction given by Youd *et al.*(1978). On the geological criteria, the susceptibility of sediments to liquefaction are determined by considering their age. The selected geological criteria are given in Table 1.

Table 2 shows the liquefaction susceptibility chart of geomorphological setting for characterizing the liquefaction potential, given by Wakamatsu (1992). These criteria, which have been made based on the site specific correlation between past liquefaction occurrances and geologic and geomorphologic settings, are possible to identify the liquefaction potential of sediments.

3.3 Seismicity criteria

The characteristics of earthquake ground motions are affected by several factors such as source, path and site effects, but it has been pointed out that variation of site effects is very large. Although the attenuation relations give the ground motion intensity on reference ground, observations during past earthquakes have suggested that variation of the intensity of shaking is significantly dependent on local site conditions. In this study the seismic intensity increments and relative site amplifications were considered as a seismic criteria in geological hazard evaluation.

The empirical correlations between the surface geology and seismic intensity increments have been established by

Table 1 Susceptibility of geological items to liquefaction (after Youd and Perkins, 1978)

Geological category (Type of deposits)	Susceptibility of sediments considering age of them (Recent	Holocene	Pleistocene	Pre-Pleistocene)
River Channel	A	B	D	E
Delta	A	B	D	E
Uncompacted fill	A	-	-	-
Flood Plain	B	C	D	E
Delta and fan delta	B	C	D	E
Lacustrine and Playa	B	C	D	E
Collvium	B	C	D	E
Dunes	B	C	D	E
Loess	B	B	B	E
Tephra	B	B	E	E
Sebka	B	C	D	E
Esturine	B	C	D	E
Beach low wave energy	B	C	D	E
Laggoonal	B	C	D	E
Foreshore	B	C	D	E
Alluvial fan and plain	C	D	D	E
Marine terraces and plains	C	D	E	E
Beach high wave energy	C	D	E	E
Talus	D	D	E	E
Glacial fill	D	D	D	E
Tuff	D	D	D	E
Residual soils	D	D	E	E
Compacted fill	D	-	-	-
Others	E	E	E	E

Note:
A=very high liquefaction susceptibility, B=high liquefaction susceptibility, C=moderate liquefaction susceptibility, D=low liquefaction susceptibility, E=very low liquefaction susceptibility

Table 2 Susceptibility of geomorphological items to liquefaction (after Wakamatsu, 1992)

Geomorphological units	Susceptibility of units to liquefaction
Natural levee(Edge)	A
Abandoned river channel	A
Former pond	A
Dry river bed consisting of sandy soils	A
Sand dune(lower slope of sand dune)	A
Artificial beach	A
Interlevee low land	A
Reclaimed land by drainage	A
Reclaimed land or filled land	A
Spring	A
Fill on boundary zone between sand dune and low land	A
Fill adjorning cliff	A
Fill on marsh or swamp	A
Fill on reclaimed land by drainage	A
Other type of fill	A
Alluvial fan with vertical gradient more than 0.5%	B
Valley plain consisted of sandy soils	B
Natural levee(Top)	B
Back marsh	B
Marsh and Swanp	B
Delta	B
Sand bar	B
Beach	C
Valley plain consisted of gravel or cobble	D
Alluvial fan with vertical gradient more than 0.5%	D
Dry river bed consisting of gravel	D
Gravel bar	D
Sand dune(Top of sand dune)	D
Others	E

Note:
A=very high liquefaction susceptibility, B=high liquefaction susceptibility, C=moderate liquefac tion susceptibility, D=low liquefaction susceptibility, E=very low liquefaction susceptibility

many investigators such as Medvedev(1962), Evernden, *et al.*(1985), Kagami, *et al.*(1988), and Astroza *et al.*(1991) which are based on observations during earthquakes in Middle Asia, California, Japan, and Chile, respectively. The Medvedev criteria is used in this study which is shown in Table 3.

Another seismic criteria is presented in terms of relative site amplifications which were proposed by Borcherdt *et al.*(1976) to evaluate the effects of site geology by measuring generated ground motions during nuclear explosions at sites with various geological conditions and calculating the spectral amplifications of the motions with respect to those at granite rock. They found a strong correlation between surface geology and the average horizontal spectral amplification which is the average of the spectral amplification in the frequency range of 0.5 to 2.5 H_Z.

The selected criteria gives the values of the relative amplifications for different soils, which were proposed by Shima (1978), based on the analytical calculation of seismic response of ground. This is the ratio of the maximum value of ground response in the frequency range of 0.1 to 10 H_Z, with respect to that at loam ground. This criteria is illustrated in Table 4.

Table 3 Intensity increments of ground (Medvedev, 1962)

Geological units	Opportunity rate
Granites	E
Limestone, sandstone, and shales	D
Gypsum, and marl	D
Coarse material ground	C
Sandy ground	C
Clayey ground	C
Fill	B
Moist ground	B
Moist fill and soil ground	A

Note:
A=very high, B=high, C=moderate, D=low, E=very low

3.4 Geotechnical criteria

The geotechnical criteria consists of thickness of sandy layers, water table, surface layer's thickness, and type of soils. These criteria are given in Table 5 which were established based on past observations of liquefied and nonliquefied sites during earthquakes.

The sandy layers with the thickness greater than 3m are assumed to have a very high likelihood of occurrance of liquefaction, and with the thickness less than 0.5m the susceptibility to liquefaction is very low. For water table depths greater than 10m, the likelihood of liquefaction in most deposits is very low, and for water table less than 1m this likelihood is very high. Also, severity of layers with more than 10m depth to liquefaction is very low, while the susceptibility of layers to liquefaction with less than 3m depth is very high. Since liquefaction is the most common phenomena during earthquakes in loose sandy soils, the type and grain size of sandy soils were considered as effective factors in liquefaction occurrance. Consequently, the distance between very high to very low likelihood to liquefaction of soil layers is divided into five intervals that are given by alphabetical symbols from A to E.

4 Hazard evaluation

In general, the rate of natural hazard potential(liquefaction and slope failure potential), the knowledge and information of the selected evaluation criteria, and the weight among them may be assessed from some qualitative evaluation scheme and recorded by descriptive terms.

When the qualitative evaluation scheme is adopted, results of the assessment are generally preferred in linguistic terms. For example, rating for the slope failure potential or liquefaction susceptibility, according to a particular criterion may be recorded by very low, low, medium, high, and very high. Similary, the weight applied to each of the adopted criteria may use one of the following terms on the natural language expression: extremely important, very important, important, moderately important, and relatively unimportant. Then the expression of the assessed hazard rate in linguistic terms must be repre-

Table 4 Relative site amplification of ground (after Shima, 1978)

Geological units	opportunity rate
Peat	A
Humus	B
Clay	B
Loam	C
Sand	D
Others	E

Note:
A=very high, B=high, C=moderate, D=low, E=very low

Table 5 Susceptibility of Geotechnical items to liquefaction

Thickness of sandy layers(T)	Grade	Water table(W.T)	Grade
$T > 3m$	A	$W.T < 1m$	A
$2m < T \leq 3m$	B	$1m \leq W.T < 3m$	B
$1m < T \leq 2m$	C	$3m \leq W.T < 5m$	C
$0.5m < T \leq 1m$	D	$5m \leq W.T < 10m$	D
$T < 0.5m$	E	$W.T \leq 10m$	E

Thickness of surface layers(T.S)	Grade	Type of soils	Grade
$T.S < 3m$	A	SP	A
$3m \leq T.S < 5m$	B	SW	B
$5m \leq T.S < 7m$	C	SM	C
$7m \leq T.S < 9m$	D	SC	D
$T.S \geq 9m$	E	Others	E

Note:
A=very high liquefaction susceptibility, B=high liquefaction susceptibility, C=moderate liquefaction susceptibility, D=low liquefaction susceptibility, E=very low liquefaction susceptibility
SP: Poor graded sand, SW: Well graded sand, SM: Silty sand, SC: Clayey sand, Others: Other type of soils

sented with fuzzy set before further processing on the system. When rating and weight terms are represented by fuzzy set and are input into a deterministic model, the output will be a fuzzy set (Juang, 1992). Then by considering the normal weighted average model and generalization of it with input fuzzy set, the rating of hazard potential according to adopted criteria will be simulated as a fuzzy set. The used fuzzy set operation for this, is fuzzy weighted average operation *FWA*, which given in Equation (12).

Using the equation of *FWA*, and according to each criteria, the rating is assessed and recorded as one of the five fuzzy subset such as: *A,B,C,D,* and *E* which are defined as follows,

A is very high, *B* is high, *C* is moderate, *D* is low and E is very low.

These fuzzy subsets are assumed to be normal and convex and they are characterized by the membership functions $f(x)$, which are refered to as π-curves(Andonyadis *et al.*, 1985). The π-curve is symetric and the area under the curve is one half of the range over which the curve or function is defined, as shown in Figure 5.

The range reflects the degree of fuziness. Figure 6 shows an example of the π-curves representing the fuzzy subsets *A,B,C,D,* and *E*. It is noted that in this study, the weight assigned to each criteria also takes as its value one of the above five fuzzy subsets. Then membership functions for the weight terms are the same as those for the rating terms (Fig. 6) which is listed in Table 6.

Then the final fuzzy set represents the total hazard potential and is calculated as follows:

1. For each membership function, one that characterizes fuzzy subsets *A,B,C,D,* and *E*, determines its cumulative function $F(x)$, by integration. The maximum value of the cumulative function $F(x)$, which is the total area under the curve depends on the over range which the π-curve is defined.

2. By the notation that every term or variable in Equation (12), is a fuzzy subset whose membership function, cumulative function, and maximum cumulative functional values are known, a uniform random number for each term or variable in the right-hand side of Equation (12), is generated. Then by normalizing the uniform random number with respect to the maximum functional value of the cumulative function, and then equating the normalized uniform random number to the cumulative function $F(x)$, a value x can be calculated for each variable or fuzzy subsets. By repeating this process for each fuzzy subset in the right-hand side of equation 12, a set of random numbers (x's) is obtained and Equation (12) is ready to be evaluated.

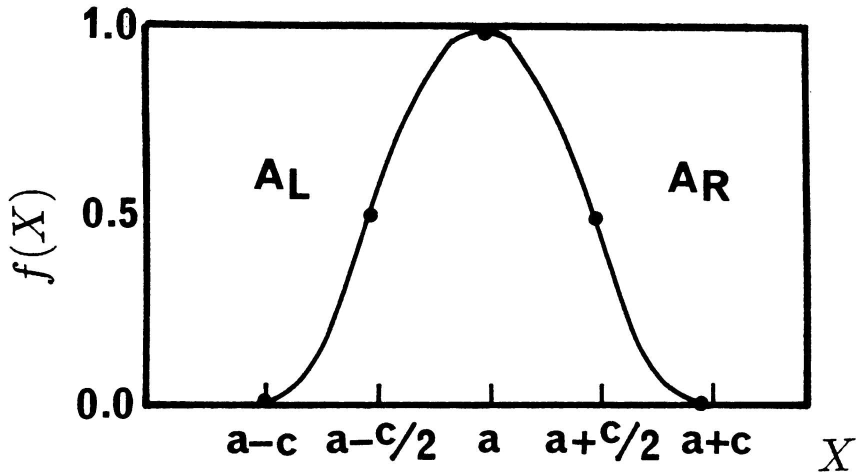

Fig. 5 π Curve and fuzzy ranking model

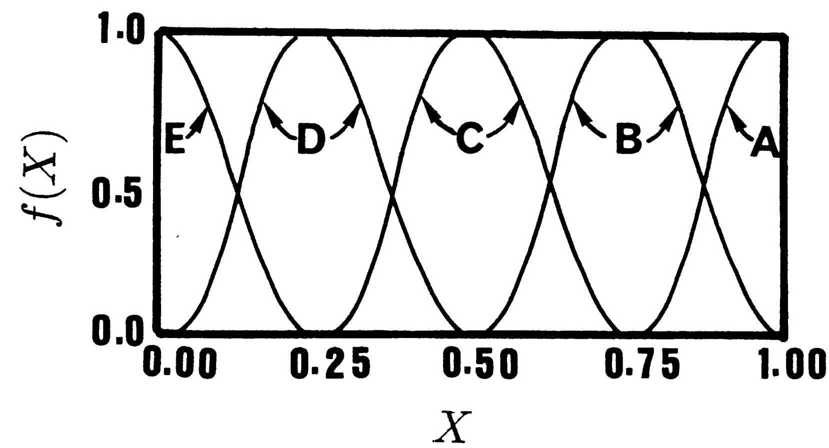

Fig. 6 Membership function of used fuzzy subsets(after Juange *et al.*, 1992)

3. Calculate the weighted average R, by entering Equation (12) with the random numbers (x's) obtained.
4. Repeat steps 2 and 3 a large number of times for satisfactory results. The number of repeatations or simulations may be estimated by trial-and error procedure.
5. Determine the minimume, maximum, mean, and standard deviation of the R values obtained from step 3 and 4. These four parameters are then fitted with a beta distribution function (Harr, 1987).
6. Normalize the curve-fitted beta distribution function with respect to its maximum functional value. This step results in the desired membership function that characterizes the final fuzzy subset obtained from Equation (12).

Using described method, the final fuzzy subset which represents the overall assessment of group of alternatives as a hazard rate of the system is obtained. Then, a mapping model is often required for ranking or converting the final fuzzy subsets into some utility.

A simple model developed by the Juang, used for ranking of final fyzzy subsets. This utility model for a hazard potential assessment defined in Equation (21).

A simple example of liquefaction potential assessment by fuzzy set theory will be described as follows:

In this simple example, the liquefaction potential of the small area is characterized by geological, seismic and geomorphological factors. These items are expressed by linguistic variables of Low, Medium, and Very high, and the relative importance of them are assumed Moderately important, Very important, and Notimportant which is given by fuzzy sets A, B, C, D, and E. These fuzzy sets are assumed:

$A = \{0/1, 0.1/2, 0.6/3, 1/4\}$

$B = \{0/1, 0.2/2, 0.9/3, 0.7/4\}$

$C = \{0.2/1, 1/2, 1/3, 0.2/4\}$

$D = \{1/1, 0.6/2, 0.1/3, 0/4\}$

$E = \{1/1, 0.2/2, 0/3, 0/4\}$

Then the liquefaction potential of the considered area based on the *FWA* and fuzzy arithmatic operations will be

Table 6 Parameters that define π curve membership function (after Juange *et al.*, 1992)

Fuzzy set (Symbol)	Parameters that define π curve functions of fuzzy sets			Descriptor for rating	Descriptor for weight
	a	c	range		
A	1.0	0.25	$a-c \leq x \leq a$	Very high	Extremely important
B	0.75	0.25	$a-c \leq x \leq a+c$	High	Very important
C	0.50	0.25	$a-c \leq x \leq a+c$	Medium	Important
D	0.25	0.25	$a-c \leq x \leq a+c$	Low	Moderately important
E	0.00	0.25	$a \leq x \leq a+c$	Very low	Relatively unimportant

Note:
These membership functions are symmetrical and take the following form: $f(x) = 2\{[x-(a-c)]/c\}^2, a-c \leq x \leq (a-c/2)$; $f(x) = 1-2[(x-a)/c]^2, (a-c/2) \leq x \leq (a+c/2)$; and $2\{[(a+c)-x]/c\}^2, a+c/2 \leq x \leq a+c$.

calcualated as follows:

$$R = (Low*Moderately\ important) + (Medium*Very\ important) + (Very\ high*Notimportant) / (Moderately\ important + Very\ important + Not\ important) \quad (24)$$

Where, R is liquefaction potential index given by *FWA*. Then the obtained fuzzy sets of (*Moderately important* + *Very important* + *Notimportant*) after normalization will be:

$(D + B + E) =$ {0/1, 0/2, 0/3, 0.22/4, 1/5, 0.77/6, 0.66/7, 0.22/8, 0.11/9, 0/10, 0/11, 0/12}

In the same manner the fuzzy sets of $(D*D) + (C*B) + (A*E)$ after normalization will be:

{0/1, 0/2, 0/3, 0/4, 0.1/5, 0.22/6, 0.22/7, 0.22/8, 0.6/9, 0.61/10, 1.0/11, 1.0/12, 1.0/13, 1.0/14, 0.93/15, 0.85/16, 0.77/17, 0.64/18, 0.6/19, 0.6/20, 0.6/21, 0.6/22, 0.5/23, 0.36/24, 0.35/25, 0.22/26, 0.2/27, 0.2/28, 0.2/29, 0.1/30, 1/31, 0.1/32, 0.1/33, 0/34, 0/35, 0/36,0/37, 0/38, 0/39, 0/40, 0/41, 0/42, 0/43, 0/44, 0/45, 0/46, 0/47, 0/48}

Using the fuzzy division of (*Low* * *Moderately important*) + (*Medium* * *Very important*) + (*Very high* * *Notimporatnt*) by (*Moderately important* + *Very important* + *Notimportant*), the calculated final fuzzy set,(R), which represents the liquefaction potential of the area after reduction the number of alternatives using Clement's method becomes:

$R =$ {0.22/1, 0.77/2, 0.93/3, 0.36/4}

and after normalization this becomes:

$R =$ {0.23/1, 0.82/2, 1/3, 0.38/4}

The liquefaction potential rate of the area is calcuated by using the ranking index model given by Juang as follows:

$$LPI = \frac{(A_L - A_R + C)}{2C} = \frac{1.45 - 1.12 + 5}{10} = 0.53 \quad (25)$$

The final fuzzy set which represents the *LPI* given in Figure 7. Then the liquefaction potential of the area is expresed as Medium in linguistic terms.

5 Practical application of method in northwestern Iran

The liquefied and nonliquefied area during the 1990 Manjil-Iran earthquake in Gilan plain (Fig. 1) was chosen for the evaluation of capability of the method on the prediction of hazard potential. For each mesh, the liquefaction potential is assessed using the established criteria given in tables 1 to 5, and existing boring data.

All the weights and ratings are expressed in linguistic terms. The weight of criterion is assigned with one of the five terms: not important, moderately important, important, very important, and extremely important for occurrence of liquefaction. In a similar manner, the rating assumed one of the following terms: very low, low, moderate, high, and very high susceptible to liquefaction.

In this study, the weights of very important, important, extremely important are considered for geological, geomorphological, seismic, and geotechnical criteria.

The obtained ratings of each criteria and considered weight of them in form of linguistic terms are translated into fuzzy sets. Using the presented procedures earlier the liquefaction potential of each point is calculated. The liquefaction potential contours can be drawn based on the calculated *LPI* values, which the result will be a liquefaction potential map of the area.

The calculated *LPI* for the Gilan plain area given in Table 7. Also, the liquefaction potential map of the Astaneh city (location 1-30 in Fig. 1) is illustrated in Figure 8. The comparison of calculated *LPI* and past observations during the 1990 Manjil-Iran earthquake reveals that, most of the locations which were characterized by liquefaction phenomena during the earthquake show the high liquefaction potential calculated by fuzzy set theory.

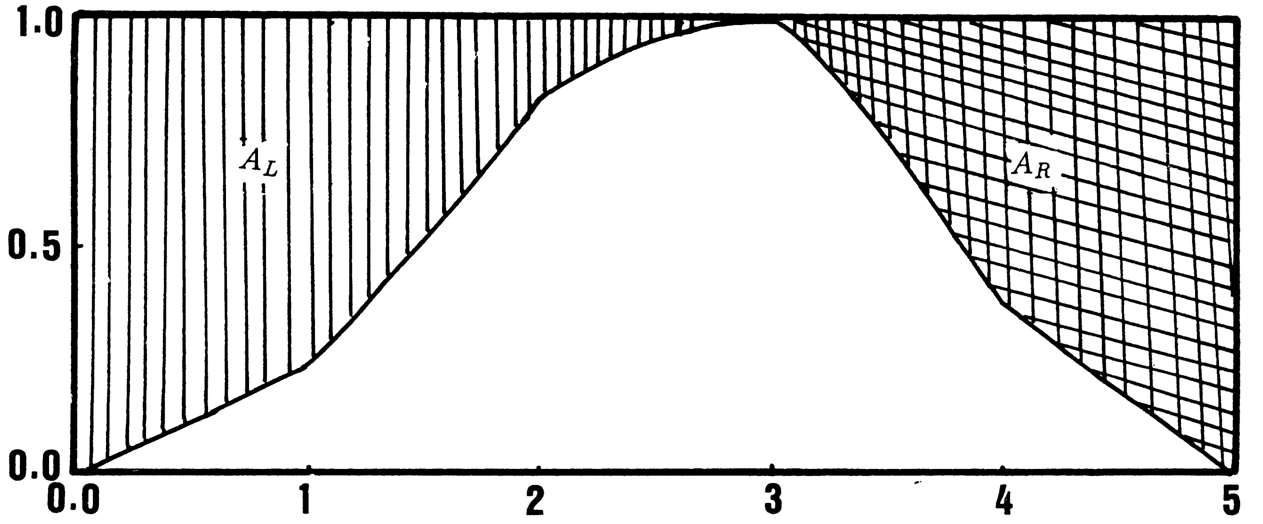

Fig. 7 Final fuzzy set of LPI

Table 7 Calculated liquefaction potential of Gilan plain area using fuzzy sets

No.	LPI	Previous observations Liquefied / Nonliquefied	No.	LPI	Previous observations Liquefied / Nonliquefied
1	0.757	Liquefied	30	0.574	Liquefied
2	0.757	Liquefied	31	0.700	Unknown area
3	0.648	Liquefied	32	0.440	Unknown area
4	0.694	Liquefied	33	0.437	Unknown area
5	0.740	Liquefied	34	0.512	Unknown area
6	0.757	Liquefied	35	0.532	Unknown area
7	0.774	Liquefied	36	0.415	Unknown area
8	0.757	Liquefied	37	0.524	Unknown area
9	0.660	Liquefied	38	0.375	Unknown area
10	0.717	Liquefied	39	0.368	Unknown area
11	0.757	Liquefied	40	0.429	Unknown area
12	0.757	Liquefied	41	0.553	Unknown area
13	0.757	Liquefied	42	0.375	Unknown area
14	0.688	Liquefied	43	0.538	Nonliquefied
15	0.688	Liquefied	44	0.449	Unknown area
16	0.648	Liquefied	45	0.483	Nonliquefied
17	0.665	Liquefied	46	0.483	Nonliquefied
18	0.803	Liquefied	47	0.500	Unknown area
19	0.688	Liquefied	48	0.635	Unknown area
20	0.648	Liquefied	49	0.578	Unknown area
21	0.631	Liquefied	50	0.506	Nonliquefied
22	0.717	Liquefied	51	0.271	Nonliquefied
23	0.757	Liquefied	52	0.224	Unknown area
24	0.757	Liquefied	53	0.592	Unknown area
25	0.649	Liquefied	54	0.443	Unknown area
26	0.717	Liquefied	55	0.515	Unknown area
27	0.677	Liquefied	56	0.448	Unknown area
28	0.701	Liquefied	57	0.448	Unknown area
29	0.701	Liquefied			

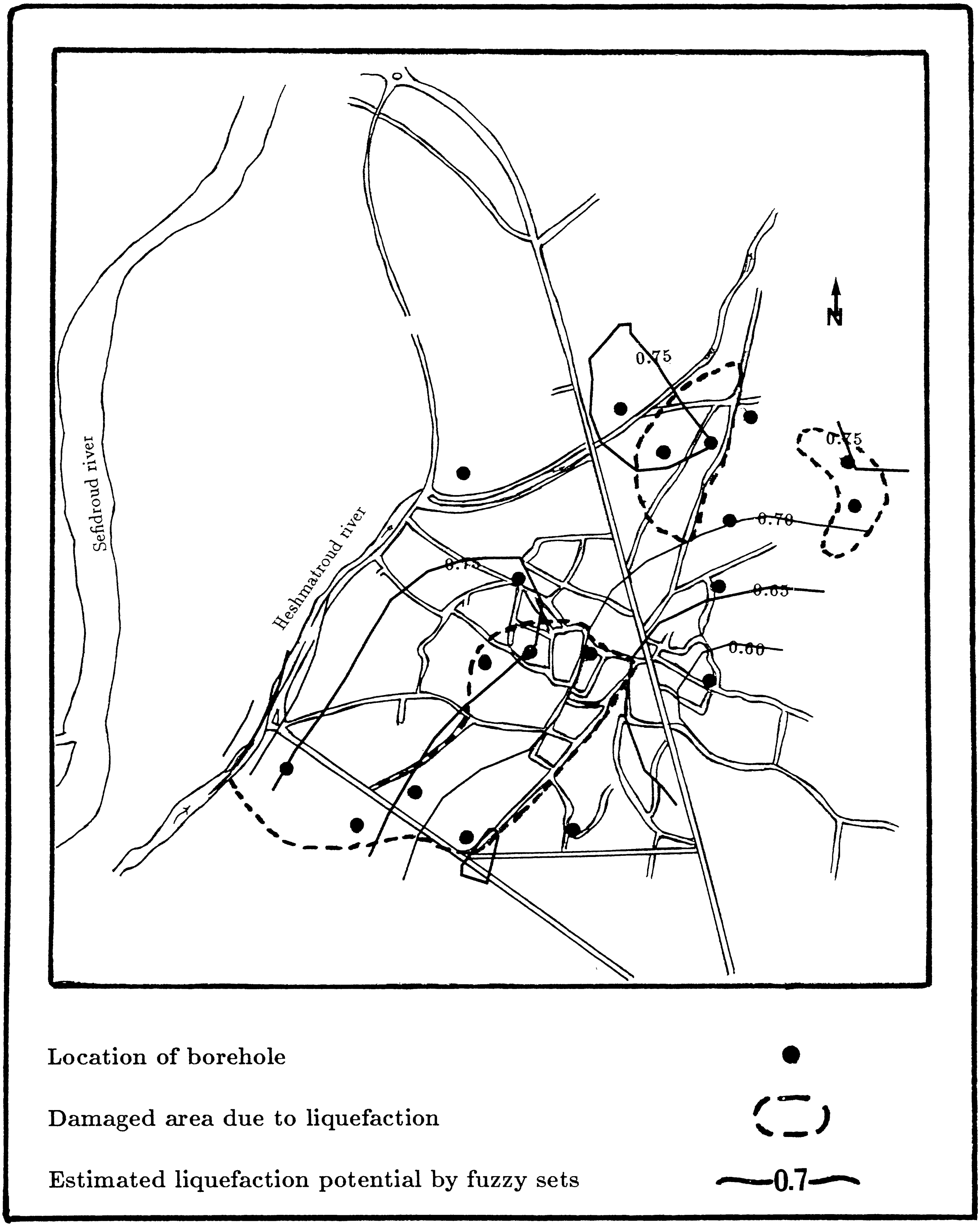

Fig. 8 Liquefaction potential map of Astaneh city (northwestern Iran)

6 Discussion and conclusion

The liquefaction hazard assessment systems including rate of hazards potential, informations, rating and weight of criteria are obtained based on the qualitative evaluation scheme and are recorded by linguistic terms or variables. The application of linguistic variables in liquefaction hazard assessment system enables hazard risk evaluation in rapid and low-cost way. But on the other hand, using these variables increases the uncertainty of the system, as discribed earlier. In order to remove this problem and evaluate hazard potential of ground on the established criteria, the multicriteria evaluation method such as fuzzy sets theory was used.

The fuzzy sets theory is the method that, when rating and weight of variables are presented by fuzzy subsets the output will be fuzzy sets by deterministic model. This advantage enables us to apply this method to quantification of linguistic descriptions. In fuzzy set method, the linguistic variables must be represented by fuzzy subsets and then using the fuzzy weighted average operation the hazard potential in system will be simulated, and then the overall of assessed hazard potential is given by a fuzzy set. The ranking index model is used for converting the final fuzzy subsets into some utilities as a mapping of fuzzy sets. By repeating this procedures for all of the concerned area, the hazard potential of the area can be evaluated and mapped.

Also, the capabilities of presented method for assessment of liquefaction hazard was examined in northwestern part of Iran. The obtained data from 56 locations on the basis of established criteria, were input into the constructed hazard evaluation system by fuzzy sets method. The simulated results by this evaluation system which was given in Table 7 revealed that the method is useful for detail hazard assessment. Furtheremore, this system enables us to represent hazards evaluation in any desired form.

The calcualted hazard rate by fuzzy set can be input into the contour line program for preparation of geological hazard maps.

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要旨：ファジィ集合理論をツールとした地質災害の危険度の評価

：ファテミ・アグダ，小池克明，鈴木敦巳，北園芳人

飽和した砂質地盤における液状化ポテンシャルの算出のために，ファジィ集合理論を用いた低コストで迅速な評価手法を検討した．このシステムでは，表層地質，地形，地盤振動の増幅率・震度を含む地震工学的特性，および砂層の厚さ・地下水位・表土層の厚さ・土質を含む地盤工学的特性の８項目を用い，各項目には液状化に対しての評価規準を設定した．個々の評価基準は自然言語で表現されるので，これにメンバーシップ関数を与え，ファジィ集合演算によって液状化ポテンシャルを算出した．

本研究で提案する評価システムを，1990年6月に地震の被害を被ったイラン北西部のGilan平野に適用した．液状化ポテンシャルの高い部分は，この地震によって液状化した砂質地盤と概ね対応しており，本システムの有用性が確かめられた．

キーワード：ファジィ集合，災害規準，災害度，災害評価，液状化

(1994年５月13日受付，1994年８月20日受理)