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Thermal Wave Propagation Phenomenon in a Thin Film Heated at Asymmetrical Wall Temperature

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Abstract: Numerical study is performed to investigate thermal wave propagation in a very thin film subjected to an asymmetrical temperature change on both sides. The non-Fourier, hyperbolic heat conduction equation is solved using a numerical technique based on MacCormak's predictor-corrector scheme. Consideration is given to the time history of thermal wave behavior before and after asymmetrical collision of wave fronts from two sides of a film. It is disclosed that in transient heat conduction, a heat pulse is transported as a wave in the film, and that non-Fourier heat conduction is extremely significant with certain range of film thickness and time. That is, sudden heating on both sides of the extremely thin film causes temperature overshoot within a very short period of time.

Key words: Thermal propagation, overheating, non-Fourier heat conduction.

Nomenclature

C	Speed of thermal wave (m/s)
c_p	Specific heat
k	Thermal conductivity (w/Km)
$Q(\eta, \xi)$	Dimensionless heat flux
$q(\xi,\tau)$	Heat flux (W/m^2)
$T(\xi,\tau)$	Temperature (K)
T_0	Reference temperature (K)
t	Time (sec.)
x	Space variable
x_0	Film thickness (m)

Greek Letters

α	Thermal diffusivity (m^2/s)
η	Dimensionless space variable
$\theta(\eta, \xi)$	Dimensionless temperature
ξ	Dimensionless time variable
ρ	Density (kg/m^3)
τ	Relaxation time (α/C^2, sec.)

Subscript

n	Time level

Superscript

i	Spatial location

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1. Introduction

Recently, several issues of basic scientific interest arise in cases such as laser penetration and welding, explosive bonding, electrical discharge machining, and heating and cooling of micro-electronic elements involving a duration time of nanosecond or even picosecond in which energy is absorbed within a distance of microns from the surface. In studying such phenomena, the classical Fourier heat conduction equation breaks down at temperatures near absolute zero or at moderate temperatures when the elapsed time during a transient is extremely short. This is because the wave nature of thermal propagation is dominant, that is, a thermal disturbance travels in the medium with a finite speed of propagation [1-4].

The above phenomena are physically anomalous and can be remedied through the introduction of a hyperbolic equation based on a relaxation model for heat conduction which accounts for a finite thermal propagation speed. Thus, considerable interest has been generated toward the hyperbolic heat conduction (HHC) equation and its potential applications in engineering and technology. A comprehensive survey of the relevant literature is available in Ref. [5]. Some dealt with wave characteristics and finite propagation speed in transient heat transfer conduction [3, 6-9]. Several analytical and numerical solutions of the HHC equation have been presented in the literature. In studying the propagation of temperature pulse in a semi-infinite medium, Baumeister and Hamill [1] used Laplace transforms to solve the HHC equation. The same method was employed by Maurer and Thompson [10], who reported the importance of the wave effect in response to a high heat flux irradiation. Carey and Tsai [11] analyzed a propagating heat wave reflected at a boundary, in which the numerical methods based on a variable formulation of the problem and the Galerkin finite-element method are employed. Vick et al. [12] and Ozisik et al. [13] predicted the growth and decay of a thermal pulse in one-dimensional solid. In particular, Ozisik et al. [13] used integral transforms to study the effect of pulses. Glass et al. [14] employed a numerical technique based on MacCormack's predictor-corrector scheme to solve the HHC equation. By using the same method, Glass et al. [15, 16] analyzed numerically hyperbolic heat conduction in a semi-infinite slab with temperature-dependent thermal conductivity and investigated the effects of Stefan number, melt temperature, and variable thermal conductivity. As the other method, Frankel et al. [17] developed a general three-dimensional constant property heat flux formulation based on the hyperbolic heat conduction approximation. They reported that the flux-formulation is more convenient to solve problems involving flux-specified boundary conditions. Tan and Yang [18] investigated heat transfer resulting from symmetrical collision of thermal waves induced by a step change in the wall temperature of the thin film by means of the method of separation of variables. They obtained theoretical results for the time history of propagation process, magnitude and shape of thermal waves and the range of film thickness and duration time. By using the same method, Tan and Yang [19] predicted wave nature of heat propagation in a very thin film subjected to an asymmetrical temperature change on both sides. It was found that: (1) when a thin film is heated on both side walls, temperature overshoot occurs within a very short period of time; and (2) in contrast, when it is cooled, temperature undershoot occurs. Furthermore, Tan and Yang [20] treated heat propagation in a very film subjected to an exponentially decaying temperature change on both sides. They reported that both temperature overshoot and temperature undershoot occur in the films within a very short period of time.

This paper treats the wave behavior during transient heat conduction in a very thin film (solid plate) subjected to an asymmetrical temperature change on both side surfaces. Analytical solutions are obtained by means of a numerical technique based on MacCormak's predictor-corrector scheme to solve the non-Fourier, hyperbolic heat conduction equation.

2. Formulation and Numerical Method

Consider a very thin film with thickness of x_0 maintained at a uniform, initial temperature T_0. The walls at $x = 0$ and x_0 are suddenly heated or cooled to a temperature T_{w1} and T_{w2}, respectively. In the present study, there is no heat generation in a film. Nonequilibrium convection and radiation are assumed negligible. Under the conditions and assumption, the modified Fourier equation [5] and the energy equation can be represented in the one-dimensional flow of heat, as

$$\tau \frac{\partial q}{\partial t} + q + k \frac{\partial T}{\partial z} = 0 \tag{1}$$

and

$$\rho\, c_p \frac{\partial T}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{2}$$

respectively. Note that the relaxation time τ is assumed constant. For convenience in analysis and computation, the initial and boundary conditions to be imposed here are given for heating case, as

$T = T_0$, $\frac{\partial T}{\partial x} = 0$ at t=0, 0<x<x0

$T = T_{w1}$ $(= 1.5T_0)$ at $t > 0$, $x = 0$

$T = T_{w2}$ (= $2.0T_0$) at $t > 0$, $x = x_0$

The following dimensionless quantities, i.e., dimensionless temperature, dimensionless heat flux, and dimensionless time and space variables are introduced

$$\theta(\xi,\eta) = \frac{T}{T_0} \tag{3a}$$

$$Q(\xi,\eta) = \frac{\alpha q}{T_0 kC} \tag{3b}$$

$$\xi = \frac{C^2 t}{2\alpha} \tag{3c}$$

$$\eta = \frac{Cx}{2\alpha} \tag{3d}$$

Eqs. (1) and (2) can be expressed in terms of the above dimensionless variables as

$$\frac{\partial Q}{\partial \xi} + 2Q + \frac{\partial \theta}{\partial \eta} = 0 \tag{4}$$

and

$$\frac{\partial Q}{\partial \eta} + \frac{\partial \theta}{\partial \xi} = 0 \tag{5}$$

Note that in the present study, Eqs. (4) and (5) are employed as the governing equations. Initial and boundary conditions are represented for wall-heating and -cooling, respectively, as

= 1, Q = 0 at $\xi = 0$, $0 < \eta < 1$,

= 1.5, at $\xi > 0$, $\eta = 0$,

= 2.0, at $\xi > 0$,

Note that the boundary condition of Q at $\xi > 0$ is derived from Eqs. (4) and (5).

In general, many investigators combine the energy and flux equations (i.e., Eqs. (4) and (5)) into a single second-order partial differential equation to solve the HHC problem. As for this solution method, Glass et al. [6, 14] reported that MacCormack's method (Anderson et al., 1983), which is a second-order accurate explicit scheme, can handle these moving discontinuities quite well and is valid for the HHC problems. Since the hyperbolic problems considered here have step discontinuities at the thermal wave front, MacCormack's prediction-correction scheme can be used in the present study. When the HHC problem is numerically solved through the scheme employed here, it is convenient to solve Eqs. (4) and (5) rather than to combine these two equations into a single second-order partial differential equation before solving [14].

Throughout numerical calculations, the number of grids is properly selected between 1,000 and 5,000 to obtain a grid-independent solution, resulting in no appreciable difference between the numerical results with different grid spacing. In solving the governing equations employed here, i.e., the HHC problem including the nonlinear nature, the stability is affected by the ratio of $\Delta\xi$ to $\Delta\eta$, $\Delta\xi/\Delta\eta$ which is called the Courant number [16]. For example, as the Courant number becomes smaller, the effect of odd derivative truncation-error terms becomes larger, and oscillations occur in the vicinity of discontinuities in the solution. Thus, $\Delta\xi/\Delta\eta$ is fixed at 0.98 in the present study. The ranges of the parameters are nondimensional plate thickness $Cx_0/2\alpha$ = 0.5 and 5.

3. Numerical Results and Discussion

Figs. 1 and 2 illustrate the timewise variation of the temperature distribution, θ, in films having $Cx_0/2\alpha$ of 0.5 and 5, respectively. (a) in both figures corresponds to numerical predictions resulting from heating a film at both side surfaces. In particular, Fig. 1 depicts, in detail, the propagation process of thermal waves in a film with the value of $Cx_0/2\alpha$ of 0.5. It is observed in Fig. 1 that: (1) sharp wavefronts exist in the thermal wave propagation, which is the same as the other wave phenomena, (2) after the wall temperatures on two sides are suddenly raised, a set of wavefronts appears and advances towards the center in the physical domain which separates the heat-affected zone from the thermally undisturbed zone, as seen in Fig. 1a, (3) at ξ = 0.5, thermal wavefronts meet and collide with each other at the center of the film, (4) after first collision, the center temperature in a film, for the case of heating, causes a significant amplification resulting a much higher temperature in this region, (5) after that, reverse thermal wavefronts take place and travel towards both

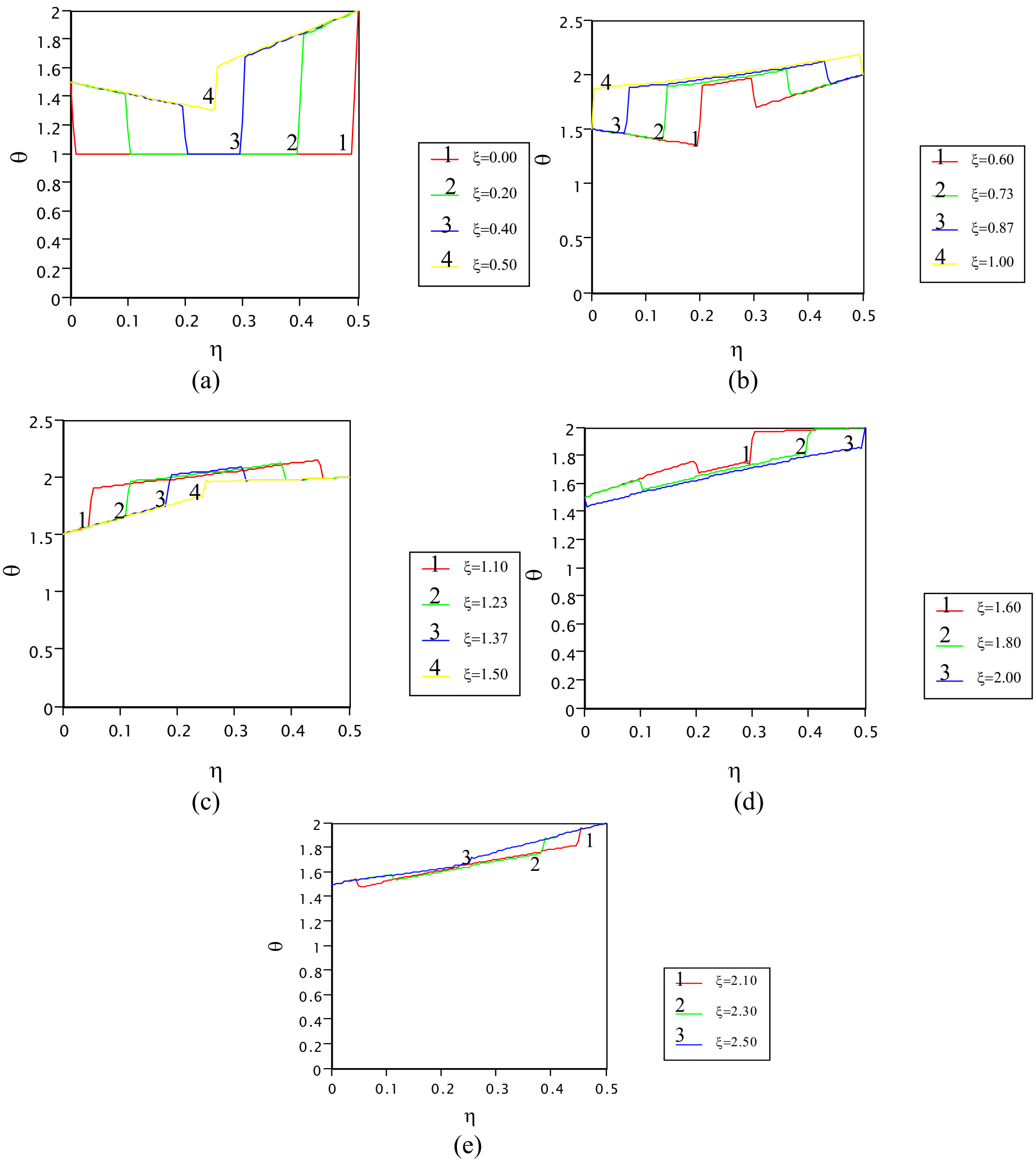

Fig. 1 **Instantaneous dimensionless temperature distribution in the film at Cx_0/α=1 with an asymmetrical temperature change.**

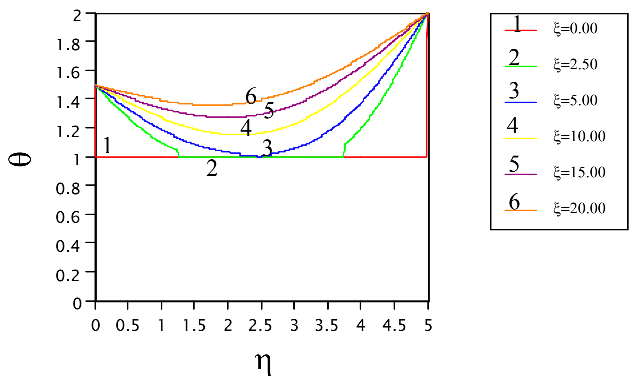

Fig. 2 **Instantaneous dimensionless temperature distribution in the film at Cx_0/α = 10 with an asymmetrical temperature change.**

side walls of the film, as shown in Figs. 1b, (6) when thermal wavefronts reach at both side walls at $\xi = 1.0$, the film temperatures at both sides of strongly heated walls exceed the imposed wall temperature, called temperature overshoot, and (7) after thermal wavefronts are reflected from both side walls of the film, the similar pattern is continued as seen in Fig. 1c through Fig. 1e. By several times of collision, reflection and continuous attenuation of the thermal waves, the wavefronts become weak. The thermal wave behavior was also predicted by Tan and Yang [17], who obtained the theoretical results by solving the hyperbolic heat conduction equation by means of the method of separation of variables.

The present numerical solution predicts the existence of thermal waves, particularly in a very thin film and presents the propagation process of thermal waves, the magnitude and shape of thermal waves, and the regular decaying process of thermal wave in films with different values of $Cx_0/2\alpha$. Such behavior is characteristic of a thermal system with a relaxation or nonlinear diffusion theory. It is found from Figs. 1 and 2 that as time progresses, the peak of the wave decays and disappears within $\xi = 10$ for any films with different thickness. One observes that after wavefronts arrive at the center of the film with $Cx_0/2\alpha = 5$, they gradually disappear in the absence of reverse temperature waves and temperature overshoot or temperature undershoot, as seen in Fig. 2. It behaves like diffusion domination. A very interesting phenomenon of temperature overshoot or temperature undershoot occurs in the very thin film, i.e., in the film of smaller values of $Cx_0/2\alpha$ over a very short period of time, which is induced by the collision of the wave fronts. Such wave behavior can't be predicted by the classical heat conduction theory, i.e., Fourier's law. This is because it allows for the immediate diffusion of heat as soon as the energy is released in the absence of a relaxation time, that is, heat propagates at an infinite speed. Thus, the presence of the thermal relaxation time yields non-Fourier effect and this trend becomes more significant when the relaxation time is longer. For example, larger reverse waves can be seen in a film with $Cx_0/2\alpha = 0.5$, with the temperature greatly exceeding the imposed wall temperatures of the film, as shown in Fig. 1.

It is found from the results that the thermal relaxation time plays a primary role in distinguishing a domain to be wave dominating or diffusion dominating. Several investigators have estimated the magnitude of thermal relaxation time τ to range from 10^{-10} second for gases at standard conditions to 10^{-14} second for metals [14] with that for liquids [21] and insulators [22], falling within this range. If τ is known, one can obtain the range of film thickness within which heat propagates as a wave. The criterion for thermal wave dominating in the present study is $Cx_0/\alpha < 10$, as seen in Figs. 1 and 2. For example, the value of silicon corresponds to the thickness of the film in the order of about 0.01 micron using 10^{-14} s and 93.4×10^{-6} m^2/s as the relaxation time and thermal diffusivity [23], respectively.

4. Summary

Heat waves have been theoretically studied in a very thin film subjected to a sudden asymmetric temperature change at two side walls. The non-Fourier, hyperbolic heat conduction equation is solved using a numerical technique based on MacCormak's predictor-corrector scheme. Results have been obtained for the propagation process, magnitude and shape of thermal waves and the range of film thickness and duration time within which heat propagates as wave.

It is revealed that only when τC is of the same order as or larger than one half the film thickness, thermal waves can appear. As the value of $x_0/2\tau C$ become smaller, the temperature is substantially pronounced, as seen in Fig. 1. The criterion for the occurrence of thermal shock waves in a thin film is the film thickness in the order of 0.01 micron for metals. If a film is strongly heated, temperature overshoot may take place in the films of smaller values of $x_0/2\tau C$ within a very short period of time, respectively.

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