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Incentive subsidy scheme design with elastic transport demand

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SUMMARY

Huge public transport subsidies caused by deficits have become a heavy financial burden on some local governments due to the decline of bus passenger numbers. It is essential to apply the performance-based contract to bus services considering maximization of social welfare. This paper constructs an incentive subsidy contract considering the decision-making powers of the service level and calculating the proper frequency elasticity aiming at two problems of performance-based contracts. Meanwhile, we consider a role of bus operators ignored by most researchers. Under the scheme, the decision-making power of the service level is discussed based on five assumptions, and meanwhile, bus operators are motivated to reduce cost and improve service level in the scheme. The case of the bus service of Arao city indicates that the optimal frequency equals to zero when bus operators decide frequency. If bus operators determine efforts, the optimal effort also equals to zero with the goal of maximizing the profit. Also, bus operators can play their roles in lessening cost and improving service level to help bus operators and the local government achieve a win-win situation, which maximizes the social benefit in this subsidy scheme when all factors are decided by the government. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS: bus services; incentive subsidy scheme; Laffont-Tirole's model; frequency elasticity

1. INTRODUCTION

External costs are incurred in urban areas that experience substantial levels of traffic congestion due to the increasing number of private cars. The existence of these external costs is reflected in governments around the world seeking more sustainable means of meeting passenger transport requirements, including support for public transport because of its capacity to meet social obligations and to reduce the external costs of private car use. Therefore, transit subsidies are necessary, given substantial economies of scale, in order to permit services to be set at a level that will result in reasonably efficient use of the public transport. Also, it is regarded as important to provide service for those who are unable to drive, in addition to helping cities find some relief from urban congestion and reducing fuel consumption.

Although transit subsidies are necessary to lower the price of public transport and diminish discrimination, they have long been controversial. Pucher *et al*. [1] noted that costs and subsidies are jointly determined, as higher costs elicit more subsidies, which in turn induce higher costs.

As a mechanism with a major objective of containing the cost to government of service provision, competitive tendering (CT) was proposed and applied. The CT contract attracted significant interest because of its attempt to apply responsibilities and incentives to contract operators to reduce cost. The three kinds of incentive contracts are gross-cost, net-cost, and cost-plus contracts. The gross-cost contract introduced in London is payment to the operator of a specified sum of provision of the specified service for a specified period, with all revenue collected being for the account of the government [2]. Under the net-cost contract, instituted to Hong Kong bus companies, revenues must cover cost and guarantee profit. Worcman [3] concluded that contract payments of

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Curitiba bus services are calculated from an analysis of each operator's cost structure, and profit margins are set at 10% of turnover (cost-plus contract).

The success of the aforementioned CT regimes is measured by cost reduction. The regimes are also used to force bus companies to improve service level. Gargett and Wallis [4] and Radbone [5] introduced the Adelaide competitive tendering and contracting that was initiated by the government of South Australia in 1994. Under this, a fixed sum, which was the basis of the tender price bid and a patronage-related amount, which was calculated according to the change in patronage from the base year, comprised the contract payment.

However, there are some criticisms of competitive tendering. Hensher and Stanley [6] pointed out weakness of competitive tendering. Although competitive tendering has lead to some noticeable gains in cost efficiency, the sizable financial gains are once-off, and subsequent retendering has delivered fewer financial benefits. Another reason is that CT is focused on individual contracts with no mechanism to ensure that the incentive payment support sums to the optimal subsidy commitment across a broader geographic area. Consequently, researchers have looked for alternative ways for the appeal of active competition. Performance-based quality contracts align with this view through incentive payments and benchmarked best practice costs.

Hensher and Stanley [6] consummated the concept of performance-based contracts (PBCs). They proposed that the PBC framework takes into account the commercial angle of operators and the obligations of government to ensure that subsidy support is spent in a way that maximizes the net benefits of society. A PBC was proposed that combines payment for delivering a minimum level of service and established the optimum subsidy on the basis of the system-wide maximization of social surplus [7]. The reward system can both meet government community service obligations and realize an incentive regime that rewards operators for passengers increases (above minimum level of service patronage levels).

Gonzalez-Dıaz and Montoro-Sanchez [8] concluded some lessons from incentive theory promoting quality in bus transport. Most performance-based contracts regulate the service level as the performance indicator; when bus operators improved the service level, public transport demand may likely be influenced.

There are two questions regarding performance-based contracts. The first is which part will determine the service level. In most contracts, the service level is usually determined by the bus operators. The reason is that a public transport operator usually has the best and most detailed knowledge of the market they serve and also better knowledge of the cost structure of public transport services to assess the cost of increasing or decreasing frequencies and so forth. However, the objective of the bus operator is maximizing profit, and political considerations have weighed heavily in the determination of a level barely sufficient to maintain a tolerable service, with little attempt at a rational determination based on costs and benefits. The second question is that of elasticity between the service level and the demand. Bus service demand generally rises when bus service quality achieves a viable and reasonable standard. There are many important aspects to be considered in increasing bus service demand, including the changes of bus service elements and characteristics of ridership factors. Suwardo *et al*. [9] concluded that frequency change is likely a more preferable recommendation than change in capacity because of the opportunity of raising load factor in the future. Also, in some of the researches of PBC, frequency was studied as an index of performance. However, there is less incentive theory research on how to get the elasticity, which they employed in their papers.

Aiming these two questions, the main purpose of this paper is to propose an optimal performance-based incentive subsidy scheme, in which the decision-making power regarding service level is discussed through the proper frequency elasticity value. Under this scheme, the bus operator is motivated not only to attract more passengers through improving service level, but also to reduce deficits to obtain the premium, a point which is ignored in existing performance-based contracts.

The structure of this paper is organized as follows: Section 2 provides an incentive subsidy scheme based on Laffont-Tirole's model, and five solutions aiming at the decision-making power of the service level are proposed; Section 3 contains results and analysis of the situation of deficit and social welfare under the five solutions through investigation of the public transport of Arao in Japan as the research object; finally, Section 4 presents conclusions and the feasibility of introducing the incentive scheme.

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2. INCENTIVE SUBSIDY SCHEME MODEL

In the present subsidy scheme in Japan, the government supplies enough subsidies to make up for the gradually increasing deficit of bus operators, and there is not any stimulation for bus operators to reduce deficit. The local government has to pay the same amount of subsidies as the deficit stated by bus operators as shown on the left side of Figure 1.

Zou and Mizokami [10] proposed a new incentive subsidy scheme as represented on the right side of Figure 1. In the scheme, the local government will give a premium to the bus operator if it successfully makes efforts to reduce deficit. The result is that bus operators get their premium and the local government lessens subsidies, a win-win situation. The goal of bus operators is maximizing profit $U = t - \psi(d)$, whereas the objective of the government is maximizing social welfare, that is, maximizing the sum of passenger surplus benefit $S - (1+\lambda)(\beta - d + t)$ and the excess profit $U = t - \psi(d)$.

$$\begin{gathered} \underset{t,d}{Max}\, S - (1+\lambda)[\beta - d + \psi(d)] - \lambda(t - \psi(d)) \\ s.t. t - \psi(d) \geq 0 \end{gathered} \tag{2.1}$$

Where, S =passenger benefit; λ = shadow cost of public fund; $\psi(d)$ = disutility of effort d; t = net monetary premium; β = deficit without effort to reduce cost; and d = bus operator's effort.

Under complete information, the government knows the deficit β and observes the bus operator's effort d. The maximum social welfare is determined by the reduced cost d^* and the excess profit U^*. When the social welfare is maximized, $U^* = 0$.

However, the service level was assumed to be the same as the current situation in Equation (2.1). Actually, one requirement of the incentive contract is improvement in the service level [11]. In order to analyze how the change in service level affects passengers and subsidies, this article improves on the aforementioned model, introducing frequency as the service level variable, and builds an incentive subsidy model with elastic transport demand.

2.1. Model description

The objective of the model is to maximize social welfare, that is, to maximize of the sum of passenger surplus benefit and excess profit, as Equation (2.2) shown.

$$Max \quad SB = \sum_h (U_h + UB_h) \tag{2.2}$$

Where, SB = the social welfare; U_h = the excess profit of line h; and UB_h = the surplus benefit of passengers of line h.

The excess profit U_h is the difference between the premium t_h and the disutility $\psi(d_h, f_h)$, as Equation (2.3) shown.

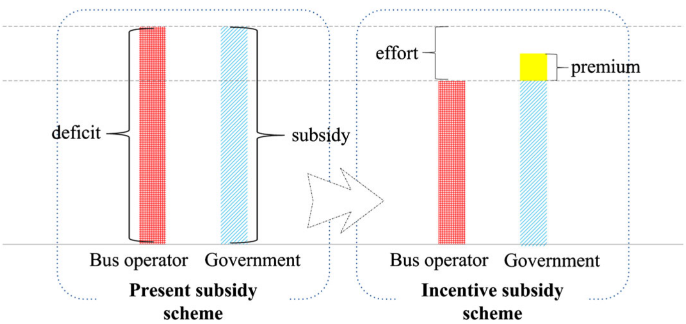

Figure 1. Mechanism design of incentive subsidy scheme model.

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$$U_h = t_h - \psi(d_h, f_h) \tag{2.3}$$

Where, t_h = the premium that rewards bus operators for improving service level and reducing cost of line $h.\psi(d_h, f_h)$ = the disutility of bus operators because of their efforts to improve the service and reduce cost of line h. When bus operators take efforts, disutility of efforts will occur. The $\psi(d_h, f_h)$ is the increasing function of d_h, which means the more is the disutility of efforts, the higher is the reduced cost.

Meanwhile, the bus operator is constrained by individual rationality. The excess profit should not be negative, as Equation (2.4) shown.

$$U_h = t_h - \psi(d_h, f_h) \geq 0 \tag{2.4}$$

The subsidies and premium supplied to the bus operator are gathered from both taxes and from passengers. Therefore, the surplus benefit UB_h is the difference between the benefit that the public transport brings to passengers and the paid. Due to the existence of social cost coefficient, UB_h is expressed as Equation (2.5).

$$UB_h = S(f_h) - (1+\lambda)\{C(d_h, f_h) + t_h\} \tag{2.5}$$

Where, $S(f_h)$ = the surplus of passengers of line h;λ = shadow cost of public fund; $C(d_h, f_h)$ = the deficit of line h after efforts taken to reduce cost and improve service.

Eventually, the social benefit SB can be written as Equation (2.6) shown.

$$\begin{aligned} Max\ \ SB &= \sum_h SB_h = \sum_h (U_h + UB_h) \\ &= \sum_h \{S(f_h) - (1+\lambda)(C(d_h, f_h) + t_h) + U_h\} \\ &= \sum_h \{S(f_h) - (1+\lambda)(C(d_h, f_h) + \psi(d_h, f_h)) - \lambda U_h\} \end{aligned} \tag{2.6}$$

2.2. *Maximizing social benefit SB_h*

This study solves the social benefit maximization problem based on the Stackelberg model [12]. The government is the leader whose object is maximizing the social welfare. The bus operator is the follower whose object is maximizing profit. The government must know ex ante that the bus operator observes his actions. In the model, the premium t_h、 reduced cost d_h and the frequency f_h is variable. The premium t_h, determined by the government, is supplied by the government to reward good performance of the operator. As we said before, there is dispute about the decision-making power of the service level. In order to solve the problem, we make several assumptions about the decision-making power of d_h and f_h shown as Table I. According to assumptions, there are five possible choices; we will respectively calculate the maximal social benefit of each case to get the right decision of the service level.

(1) Case 0

Under Case 0, the existing service level continues unchanged, whereas the reduced cost d_h and the premium t_h are determined by the government. When the social benefit is maximal, the excess profit equals to 0. $t_h^* = \psi\left(d_h^*\right)$. And d_h^* fits $-(1+\lambda)*\left\{\frac{dC(d_h)}{dd_h} + \frac{d\psi(d_h)}{dd_h}\right\} = 0$.

(2) Case 1

Under Case 1, t_h is determined by the government, whereas d_h and f_h are determined by bus operators. As we said, $\psi(d_h, f_h)$ is the decreasing function of d_h, but we do not know its relationship with f_h. Therefore, the model is solved based on two hypotheses.

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Table I. Decision-making power of variables and the optimal solution.

	Case 0	Case 1: $\frac{\partial\psi(d_h,f_h)}{\partial f_h}\leq 0$	Case 1: $\frac{\partial\psi(d_h,f_h)}{\partial f_h}\geq 0$	Case 2	Case 3: $\frac{\partial\psi(d_h,f_h)}{\partial f_h}\leq 0$	Case 3: $\frac{\partial\psi(d_h,f_h)}{\partial f_h}\geq 0$	Case 4
t_h	Government t_h^*	Government t_h^*	Government $t_h^*=0$	Government t_h^*	Government t_h^*	Government $t_h^*=0$	Government t_h^*
d_h	Government d_h^*	Operator $d_h^*=0$	Operator $d_h^*=0$	Operator $d_h^*=0$	Government d_h^*	Government $d_h^*=0$	Government d_h^*
f_h		Operator f_h^*	Operator $f_h^*=0$	Government f_h^*	Operator f_h^*	Operator $f_h^*=0$	Government f_h^*

If $\psi(d_h,f_h)$ is the decreasing function of f_h, the first optimal order condition that makes the excess profit maximum is $d_h^*=0$. When the optimal frequency f_h^* satisfies $t_h^*=\psi\left(0,f_h^*\right)$, the social benefit is maximal.

If $\psi(d_h,f_h)$ is the increasing function of f_h, the first optimal order condition that makes the excess profit maximum is $d_h^*=0\,f_h^*=0$. When $t_h^*=\psi(0,0)=0$, the social benefit is maximal.

(3) Case 2

Under Case 2, t_h and f_h are determined by the government, whereas d_h is determined by the bus operator. When the first optimal order is $d_h^*=0$, the excess profit of the bus operator is maximal.

The optimal solution of social benefit is $t_h^*=\psi\left(0,f_h^*\right)$. And f_h^* satisfies $\frac{dS(f_h)}{df_h}-(1+\lambda)*\frac{\partial C(0,f_h)}{\partial f_h}$-$(1+\lambda)*\frac{\partial\psi(0,f_h)}{\partial f_h}=0$.

(4) Case 3

Under Case 3, t_h and d_h are determined by the government, whereas f_h is determined by the bus operator. Because the relationship between $\psi(d_h,f_h)$ and f_h is unknown, the model is solved based on the same hypotheses.

If $\psi(d_h,f_h)$ is the decreasing function of f_h, the first order condition that makes the excess profit maximum is $f_h^*=f_h(d_h)$.

And when $t_h^*=\psi\left(d_h^*,f_h^*\right)$, the social benefit is maximal. The optimal frequency d_h^* satisfies $-(1+\lambda)\cdot\left\{\frac{dC(d_h,f_h(d_h))}{dd_h}+\frac{d\psi(d_h,f_h(d_h))}{dd_h}\right\}=0$.

If $\psi(d_h,f_h)$ is the increasing function of f_h, the first order condition that makes the excess profit maximum is $d_h^*=0\,f_h^*=0$. When $t_h^*=\psi(0,0)=0$, the social benefit is maximal.

(5) Case 4

Under Case 4, t_h, d_h, and f_h are determined by the government.

The optimal first condition is $t_h^*=\psi\left(d_h^*,f_h^*\right)$, $d_h^*=d(f_h)$. The optimal frequency f_h^* satisfies $\frac{dS(f_h)}{df_h}-(1+\lambda)\cdot\left\{\frac{dC(d(f_h),f_h)}{df_h}+\frac{d\psi(d(f_h),f_h)}{df_h}\right\}=0$

From the aforementioned calculation, we can see that when f_h is determined by the bus operator under Case 1 and Case 3, the model is solved based on two situations because of the relationship between $\psi(d_h,f_h)$ and f_h. When $\psi(d_h,f_h)$ is the increasing function of f_h, the bus service is terminated $f_h^*=0$. This means operators will stop the bus service. When $\psi(d_h,f_h)$ is the decreasing function of f_h, the optimal frequency f_h^* can be obtained through the formulation. When d_h is determined by bus operators under Case 1 and Case 2, $d_h^*=0$. This means the best behavior of the bus operator is to take no efforts to reduce cost.

3. APPLICATION OF THE INCENTIVE SUBSIDY MODEL BASED ON ELASTIC DEMAND

3.1. *The bus services situation of Arao city*

This paper aims at the bus services in Arao city as the research project. Arao is a city located in Kumamoto, Japan. As of 2011, the city has an estimated population of 56 144, with the density of

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982.40 persons per km^2. At present, the public transport of Arao city is operated by Kyushu Sanko Bus Co., Ltd and Nishi-Nippon Railroad Co.,Ltd (NNR). Figure 2 shows the trend of the passengers and the subsidy from 2003 to 2009.

Due to the growing popularity of private vehicles, the operation situation has sharply deteriorated. In order to avoid the lessening of service levels bus operators—and thus to avoid further deficit—the local government supplies sufficient subsidies. These subsidies reached nearly ¥160m in 2003, causing a large financial burden on the government. In order to relieve the burden, transit privatization began in 2004. Savage [13] discussed the effects of privatization and reported that operating costs and subsidization had decreased after privatization. However, he mentioned that demand had declined as a result of service changes and that services had been concentrated to the most popular routes. The transit privatization in Arao also faced these problems, and it resulted that the amount of subsidies fell from ¥154m to ¥55m and in passenger number by about 35% in the same year. The object of the bus operator is to maximize profit through controlling costs [14] and cutting bus lines with large deficits. Bus service was considered by the bus operator as a business and not as a merit good, and the situation continued to deteriorate in 2005. The government began to increase subsidies from 2006 in order to attract more passengers. However, the number of passengers continued to decrease. Thus, transit privatization in Arao reduced subsidies at the expense of a decrease in demand. Application of a subsidy mechanism considering passenger demand is thus crucial for bus services in Arao city.

3.2. *Parameter calibration*

The bus service is operated by two companies in Arao city. Kyushu Sanko Bus Co., Ltd owns 20 bus lines, whereas NNR has two bus lines. The bus lines of Kyushu Sanko Bus Co., Ltd cover the whole city, whereas bus lines of NNR are running in the suburbs with very low frequencies. Therefore, the paper takes bus lines of Kyushu Sanko Bus Co., Ltd as the research object. First, we need to confirm $C(f_h,d_h)$,$D_{ij}(f_h)$, $S(f_h)$and $\psi(d_h,f_h)$ that are needed to get the social welfare.

3.2.1. *The deficit* $C(f_h,d_h)$ of line h

The deficit of line h is the difference among the cost of line h before effort, the revenue of line h and the reduced cost due to efforts. The formulation is as Equation (3.1) shown.

$$C(d_h,f_h) = P(f_h) - \sum_{i,j}\left\{D_{ij}(f_h)\cdot c_{h(ij)}\right\} - d_h\cdot 365\cdot 2\cdot f_h\cdot L_h \tag{3.1}$$

Where $P(f_h)$ = the cost of line h before effort; $D_{ij}(f_h)$ = estimated demand between stop ij of line h after effort; $c_{h(ij)}$ = the ticket price between stop ij of line h; d_h = the reduced cost per mile of line h; f_h = the frequency per day of line h; and L_h = the length of line h.

$P(f_h)$ is related to the revenue kilometer of line h. Regression analysis is carried out to calculate $P(f_h)$ through using 20 bus lines data. The result is shown in Table II. The revenue kilometer is the function of f_h, and is expressed as $365 \times 2 \times f_h \times L_h$.

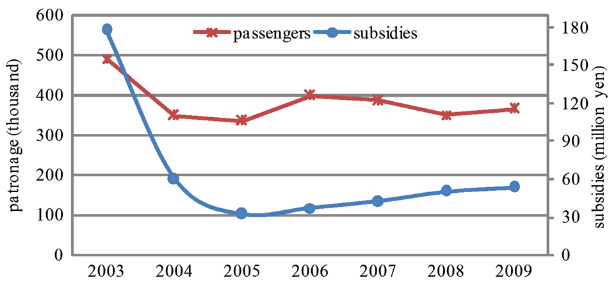

Figure 2. The current situation of bus services in Arao.

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Table II. The calibration results.

Variant	Parameter	t-value
Constant	5.58×10^5	0.95
Revenue kilometer	191.4	13.6
Relevance	0.91	

3.2.2. Passengers between stop i and stop j of line h after effort $D_{ij}(f_h)$

If the frequency changes, the demand of passengers must change with it. The new demand is estimated based on the current passengers between stop i and stop j as Equation (3.2) shown. The data of $D^B_{h(ij)}$ is from the origin–destination survey that was carried out to passengers in 2010.

$$D_{ij}(f_h) = D^B_{h(ij)} + \varepsilon \cdot \left(\frac{D^B_{h(ij)}}{f^B_h} \right) \cdot \left(f_h - f^B_h \right) \tag{3.2}$$

Where, $D^B_{h(ij)}$ = passengers of line h between stop ij; ε = frequency elasticity; f^B_h = the frequency before; and f_h =the frequency after changing the service level.

There has been much research on frequency elasticity. However, there are fewer studies on how to calculate the elasticity. Most of the elasticity value was obtained from data statistics [15]. This paper built a model to calculate frequency elasticity while considering the trend of public transport and other factors. Effective factors considered by this paper include bus services, population level and number of private cars. Factors for bus services include frequency and the inherent trend of bus services. We also consider influences that the adding or cancelling of lines may have on demand. The local government respectively reorganized the bus network in 2008 and 2009. Some bus lines were added and some canceled in each reorganization of the network, and these changed bus lines may affect demand in the subsequent year. The factors of the previous year may influence the demand in the subsequent year, as Equation (3.3) shown. The affecting factors are as presented in Equation (3.4).

$$D_{ht} = A\left(x_{h(t-1)}\right) * D_{h(t-1)} \tag{3.3}$$

Where, D_{ht} = the number of passengers of line h in t year; $A(x_{h(t-1)})$ = the index that affecting number of passengers of line h in $t-1$ year; $N_{h(t-1)}$ = the number of passengers of line h in $t-1$ year.

$$\begin{aligned} \ln A\left(x_{i(t-1)}\right) &= \mu_p \ln x_{p(t-1)} + \mu_{car} \ln x_{car(t-1)} + \\ &\left(\mu_{bus} + \eta_{bus} * \delta_{i(t-1)} + \varepsilon_{bus} * f_{i(t-1)}\right) \ln N_{i(t-1)} \end{aligned} \tag{3.4}$$

Where, μ_p = the influence coefficient of population; $x_{p(t-1)}$ = the population in $t-1$ year; μ_{car} = the influence coefficient of car; $x_{car(t-1)}$ = the number of private cars in $t-1$ year; μ_{bus} = the inherent influence coefficient of bus; η_{bus} = the influence coefficient of cancelling or adding lines;$\delta_{h(t-1)}$ = passengers of line h that were influenced by cancelling or adding lines in $t-1$ year; ε_{bus} = the elasticity of frequency; $f_{h(t-1)}$ = the changed frequency of line h in $t-1$ year.

Utilizing the public transport data, the size of population, and the number of private cars from 2005 to 2010, the SPSS (IBM, Armonk City, NY, USA) was employed to calibrate parameters with 95% confidence level. Results are shown in Table III.

The variant of population was excluded as a calibration parameter by the software, perhaps because the population size has less influence on the demand because of low population growth rate. In 2005, the population of Arao was 56 420 and the growth rate −0.3%. There is scarcely population change in Arao. Therefore, the parameter of size of population was excluded by the software.

The parameter of frequency is 0.349. Currie and Wallis [16] concluded that the average elasticity is 0.35. Paulley *et al*. [17] calculated the frequency elasticity of bus demand as approximately 0.4 in the

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Table III. The estimation results.

Variant	Parameter	Significant values
Constant	1.12	0.037
Influence coefficient of car	−0.331	0.019
Inherent coefficient of bus	−0.064	0.020
Coefficient of changed lines	7.50*10^{-6}	0.001
Elasticity of frequency	0.349	0.001

short run. Therefore, the frequency elasticity of Arao is reasonable. The value of elasticity is positive, and that means the passenger demand is the increase function of the frequency.

3.2.3. The surplus $S(f_h)$ of passengers of line h

The generalized cost borne by passengers is defined as the sum of ticket price and the cost of waiting and trip times. When bus operators increase frequency, passenger generalized cost will decrease from g^B to g due to the reduction of the waiting time, as shown in Figure 3. As such, the demand will increase from D^B to D. The area g^BBAg that is derived from a reduction in the generalized cost and the rise in demand is the passenger surplus measured as Equation (3.5).

$$
\begin{aligned}
S(f_h) &= \frac{1}{2}\cdot\sum_{ij}\left\{D^B_{h(ij)} + D_{ij}(f_h)\right\}\cdot\left\{g^B_{h(ij)} - g_{h(ij)}\right\} \\
&= \frac{1}{2}\cdot\sum_{ij}\left\{D^B_{h(ij)} + D_{ij}(f_h)\right\}\cdot\left\{g^B_{h(ij)} - \left(c_{h(ij)} + \omega\cdot Time_{h(ij)} + \omega\cdot\frac{60\cdot 13}{2^* f_h}\right)\right\} \quad (3.5)
\end{aligned}
$$

Where, $D^B_{h(ij)}$ = the demand between stop ij of line h before changing the frequency; $D_{ij}(f_h)$ = the demand between stop ij of line h after changing the frequency; $g^B_{h(ij)}$ = the generalized cost between stop ij of line h before; $g_{h(ij)}$ = the generalized cost between stop ij of line h after; $c_{h(ij)}$ = the ticket price between stop ij of line h; ω = the time value of passengers (ω is set 24.94 (yen/min) based on the Japanese national standard); and $Time_{h(ij)}$ = the trip time between stop ij of line h.

3.2.4. Disutility of efforts $\psi(d_h,f_h)$

When the bus operator takes efforts to increase frequency or reduce cost, these efforts would incur the cost $\psi(d_h,f_h)$. A reasonable subsidy mechanism applied to public transport operators in Kyushu, Japan is used to calculate the disutility. The mechanism, which acts as an incentive to promote bus operators

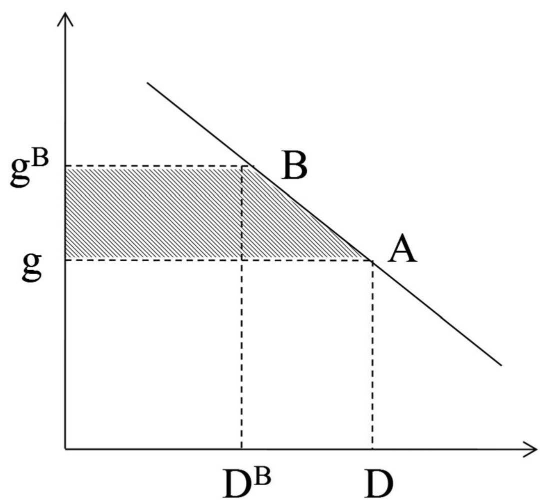

Figure 3. Composite demand function.

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to improve the service level while controlling cost, includes two conditions. One is the cost of the bus operator should be below the average cost of bus operators in the area; the other is the operating situation should be improved compared with the previous year by either reducing cost or increasing revenues. When both conditions are satisfied, the bus operator can obtain the subsidies.

A questionnaire survey was carried out to bus operators that had obtained the subsidy to gather data. The content of the questionnaire is revealed as Table IV. The data was used to calibrate the function of $\psi(d_h,f_h)$ defined as Equation (3.6).

$$\psi(d_h,f_h) = \exp(\alpha_0 + \alpha_1 \cdot gap + \alpha_2 \cdot d_h + \alpha_3 \cdot 365 \cdot 2 \cdot L_h \cdot f_h) \tag{3.6}$$

Where, *gap*= the difference between the average operating cost of all bus operators and the operating cost of one bus operator, that is, a kind of yardstick cost.

Table V presents the calibration results. The parameter of the revenue kilometer is positive in Table V. That means $\psi(d_h,f_h)$ is the increasing function of f_h. When the bus operator decides f_h and $\psi(d_h,f_h)$ is the increasing function of f_h under Case 1 and Case 3, the optimal frequency f_h^* is zero. That means the bus operator stops supplying the bus service under this situation, and this is unreasonable. Therefore, the two situations will not be discussed during the following analysis.

3.3. Result analysis

This paper takes the bus services of Arao city as an example to verify the incentive subsidy model on the basis of elastic demand. Results are shown as Table VI.

Table IV. The questionnaire survey content.

Characteristics of bus lines	Name of the bus line
	Starting/ terminal stop
	Length of the bus line
	Frequency (workday, day off)
Operating situation	Revenue, cost
	Average operating cost in the area (yen/km)
	The change of the operating cost compared with that in last year (yen/km)
	The change of revenues compared with that in last year (yen/km)
	The maintaining lines reasonable subsidies

Table V. The calibrating results.

Variant	Parameter	*t*-value
Constant	10.82538	73.19899
Difference	0.007861	12.40475
Reduced deficit	0.049383	6.718688
Revenue kilometer	8.71E-06	15.49588
Relevance	0.97	

Table VI. The calculating results

	f^* (runs/day)	$D(f)$ (thousand persons/year)	d^* (million yen/year)	C (million yen/year)	t^* (million yen/year)	$C+t^*$ (million yen/year)	$S(f)$ (million yen/year)	$SB(f)$ (million yen/year)
Current	62.5	382.3	0	72.6	0	72.6	0	0.0
Case 0	62.5	382.3	26.2	46.4	13.3	59.7	0	13.5
Case 2	143.9	505.8	0	176.4	3.2	179.6	1299.8	1111.1
Case 4	156.4	520.6	70.3	147.0	31.1	178.1	1484.0	1329.7

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3.3.1. The optimal frequency f_h^*

Figure 4 shows the optimal frequency of the bus network under different cases. The frequency stays constant and is 62.5 runs/day under Case 0, whereas the optimal frequency under Case 2 and Case 4 is sharply increased. The number of runs is 143.9 per day under Case 2, 2.3 times that of the current frequency; the number is 156.4 per day under Case 4, 2.5 times. The frequency under the Case 4 is higher than that under Case 2.

3.3.2. Reduced deficit d_h^*

At present, the average operating cost per kilometer in Arao city is ¥187. The average operating cost is reduced by ¥32/km under Case 0, whereas it is reduced by ¥43/km under Case 4. The situation of reduced cost of bus lines under Case 0 and Case 4 is presented as Figure 5. The fluctuant range of reduced cost of each bus line is ¥13–48 under Case 0, whereas the fluctuant range under Case 4 is ¥28–50. The reduced cost is relevant with the revenue kilometer. Taking the Case 0 as an example, it is seen that the reduced cost is more when the revenue kilometer is higher.

Figure 6 shows the relationship between the frequency and the reduced cost. As stated earlier, the frequency under Case 4 is higher than that under Case 2. Through Figure 6, it is seen the frequency of the line under Case 4 is higher than the frequency of same line under Case 2 when the reduced cost of the bus line is larger. That means the bus operator would prefer to improve the service of the bus line with larger cost reduction. In such a case, the bus operator can obtain a higher premium with less cost.

3.3.3. The premium t_h^*

The premium under different cases is presented in Table VI. The premium under Case 0 is ¥13.3m per year due to the reduced cost of 26.2 million. Under Case 2, the bus operator gets ¥3.2m because of the increase of frequency; the premium under Case 4 is ¥31.1m, which is 2.3 times the premium under Case 0. The reason is that the frequency under Case 4 has a sharp augment, reaching 156.4 runs a day, and meanwhile the bus operator takes efforts to reduce costs to ¥70.3m. In conclusion, when

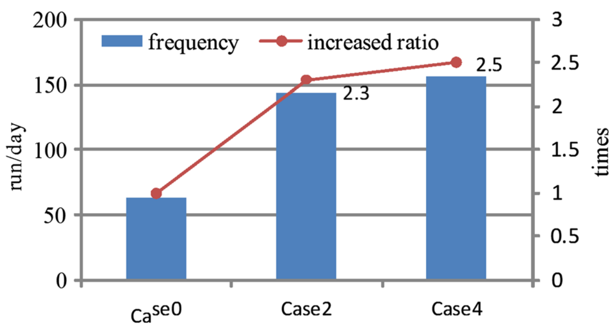

Figure 4. The frequency under cases.

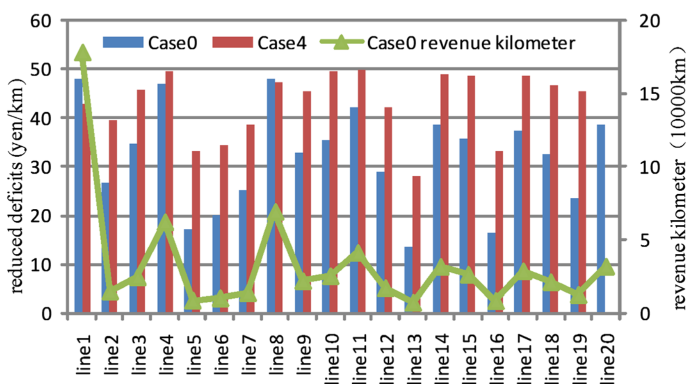

Figure 5. Optimal reduced cost of each bus line.

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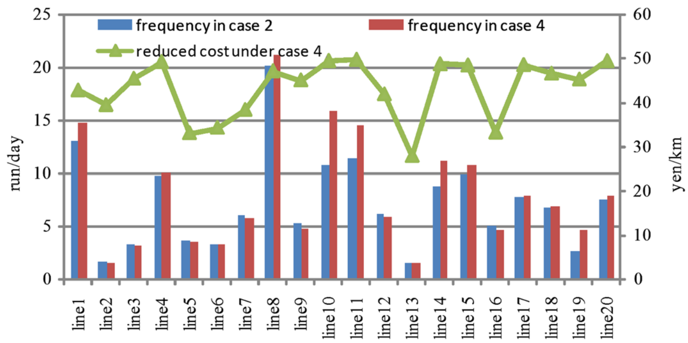

Figure 6. Relationship between the frequency and the reduced cost.

the bus operator takes more efforts to increase the frequency and reduce cost, it will obtain a higher premium from the government as a reward.

3.3.4. *The deficit C_h and total subsidies*

Figure 7 shows a comparison of deficit and subsidies. Total deficit of bus services in Arao city is ¥72.6m. When the bus operator takes efforts to reduce deficit to ¥46.4m under Case 0, the total amount of subsidies declines to ¥59.7m, which is ¥12.9m lower compared with the current total subsidies. The bus operator increases frequency to attract more passengers and get more revenues under Case 2 and Case 4, but it pays more cost. Therefore, deficit and total subsidies have a significant increase under the two situations. The deficits rise to ¥176.4m, and the total subsidies are ¥179.6m under Case 2. Because the bus operator not only increases the frequency but also takes efforts to reduce cost under Case 4, the bus operator supplies more service with fewer deficit compared with that under Case 2. The deficit is ¥147m, and the total subsidies are ¥178.1m under Case 4.

3.3.5. *The passenger surplus $S(f_h)$*

Under Case 0, the bus operator keeps the frequency constant, and there is no passenger surplus. The passenger surplus benefit increases to ¥1299.8m under Case 2 due to the augment of the frequency, whereas the increase of passenger surplus benefit is ¥1484.0m under Case 4. The surplus under Case 4 is larger than Case 2 because of the higher frequency and less subsidies.

3.3.6. *The social welfare SB*

The social welfare under Case 0 is ¥13.5m because the bus operator takes efforts to reduce deficit, so the amount of subsidies eventually decreased. Under Case 2, the social welfare reaches ¥1111.1m because the bus operator takes measures to increase frequency to about 2.3 times that under the current situation. Improving frequency increases the number of passengers, and subsidies thus increase. It is found that the effect to passengers caused by increasing frequency is more significant compared with the effect on subsidies; as such, social welfare has a positive relationship with frequency.

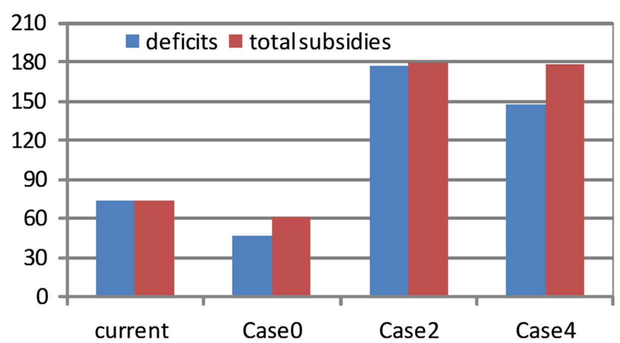

Figure 7. Deficit and total subsidies under different cases.

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Social welfare under Case 4 is the highest because of the bus operator not only taking efforts to reduce cost but also taking measures to increase frequency.

3.3.7. *fVSD(f) and fVSB*

Figure 8 shows the relationship between the frequency and the demand under Case 4. The current frequency is 62.5 runs/day, whereas the frequency under Case 4 is 156.4 runs/way. This paper takes 10 frequency points between 62.5 runs/day and 200 runs/day to do the sensitivity analysis. It is seen that the demand increases as the frequency rising. The increasing range gets smaller when the frequency exceeds the optimal frequency 156.4 runs/day. The reason is that the demand may be saturated as the frequency increases.

Figure 9 shows the relationship between the frequency and the social welfare under Case 4 through taking 10 frequency points. The frequency of the first point is the current level, and the social welfare is caused by the reduced subsidies. The social welfare of the other 9 points is caused by the improved service frequency and the reduced subsidies. There is a positive relationship between the frequency and the social welfare. However, it is seen that the social welfare of points 9 and 10 increases very slow when the frequency is higher than the optimal frequency 156.4 runs/day. The reason is that the demand of the two points increases fewer shown as Figure 8 while there is still significant growth of subsidies due to the increasing frequency.

Above all, the relationship between the frequency and the demand and the relationship between the frequency and the social welfare are positive. However, the increase trend of the demand and the social welfare of points 9 and 10 is not significant when the frequency is higher than the optimal frequency 156.4 runs/day. This means the larger frequency is not better, and it should be considered with the local public transport demand.

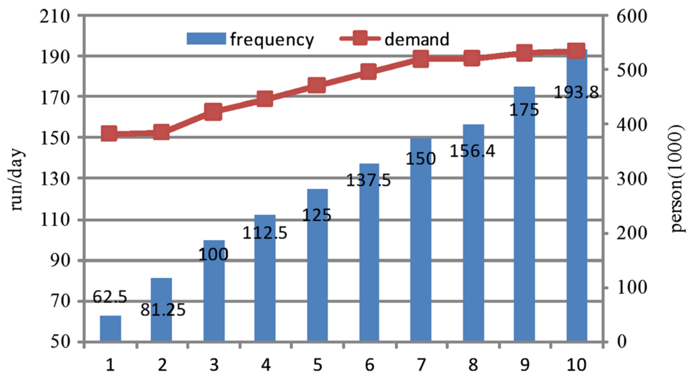

Figure 8. The relationship between frequency and demand.

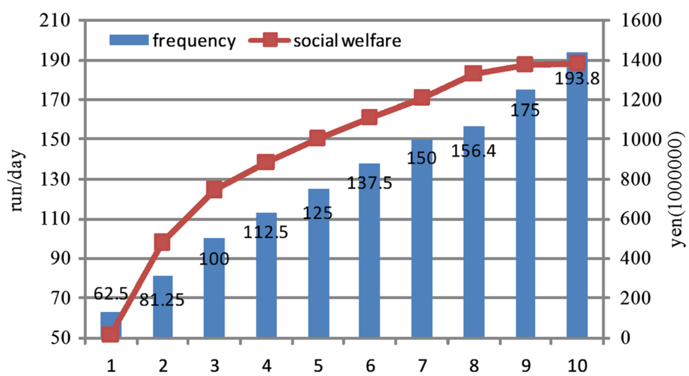

Figure 9. The relationship between frequency and social welfare.

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4. CONCLUSION

Public transport subsidies caused by deficit have become a huge financial burden of the local governments in Japan. Most governments support the loss-making subsidy scheme. Meanwhile, passengers are decreasing year by year because of the popularity of private cars and decline of population. There is no motivation to encourage bus operators to improve the service level to attract more passengers under the present subsidy scheme. Therefore, many researchers propose reformation of public transport contracts. During the development process of the contract, competitive tendering contracts and performance-based contracts are respectively put forward. The performance-based contract suggested that the subsidy mechanism should be related to the service level of bus operators. However, all the mechanisms ignore the role of bus operators in lessening subsidies. Therefore, it is essential to design a new kind of subsidy scheme, which will motivate bus operators not only to improve service level to attract more passengers, but also to take efforts to reduce cost. The reward of efforts is the premium based on the improved service level and the reduced costs supplied by the government.

Referring Zou and Mizokami [10], this paper proposed an incentive subsidy model based on elastic demand. Under the scheme, two questions of the PBC are considered. One is the decision-making power of the service level; the other is how to measure the elasticity of the service level. Referring Suwardo *et al*. [9], this paper uses frequency as the index of service level and calculates its elasticity while considering intrinsic trends of public transport and other factors. Meanwhile, decision-making power is respectively assumed to yield the maximizing social welfare based on elastic demand. It is proved that the model can help local governments and bus operators reach a win-win solution by taking public transport in Arao as the research object.

The paper gives five cases based on different powers of decisions of the service level. Premium and effort are determined by the government with frequency invariant under Case 0. The premium is determined by the government, whereas the effort to reduced cost and the frequency are determined by bus operators under Case 1. In Case 2, the premium and the frequency are determined by the government, whereas the effort is determined by bus operators. In the Case 3, besides the frequency, other two factors are determined by the government. All factors are determined by the government under Case 4. Case 1 and Case 3 are eliminated because of inconsistent with the actual through taking public transport in Arao as research object to calculate.

The current deficit is ¥72.6m. Under Case 0, the bus operator takes efforts to reduce costs ¥26.2m. The government not only makes up deficit of ¥46.4m, but also supplies a premium of ¥13.3m. Total subsidies are ¥59.7m, ¥12.9m less than the current subsidies. The number of passengers does not change because of frequency invariant. The social welfare increases ¥13.5m.

The optimal condition under Case 2 is $d_h^* = 0$, meaning the bus operator would make no efforts to reduce cost. The optimal frequency is 143.9 runs/day, 2.3 times of the current frequency. Due to the increasing frequency, deficit of the bus operator sharply increases to ¥176.4m. The bus operator also gets ¥3.2m premium because of the improved service. The total subsidy supplied by the government is ¥179.6m, an increase of ¥107m from current deficit. The number of passengers also rises 1.3 times, and the passenger surplus is ¥1299.8m. The social welfare is ¥1111.1m.

The optimal frequency is 156.4 runs/day, 2.5 times the current frequency under Case 4. The bus operator takes efforts to reduce cost by ¥70.3m, and the deficit after efforts is ¥147.0m. The degree of efforts is positively related to the revenue kilometer. The government supplies subsidies to make up the deficit, and pays a ¥31.1m premium at the same time. Total expenses reach ¥178.1m, a ¥105.5m increase from current subsidies. The number of passengers also rises more than 1.5 times, and the passenger surplus is ¥1484.0m. The social welfare is ¥1329.7m.

It is seen that the passenger surplus and the social welfare are both greatly improved under Cases 2 and 4, when the frequency is determined by the government. In Case 2, the bus operator determines the effort, and the optimal choice for it is taking no effort to reduce cost. Therefore, the total subsidy under Case 4 is lower than that under Case 2, whereas the frequency is higher under Case 4 compared with that under Case 2. The reason is that all parameters are determined by the government under Case 4. As stated earlier, the government will make decisions considering maximization of social welfare. Therefore, the social welfare is maximizing under Case 4. Under this scheme, the bus operator is encouraged to take efforts to reduce cost and improve service level. Meanwhile, social welfare can be maximized.

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Although the subsidy sharply increases in Case 4, the total subsidy is almost the same as the subsidy in 2003, as Figure 2 shows. However, the number of passengers is less than the number under Case 4, and the passenger surplus and social benefit are also greatly improved. Figure 8 and Figure 9 present that the optimal frequency 156.4 runs/day is reasonable, and we consider Case 4 to be feasible for use in real public transport.

However, there are two questions needed to be discussed. The first question is that the total subsidies are still not decreased a lot under the optimal Case 4, although the number of passengers increases compared with the current situation. It is withal high to the local government. The reason is that the aforementioned model is a theoretical model, and the final result is the optimum solution without the constraint of subsidy cap. When the incentive model is applied into the public transport, the subsidy cap is suggested to be considered by the government. Another question is the whole model assumes complete information to obtain optimal frequency. That is a very strong assumption especially in the presence of a private operator. Zou and Mizokami [10] analyzed the subsidy situation under incomplete information, and the service level is assumed the same as the current in the scheme. It is found that bus operators take fewer efforts to reduce deficits when the information is more nontransparent. More research is required on how the incentive scheme under a situation of incomplete information affects bus service level and subsidy amount.

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APPENDIX

These data U_h and UB_h are needed to calculate the social welfare SB shown as Equation (2.2). Equation (2.3) shows $\psi(d_h,f_h)$ and t_h are used to get U_h, while $S(f_h)$,$C(f_h,d_h)$ and t_h are used to get UB_h shown as Equation (2.5).

The parameter notation table explains the parameters which are used to calculate the aforementioned data.

Parameter notation

Parameter	Description	Unit
SB	The social welfare	yen
U_h	The excess profit of line h	yen
UB_h	The surplus benefit of passengers of line h	yen
$\psi(d_h,f_h)$	The disutility of bus operators due to their efforts to improve the service and reduce cost of line h	yen
$S(f_h)$	The surplus of passengers of line h	yen
$C(d_h,f_h)$	The deficit of line h after efforts taken to reduce cost and improve service	yen
t_h	The premium rewarding bus operators for improving service level and reducing cost of line h	yen
λ	Shadow cost of public fund The value is assumed 5% in the paper referring Laffont and Tirole [18].	—
d_h	The reduced cost line h	yen/km
$P(f_h)$	The cost of line h before effort	yen
$D_{ij}(f_h)$	Estimated demand between stop ij of line h after effort	person
$D^B_{h(ij)}$	Passengers of line h between stop ijThe data is got from the origin–destination survey done in 2010.	person
f_h	The frequency after	runs/day
f^B_h	The frequency before	runs/day
$c_{h(ij)}$	The ticket price between stop ij of line h	yen
$Time_{h(ij)}$	The waiting time and trip time between stop ij of line h	min
ω	The time value of passengers (set 24.94 (yen/min) based on the Japanese national standard)	yen/min
$g_{h(ij)}$	The generalized cost between stop ij of line h after	yen
$g^B_{h(ij)}$	The generalized cost between stop ij of line h before it is comprised by ticket price, waiting time and trip time between stop ij of line h	yen
L_h	The length of line h	kilometer
ε	Frequency elasticity	—
D_{ht}	The demand of passengers of line h in t year	person
$A(x_{h(t-1)})$	The index that affecting demand of passengers of line h in $t-1$ year	—
$N_{h(t-1)}$	The demand of passengers of line h in $t-1$ year	person
μ_p	The influence coefficient of population	—
$x_{p(t-1)}$	The population in $t-1$ year	person
μ_{car}	The influence coefficient of car	—
$x_{car(t-1)}$	The number of private cars in $t-1$ year	vehicle
μ_{bus}	The inherent influence coefficient of bus	—
η_{bus}	The influence coefficient of cancelling or adding lines	—
$\delta_{h(t-1)}$	Passengers of line h that were influenced by cancelling or adding lines in $t-1$ year	person
$f_{h(t-1)}$	The changed frequency of line h in $t-1$ year	runs/day
gap	The difference between the average operating cost of all bus operators and the operating cost of one bus operator the data is got through survey	yen/km
α_0	The constant of the disutility	—
α_1	The parameter of the gap	—
α_2	The parameter of efforts	—
α_3	The parameter of the revenue kilometer	—

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