

# Signal-Limitation Filters to Simultaneously Satisfy Constraints of Velocity and Acceleration Signals

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**Abstract :** In this paper, we propose a filter structure whose output satisfies the velocity and acceleration constraints for any input signals. In the field of factory automation, step signals are sometimes converted into trapezoidal waves to satisfy the intended velocity limitations. This helps the operators avoid overload in industrial robots. In some cases, physical protection of the equipment, safety, and ride quality can be ensured by limiting the characteristics of input signals in actual plants. A signal-limitation filter is proposed for the input signal to satisfy the intended signal limit. In the previous study, a signal-limitation filter structure was provided as a simple unit-feedback control with a saturation function. The filter structure in the previous study had a delay between the input and output signals because it is difficult to design gains considering both the saturating and non-saturating cases. To solve this problem, we propose a novel filter structure that includes feedforward and feedback components. Applying this filter structure with feedforward terms including saturation enables us to fulfill the desired limitations for arbitrary input signals. We evaluated the proposed structure in a signal-limitation filter that simultaneously limits the velocity and acceleration. The simulation results demonstrate the effectiveness of our proposed filter.

**Key Words :** saturation, nonlinear filter, velocity and acceleration limit

## 1. Introduction

In tracking and positioning control, a time-varying signal is used as a reference signal. In the field of factory automation, a trapezoidal signal is used to limit the maximum velocity value to avoid overloading robots. Additionally, acceleration and jerk are important factors for improving operability and riding quality. Therefore, signals whose acceleration and jerk are limited are widely used in various applications [1]–[5]. Given a particular input signal pattern, a preliminary design that satisfies the above limitations can be developed.

However, when a person manipulates devices, or when a controller provides a control input signal through a feedback controller based on sensor signals, preliminary design signals that satisfy the above constraints cannot be generated. For example, in the accelerator/brake operation of automobiles, limitations cannot be added easily because the driver's operation is not known in advance. Thus, to prevent erroneous operations by the driver, the operation signal must be filtered online to meet the limitations.

In this study, we develop a signal-limitation filter that limits the velocity and acceleration components of signals in real time. Previous studies have proposed signal-limitation filters that limit the velocity[1]–[3], acceleration[1], jerk[4], and torque[5]. Such filters are effective for use in flexible arms and are developed with a simple feedback structure with a saturation function. However, these structures have the problem of delays between the input and output signals, as gain design considering both the saturation and non-saturation cases is difficult. Therefore, such filter structures make it difficult to deal with an

arbitrary input, such as the case in which a person performs an operation.

Let  $u$  be the desired signal to be applied, which is guided by the operator or controller. In addition, let  $v$  be the signal that is to be actually applied to the system. If  $u$  satisfies the signal limitation, it is ideal to satisfy  $v = u$ . On the other hand, if  $u$  does not satisfy the signal limitation, a signal  $\tilde{u}$  that satisfies the limitation must be generated; it has a trajectory approximately identical to that of  $u$ . This study aims to design filters to generate an appropriate  $\tilde{u}$  based on the above conditions.

In this study, we consider a filter structure that simultaneously satisfies the desired velocity and acceleration limitations for arbitrary input signals based on the structure of the model error compensator (MEC)[6],[7] proposed by the authors. The MEC is a specialized structure that compensates for model errors when they occur. In a velocity-limitation filter, the velocity component of the filter output satisfies the desired velocity limit. We propose a structure for a signal-limitation filter that simultaneously limits the velocity and acceleration. Furthermore, we verify the effectiveness of this structure through a numerical simulation.

This paper is an extended version of the proceedings of the SICE Annual Conference 2017 [8]. We include the results from our evaluation of the proposed signal-limitation filter and discuss its ability to limit the velocity and acceleration, simultaneously.

## 2. Previous Studies

Figures 1 and 2 respectively show the velocity and acceleration-limitation filter proposed in a previous study [4]. Here,  $u$  is the input signal,  $\tilde{u}$  is the output signal,  $v$  is the velocity component of the output signal, and  $a$  is the acceleration component of the output signal. The same signal notations are used for the proposed filters described below.

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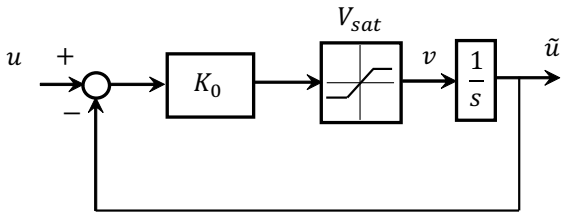


Fig. 1 Previous velocity-limitation filter.

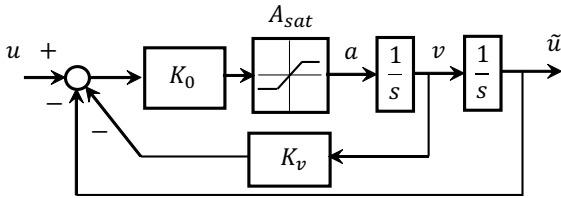


Fig. 2 Previous acceleration-limitation filter.

Currently, signal-limitation filters have a simple feedback structure to which saturation functions are added, as shown in Figs. 1 and 2. The velocity component of the output signal is used as a part of the feedback signal, and a compensation input is added to the reference signal to generate the optimum target trajectory. Each gain is determined by the time required to reach the target value.

However, setting the gain is difficult in these filters. If the gain is set such that the difference between  $u$  and  $\tilde{u}$  is small, the response waveform when the limit is satisfied is good, but the response waveform is delayed otherwise. For these reasons, the gains are set in a comprehensive manner under the conditions of saturation and non-saturation. Therefore, even if a signal is input such that  $u$  satisfies the velocity and acceleration limits, the output signal is delayed relative to the input signal. Therefore, using such filter structure designs makes it difficult to deal with an arbitrary input signal, such as in the case when a person operates a control system.

### 3. Structure of Proposed Signal-Limitation Filter

#### 3.1 Filter Design Motivation and Proposed Filter Structure

Based on the results of previous research, it is indispensable to design a filter that does not cause delays. In this study, the signal-limitation filter is designed to satisfy the following requirements:

- The maximum and minimum velocities of the output signal satisfy the designed velocity limit.
- The maximum and minimum accelerations of the output signal satisfy the designed acceleration limit.
- The signal difference between the input signal  $u$  and output signal  $\tilde{u}$  is small.
- If signal  $u$  satisfies the intended limitation, trajectories  $\tilde{u}(t)$  and  $u(t)$  are extremely similar.
- The filter can be realized with a simple feedback structure.

The actual control system has various limitations, and it is necessary to satisfy these limitations for the safety and physical protection of the equipment. The first and second objectives

are used to satisfy the limitations on the velocity and acceleration for the control input signal. The third objective attempts to maintain the shape of the original signal while satisfying the signal limitation. Therefore, in tracking or positioning control, the signal-limitation filter should preferably output a signal that is as close as possible to the input signal within the limit. The fourth objective is similar to the second, especially when the input signal satisfies the limitations. In the previously proposed filter structure, this objective cannot be achieved, irrespective of how the gain is chosen. The fifth objective is to design a simple structure for filtering and saturation that can be implemented easily in a microcomputer. This should ultimately facilitate implementation in various systems. This is a great merit at the design stage, even compared with existing methods such as the reference governor.

The generalized form of the proposed signal-limitation filter is shown in Fig. 3. Because the purpose of this study is the simultaneous constraint of velocity and acceleration, an anti-windup mechanism is added to the general system of the signal-limitation filter in reference [9]. As shown in the figure, the filter applies compensation only when the limit is not satisfied; the input signal is nearly equal to the output signal. Here,  $F_i (i = 1, \dots, n)$  is a filter,  $\hat{F}_i^{-1}$  is its inverse or pseudo-inverse filter, where  $F_i$  is mainly an integrator and  $F_i^{-1}$  is predominantly a differentiator. Figure 4 shows the form of the saturation function. The limit value of the saturation function is set to a desired value, and when that limitation is satisfied, the input value is generated (the function within the limit is a linear function of slope 1).

Let  $u$  be the input signal. First, it is converted to the velocity or acceleration component of the input signal by the differentiator. At this time, the differentiator is overlapped such that it becomes the signal to be limited. For example, when converting the signal to an acceleration signal, the differentiator is superimposed twice. After being converted into a signal to be limited, a desired limitation is satisfied by a saturation function.

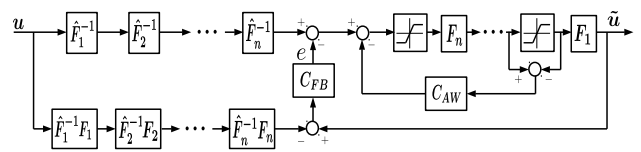


Fig. 3 Generalized form of the proposed filter.

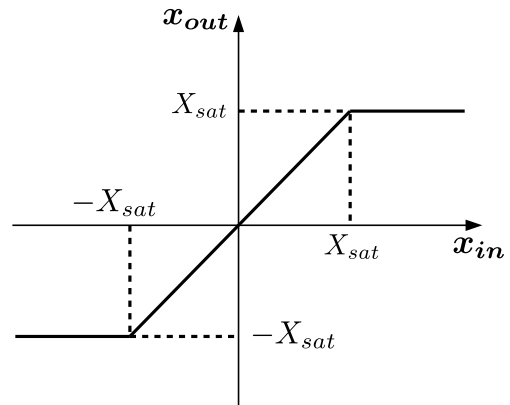


Fig. 4 Saturation function.

The signal  $\tilde{u}$  satisfying the limitation is generated by the integrator. At this time, it is shaped by adding the compensation input  $e$  to the velocity or acceleration signal, making it possible to output a signal close to the input signal while satisfying the limitation.

Because it is a structure based on the MEC[6],[7], if the input signal satisfies the limitation, the input signal to the compensator  $C_{FB}$  is a zero signal. As such, the compensator does not work, and the compensation input  $e \approx 0$ . The anti-windup controller[10]–[15] was also used as a compensator. In this case, if the signal satisfies the limitation, the signal by the compensator  $C_{AW}$  becomes a zero signal. At this time, the input/output transfer function when the limit is satisfied is

$$\frac{\tilde{u}}{u} = \prod_{i=1}^n F_i \hat{F}_i^{-1} \quad (1)$$

If  $F_i = \hat{F}_i$  for all  $i = 1, \dots, n$ ,  $\tilde{u}/u = 1$  is satisfied and the input signal  $u$  is generated as the output signal  $\tilde{u}$ . If  $\hat{F}_i^{-1}$  is a pseudo-inverse filter to ensure propriety and is set to function as an inverse filter in the signal band of  $u$ ,  $\tilde{u}$  converges to  $u$ . On the other hand, when the input signal does not satisfy the limit, the compensator  $C_{FB}$  acts to suppress the signal difference between the input signal  $u$  and output signal  $\tilde{u}$ .

The design of  $C_{FB}$  must compensate for this difference. Furthermore, the second objective—that the signal difference between input signal  $u$  and output signal  $\tilde{u}$  is small—depends on the magnitude of the constraint and the input signal. For example, when it is desirable to shape the signal such that it satisfies the velocity limit of the lamp input, the difference between the output satisfying the limit and the lamp input cannot be fulfilled with any compensation. In the design of  $C_{FB}$ , although the input signal sequence itself is unnecessary, the frequency characteristic of the applied signal and the magnitude of the limit of the signal become important.

Because we focus herein on the structure of the proposed signal-limitation filter, its specific design will be the subject of future research. For example, when the frequency distribution of the input signal is known, it is effective to investigate the frequency characteristic of the signal passed through the filter  $\prod_{i=1}^n \hat{F}_i^{-1}$  and to set the gain based on the comparison between the frequency distribution and the limit amount; in this case, the gain is considered effective.

As a design guideline, if the proportional gain of  $C_{FB}$  is large, the compensation effect for suppressing the error increases. However, windup is likely to occur in the case of saturation. On the other hand, reducing the gain reduces the compensation effect. Further, it is difficult to design  $C_{FB}$  so as to include the integrator insofar as windup is likely to occur. Thus, we conclude that it is better not to include the integrator in  $C_{FB}$ .

As an example, consider the case of selecting the gain  $C_{FB}$  of the signal-limitation filter. First, set the test signal by selecting a signal that does not satisfy the intended signal limit, such as the velocity limit, in part of the waveform. Let  $[t_1, t_2]$  be a time range in which the test signal does not satisfy the signal limit. Then, the test signal is applied to a signal-limitation filter with certain initial gains. We focus on the output of the velocity-limitation filter at time  $t_2$  and later. Because the waveform of the output depends on the settled gains in the signal-limitation filter, we can change the transient response after time  $t_2$  by tun-

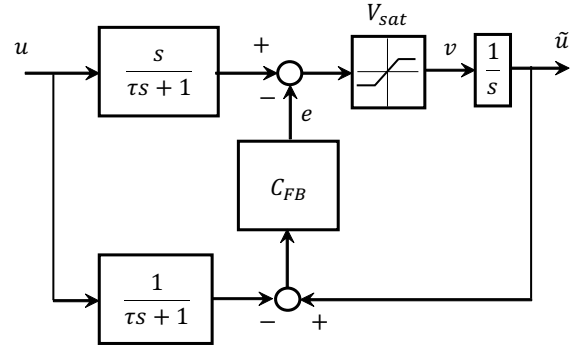


Fig. 5 Velocity-limitation filter.

ing the gain  $C_{FB}$ . Therefore, it is better to choose the gains based on the transient response after  $t_2$  with trial and error.

### 3.2 Velocity-Limitation Filter[9]

We construct a velocity-limitation filter based on the generalized form of the filter in Fig. 3. Figure 5 shows the construction of the proposed velocity-limitation filter. The filter structure contains an integrator, differentiator, and saturation function. The limit value of the saturation function is set to a desired limit value of the velocity component of the signal.

The input signal  $u$  is converted into a velocity component of the input signal by the differentiator. After this conversion, the desired limitation is satisfied by the saturation function. Next, by returning to the output signal by the integrator,  $\tilde{u}$  satisfies the velocity limitation.

In Fig. 5, the P controller is used as the compensator, and  $K_x$  is the proportional gain. The time constant  $\tau$  of the differentiator is assumed to be a sufficiently small value,  $\tau = 0.001$ , to approximate the exact value of the differential value.

When the signal  $u$  satisfies the velocity limitation shown in Fig. 5, the signal applied by the compensator is expressed as

$$e(s) = K_x \left( \tilde{u}(s) - \frac{1}{\tau s + 1} u(s) \right). \quad (2)$$

In (2), if the limit is satisfied when the input signal to the saturation function is equal to or less than  $X_{sat}$ ,  $e(t) = 0$  is satisfied; therefore, the input signal to be compensated is 0. The filter dynamics when the limitation is satisfied is given as follows:

$$\frac{\tilde{u}}{u} = \frac{1}{\tau s + 1}. \quad (3)$$

If  $\tau$  is set to be small in consideration of the band of the input signal,  $\tilde{u}$  is extremely close to  $u$ . On the other hand, when the input signal does not satisfy the velocity limitation, the compensator  $C_{FB}$  shapes the signal's velocity component. The proportional gain  $K_x$  is adjusted, and its value is set such that windup cannot occur.

### 3.3 Acceleration-Limitation Filter[9]

Figure 6 shows the configuration of the acceleration-limitation filter based on the generalized form of the filter in Fig. 3. The acceleration is limited by adding one differentiator and one integrator to the velocity-limitation filter, as shown in Fig. 5. The proportional-derivative (PD) controller is used for compensator  $C_{FB}$ . The value  $K_x$  is the proportional gain, and  $K_v$  is the differential gain. The time constant  $\tau$  is the same as

that of the velocity-limitation filter. The input signal is converted into an acceleration signal by two differentiators. Next, the desired acceleration limitation is satisfied through the saturation function. The acceleration component is returned to the output signal satisfying the limitation by the two integrators.

In Fig. 6, the compensation input signal by the compensator  $C_{FB}$  is expressed as

$$e(s) = \left\{ K_x + K_v \frac{s}{\tau s + 1} \right\} \left\{ \tilde{u}(s) - \frac{1}{(\tau s + 1)^2} u(s) \right\}. \quad (4)$$

When the limitation is satisfied in (4), because  $e(t) = 0$ , the input signal to be compensated is 0. Here, the filter dynamics when the signal satisfies the limitation is

$$\frac{\tilde{u}}{u} = \frac{1}{(\tau s + 1)^2}. \quad (5)$$

As in the case of the velocity-limitation filter, if  $\tau$  is set to be small in consideration of the band of the input signal,  $\tilde{u}$  is as close as possible to  $u$ . The compensator  $C_{FB}$  modifies the acceleration component when the limitation is not satisfied. The proportional gain  $K_x$  and differential gain  $K_v$  are adjusted, and error feedback is performed such that the output signal becomes close to the input signal.

It is necessary to design each gain by considering the stability of the feedback loop in the linear region of the saturation function. When the frequency distribution of the input signal is known, the design based on the signal passed through the second-order differentiator is effective.

### 3.4 Proposed Velocity-Acceleration-Limitation Filter

In actual applications, there are cases limiting the velocity, acceleration, etc. simultaneously. In addition to the velocity- and acceleration-limitation filters, we consider a filter that simultaneously achieves velocity and acceleration limitation based on the generalized form shown in Fig. 3. Figure 7 shows the constructed velocity-acceleration-limitation filter. This filter uses an anti-windup component[10]–[15] as a compensator in order to increase the degree of freedom of the compensator and compensation capability.

The input signal  $u$  is first converted into its acceleration component by the differentiator. After conversion into the signal to be limited, the desired limitation is satisfied using the saturation function  $A_{sat}$ . Next, the limited acceleration component  $a$  is returned to the velocity component  $\dot{v}$  through the integrator, and the saturation function  $V_{sat}$  limits the velocity magnitude. The limited velocity component  $v$  is used as the feedback signal for compensation.

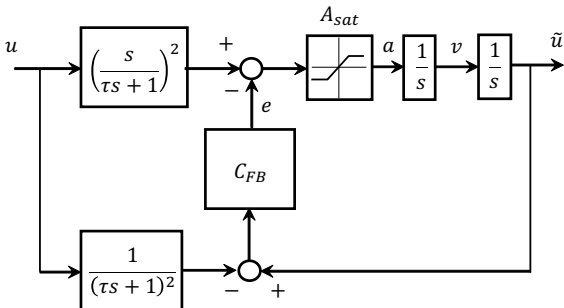


Fig. 6 Acceleration-limitation filter.

In Fig. 7, the compensator is used as PD control for the input/output deviation. The anti-windup controller is used for PD control. The values  $K_x$  and  $K_v$  are the proportional gains, and  $K_D$  and  $K_{D2}$  are the differential gains.

When the signal  $u$  satisfies the velocity and acceleration limitation shown in Fig. 7, the signal applied by the compensator is

$$e_x(s) = \left( K_x + K_D \frac{s}{\tau s + 1} \right) \left( \tilde{u}(s) - \frac{1}{(\tau s + 1)^2} u(s) \right), \quad (6)$$

$$e_v(s) = \left( K_v + K_{D2} \frac{s}{\tau s + 1} \right) (\dot{v}(s) - v(s)). \quad (7)$$

If the input to the saturation function is less than or equal to  $X_{sat}$ , the compensation input signals  $e_x(t)$  and  $e_v(t)$  are equal to 0.

Here, (5) provides the filter dynamics when the limitation is satisfied. If  $\tau$  is set to be small in consideration of the band of the input signal,  $\tilde{u}$  is as close as possible to  $u$ . The compensator  $C_{FB}$  modifies the acceleration component when the limitation is not satisfied. The gains are adjusted and error feedback is performed so that the output signal is close to the input signal.

When the signal passed through the saturation function is integrated and saturated, the influence of the original saturation function must be preserved, as shown in Fig. 7. For the acceleration component of the output signal to be within the limitation,  $\frac{d^2}{dt^2} \tilde{u}$  must satisfy  $a_{min} \leq \frac{d^2}{dt^2} \tilde{u} \leq a_{max}$ . Here,  $\tilde{u}$  is the second-order integral of the limited acceleration.

$$\begin{aligned} \tilde{u}(t) &= \int V_{sat} \left( \int A_{sat}(\dot{a}(t)) dt \right) dt, \\ \frac{d}{dt} \tilde{u}(t) &= V_{sat} \left( \int A_{sat}(\dot{a}(t)) dt \right), \\ \frac{d^2}{dt^2} \tilde{u}(t) &= \frac{d}{dt} V_{sat} \left( \int A_{sat}(\dot{a}(t)) dt \right) \\ &= \begin{cases} \frac{d}{dt} v(t) = \frac{d}{dt} \dot{v}(t) = a(t) & \text{(unsaturated case)} \\ 0 & \text{(saturated case)} \end{cases} \end{aligned}$$

From the above,  $\frac{d^2}{dt^2} \tilde{u}(t)$  is  $a(t)$  or 0. The acceleration signal  $a(t)$  satisfies the acceleration limitation and  $\frac{d^2}{dt^2} \tilde{u}$  satisfies  $a_{min} \leq \frac{d^2}{dt^2} \tilde{u} \leq a_{max}$  at all times.

## 4. Simulation

### 4.1 Simulation of Velocity-Limitation Filter

We simulated the velocity-limitation filter. In this example,  $K_x = 86$  is selected by trial and error using a test signal. In Fig. 8,  $u(t) = \sin(\frac{1}{2}t)$  is used as the input signal. The dashed and solid lines indicate the input and output signals, respectively. The first and second rows from the top in the figure respectively indicate the in/out and velocity signals. Figure 8

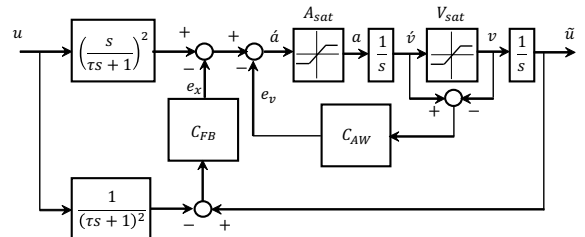


Fig. 7 Velocity-acceleration-limitation filter.

shows the response without velocity limitation. Because there is no limitation, it can be confirmed that the trajectory of the input signal corresponds to that of the output signal.

Figure 9 shows the transient response with the velocity-limitation filter. We set the maximum and minimum velocity limit of the velocity-limitation filter as 80% of the maximum velocity amplitude of the input signal. It is confirmed that the input signal is output at the part where the limit is satisfied and that the maximum velocity can be followed to the input signal at the part that does not satisfy the limitation. Because the limitation is not strict, a signal close to the input signal can be output without delay.

Figure 10 shows the case in which the maximum and minimum velocity limit of the velocity-limitation filter is set as 50% of the maximum velocity amplitude of the input signal. It can be confirmed that switching from the maximum to the minimum velocity is performed instantaneously. By the velocity-limitation filter, the input and output trajectory are as close as possible despite the severe signal limit.

Next, Fig. 11 shows the synthetic wave of the trigonometric function  $u(t) = \sin(3t) + \sin(5t) + \sin(7t)$ . Here, the limitation rate is 50%. Compared with  $\sin(t)$ , the maximum value of the velocity is a very large signal, but it can follow the input signal as close as possible within the given limit. Furthermore, the presence or absence of compensation can be confirmed in the part where the limitation is satisfied and in that where it is not.

#### 4.2 Simulation of Acceleration-Limitation Filter

Similar simulations were performed for the acceleration-limitation filter. In this example,  $K_x = 75$  and  $K_v = 220$  are selected by trial and error using a test signal. Figure 12 shows the response waveform when  $u(t) = t^3 \sin(2t)e^{-t}$  is applied. From the top in Fig. 12, the first, second, and third rows respectively show the in/out, velocity, and acceleration signals. It is a response with no limitation, and it can be confirmed that the output signal is similar to the input signal.

Figure 13 shows the case in which the maximum and minimum acceleration limit of the acceleration-limitation filter is set as 80% of the maximum acceleration amplitude of the input signal. It can be confirmed that the output signal is close to the input signal without delay because the limitation is not strict. In addition, the input signal is generated at the part that satisfies the limitation, and it can follow the input signal with the maximum acceleration in the part that does not satisfy the limitation.

Figure 14 shows the response when  $u(t) = t^3 \sin(2t)e^{-\frac{1}{2}t}$  is applied. Here, 50% of the maximum acceleration of the input

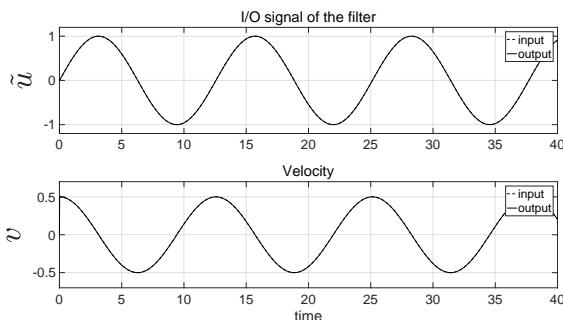


Fig. 8 Sine wave response:  $|v_{sat}| = \infty$ .

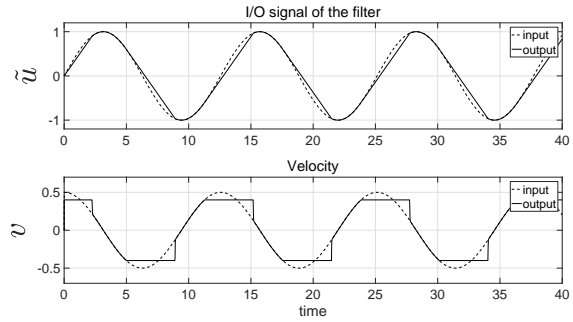


Fig. 9 Sine wave response:  $|v_{sat}| = 0.8v_{max}$ .

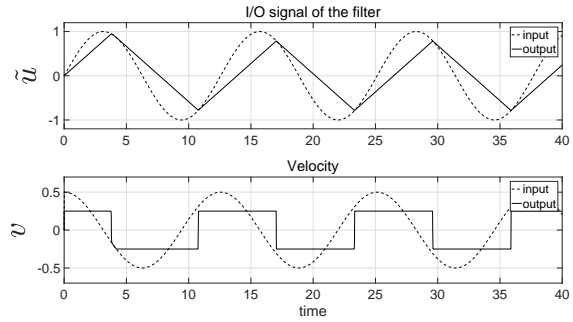


Fig. 10 Sine wave response:  $|v_{sat}| = 0.5v_{max}$ .

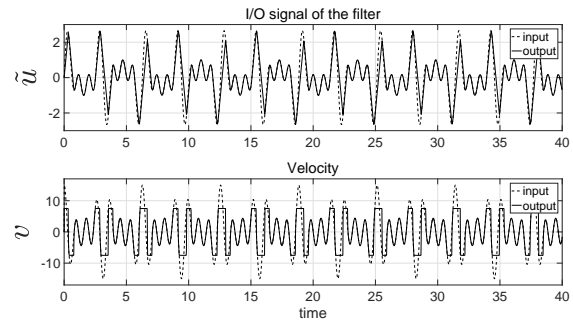


Fig. 11 Synthetic sine wave response:  $|v_{sat}| = 0.5v_{max}$ .

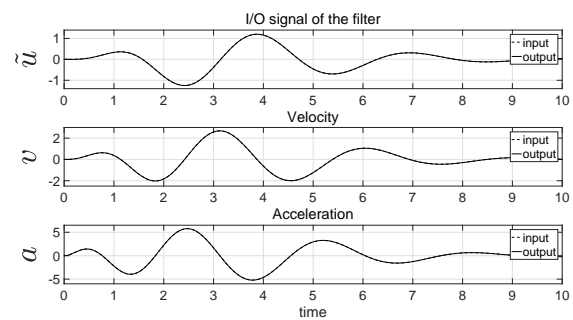
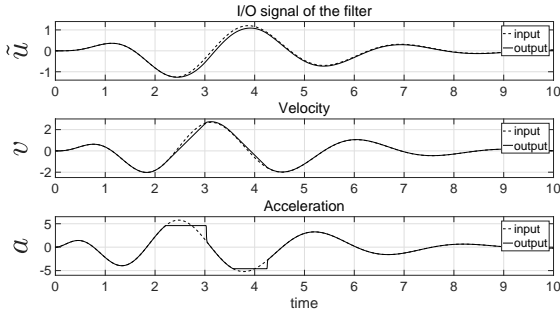
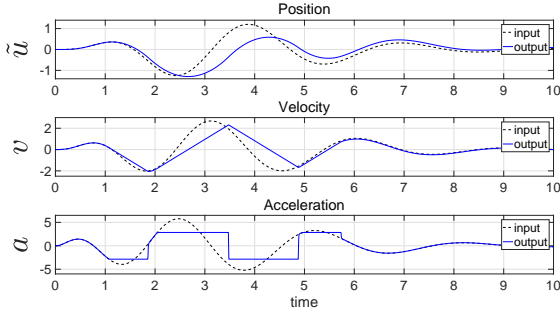
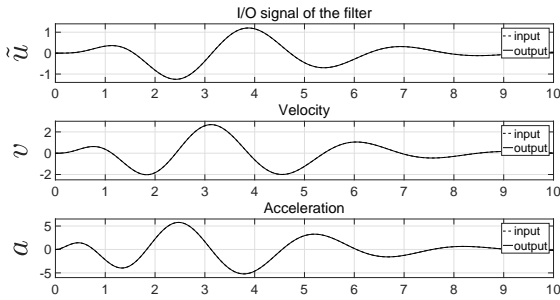
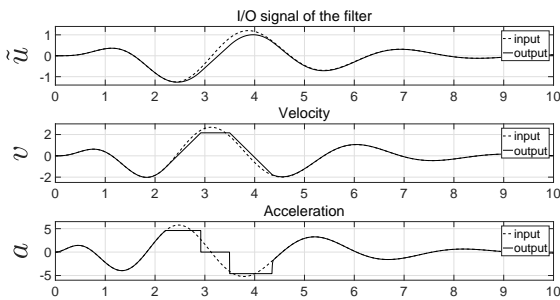


Fig. 12 Synthetic wave response:  $|a_{sat}| = \infty$ .

signal is set as the limitation in the filter. It can converge to the target value without overshoot and confirm the presence or absence of compensation both in the part that satisfies the limitation and in that which does not.

#### 4.3 Simulation of Velocity-Acceleration-Limitation Filter

We simulated the velocity-acceleration-limitation filter. In this example,  $K_x = 100$ ,  $K_D = 120$ ,  $K_v = 100$ , and  $K_{D2} = 1$  are selected by trial and error using a test signal. Figure 15 shows the response without limitation. The following input signal is

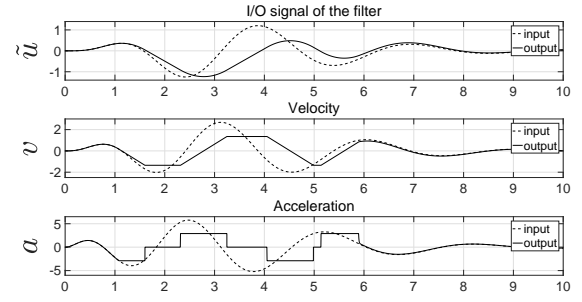
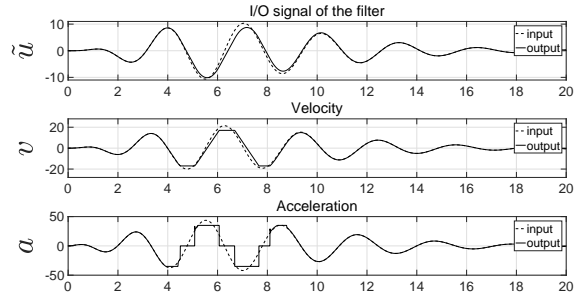
Fig. 13 Synthetic wave response:  $|a_{sar}| = 0.8a_{max}$ .Fig. 14 Synthetic wave response:  $|a_{sar}| = 0.5a_{max}$ .Fig. 15 Synthetic wave response:  $|v_{sar}| = \infty, |a_{sar}| = \infty$ .Fig. 16 Synthetic wave response:  $|v_{sar}| = 0.8v_{max}, |a_{sar}| = 0.8a_{max}$ .

used in Fig. 15:

$$u(t) = t^3 \sin(2t)e^{-t}. \quad (8)$$

Because there is no limitation, it can be confirmed that the trajectory of the input signal corresponds to that of the output signal.

Figure 16 shows the case in which the maximum and minimum velocity limit of the filter is set as 80% of the maximum velocity amplitude of the input signal. The maximum and minimum acceleration limit of the filter is also set as 80% of the

Fig. 17 Synthetic wave response:  $|v_{sar}| = 0.5v_{max}, |a_{sar}| = 0.5a_{max}$ .Fig. 18 Synthetic wave response:  $|v_{sar}| = 0.8v_{max}, |a_{sar}| = 0.8a_{max}$ .

maximum acceleration amplitude of the input signal. It is confirmed that the input signal is generated at the part where the limit is satisfied, and the output signal can follow the input signal at the part that does not satisfy the limitation. The input signal  $u$  and the output signal  $\tilde{u}$  are similar in Fig. 16.

Figure 17 shows the case in which the maximum and minimum velocity limit of the filter is set as 50% of the maximum velocity amplitude of the input signal. The maximum and minimum acceleration limit of the filter is also set as 50% of the maximum acceleration amplitude of the input signal. The input and output trajectories are as close as possible despite the severe signal limit.

Next, the input signal is changed in the simulation. Figure 18 shows the response of  $u(t) = t^3 \sin(2t)e^{-\frac{1}{2}t}$  with the 80% limitation. The maximum value of the velocity and acceleration increases, and it can be confirmed that switching is instantaneous.

Figure 19 shows the case in which the maximum and minimum velocity limit of the filter is set as 50% of the maximum velocity amplitude of the input signal. The maximum and minimum acceleration limit of the filter is also set as 50% of the maximum acceleration amplitude of the input signal. By our proposed velocity-acceleration-limitation filter, the input and output trajectories are as close as possible despite the severe signal limit.

#### 4.4 Discussion

The filter structure proposed in this paper can be thought of as improving the degree of freedom in the previously proposed velocity-limitation filter and acceleration-limitation filters, respectively, shown in Figs. 1 and 2. For example, if  $K_0 = 1/\tau$  in Fig. 1,  $\tilde{u}$  is equal to that in Fig. 5 when the input signal satisfies the velocity limit. At this time, the degree of freedom for adjusting  $C_{FB}$  exists separately in our proposed filter. Similarly, for the acceleration-limitation filter, if we set  $K_0 = 1/\tau^2$ ,

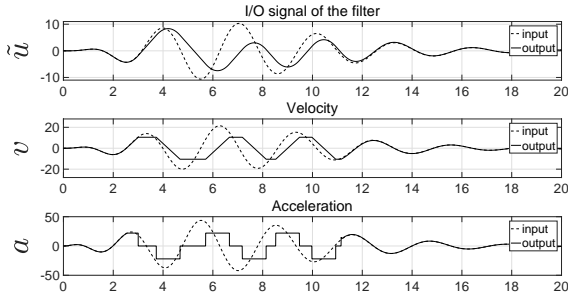


Fig. 19 Synthetic wave response:  $|v_{sat}| = 0.5v_{max}$ ,  $|a_{sat}| = 0.5a_{max}$ .

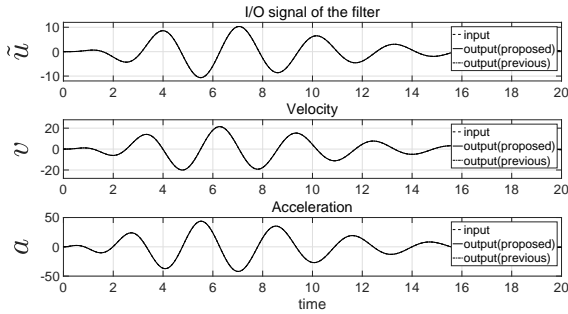


Fig. 20 Comparison of previous and proposed method:  $|a_{sat}| = \infty$ .

$K_v = 2\tau$  in Fig. 2,  $u/\tilde{u} = 1/(\tau s + 1)^2$  is equal to that in Fig. 6 when the input signal satisfies the acceleration limit.

Figures 20 and 21 show the comparison simulation results of the acceleration-limitation filters shown in Figs. 2 and 6. In Fig. 2,  $\tau$  is made sufficiently small with  $K_0 = 1/\tau^2$  and  $K_v = 2\tau$ . In Figs. 20 and 21, the dashed line denotes the input signal, the solid line is the output of the proposed method, and the thick dotted line is the output signal of the previous method, simulated with the input as Figs. 18 and 19.

Figure 20 shows the case where the input signal satisfies the acceleration limit. Figure 21 shows the case where the acceleration limit is 80% of the acceleration amplitude of the input signal. When the input signal satisfies the acceleration limit, the output signal nearly coincides with the input signal  $u$  in both the previous and proposed methods. On the other hand, in Fig. 21, both the previous and proposed methods satisfy the acceleration limit. However, the results confirm that the output signal of the previous method is completely different from the waveform of the input signal.

In the proposed method, by contrast, although there is some delay, the output signal is close to the input signal. As described above, no delay occurs in the signal satisfying the acceleration limit in the previous method, although windup occurs when a signal that does not satisfy the limit is applied. Conversely, if the gains  $K_0$  and  $K_v$  for suppressing windup are given, a delay also occurs when the acceleration limit is satisfied. On the other hand, in the proposed acceleration-limitation filter, by using the design freedom of  $C_{FB}$ , there is no delay for the signal satisfying the limit. The response to a signal that does not satisfy the limitation is closer to  $u$  than with the previous method.

Figures 22 and 23 show the comparison simulation results of the velocity-acceleration-limitation filters. The value  $\tau$  is made sufficiently small with  $K_0 = 1/\tau^2$  and  $K_v = 2\tau$  in the previous method. In Figs. 22 and 23, the dashed line is the in-

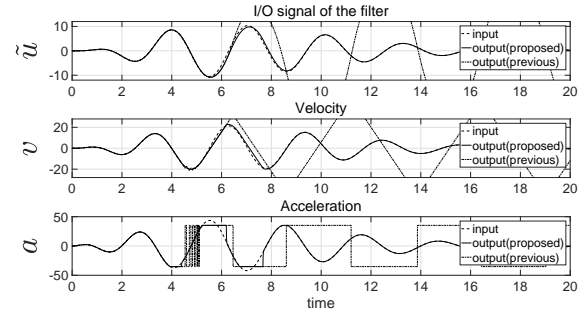


Fig. 21 Comparison of previous and proposed method:  $|a_{sat}| = 0.5a_{max}$ .

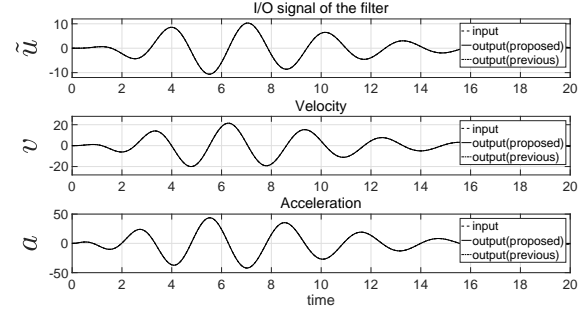


Fig. 22 Comparison of previous and proposed method:  $|v_{sat}| = \infty$ ,  $|a_{sat}| = \infty$ .

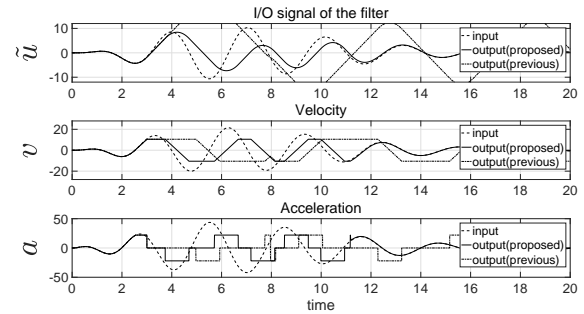


Fig. 23 Comparison of previous and proposed method:  $|v_{sat}| = 0.5v_{max}$ ,  $|a_{sat}| = 0.5a_{max}$ .

put signal, the solid line is the output signal of the proposed method, and the thick dotted line is the output signal of the previous method. The simulation proceeded with the same input as shown in Figs. 18 and 19.

Figure 22 shows the case where the input signal satisfies the velocity and acceleration limits. Figure 23 shows the case where the velocity and acceleration limit is 50% of the acceleration amplitude of the input signal. When the input signal satisfies both limits, the output signal almost coincides with the input signal  $u$  in both the previous method and the proposed method. On the other hand, in Fig. 23, both the previous and proposed methods satisfy both limits, but the output signal of the previous method is completely different from the waveform of the input signal. In the proposed method, by contrast, although there is some delay, the output signal is close to the input signal.

## 5. Conclusion

In this study, we proposed a novel filter structure to satisfy the desired limitation on signals. The proposed filter structure

is designed such that  $\tilde{u} \approx u$  holds when the signal  $u$  satisfies the limitation and  $\tilde{u}$  is close to  $u$  when the signal  $u$  does not satisfy the limitation.

By designing and simulating the proposed signal-limitation filter, we confirmed its effectiveness. Simulations with various input signals showed that the proposed method is effective for various types of signals.

In this study, we discussed a continuous-time system. However, the design problem can easily be extended to discrete-time systems, and we shall investigate this in subsequent research. The time response based on the characteristics of the input signal, constraint of the saturation function, and gain of the compensator must be strictly mathematically proven, but this is a subject for future work. Furthermore, we will discuss the configuration of a jerk-limitation filter in future work.

### References

- [1] R. Zanasi, C. G. L. Bianco and A. Tonielli: Nonlinear filters for the generation of smooth trajectories, *Automatica*, Vol. 36, No. 2000, pp. 439–448 (2000)
- [2] Y. Yuasa, N. Sakamoto and Y. Umehara: Optimal Control Designs for Systems with Input Saturations and Rate Limiters, *Proceedings of SICE Annual Conference*, pp. 2042–2045 (2010)
- [3] Y. O. Kim and I. J. Ha: Time-Optimal Control of Single-DOF Mechanical System Considering Actuator Dynamics, *IEEE Transactions on Control Systems Technology*, Vol. 11, No. 6, pp. 919–932 (2003)
- [4] G. Chen, H. Hayashi and I. Takami: Time-Optimal Trajectory Generator Under Jerk Constraints, *Proceedings of the 17th World Congress of The International Federation of Automatic Control*, Vol. 41, No. 2, pp. 6033–6038 (2008)
- [5] L. Biagiotti and R. Zanasi: Online Trajectory Planner with Constraints on Velocity, Acceleration and Torque, *Proceedings of IEEE International Symposium on Industrial Electronics*, pp. 274–279 (2010)
- [6] H. Okajima, H. Umei, N. Matsunaga and T. Asai: A Design Method of Compensator to Minimize Model Error, *SICE Journal of Control, Measurement, and System Integration*, Vol. 6, No. 4, pp. 267–275 (2013)
- [7] H. Okajima, Y. Nishimura and N. Matsunaga: A Feedback Linearization Method for Non-linear Control Systems Based on Model Error Compensator, *Transactions of the Society of Instrument and Control Engineers*, Vol. 50, No. 12, pp. 869–874 (2014) (in Japanese)
- [8] Y. Nakabayashi, H. Okajima and N. Matsunaga: Signal Limitation Filter to Satisfy Velocity and Acceleration Constraints for Arbitrary Input Signals, *Proceedings of SICE Annual Conference 2017*, FrA03.1 (2017)
- [9] H. Okajima, Y. Nakabayashi and N. Matsunaga: Signal limitation filter to satisfy velocity and acceleration limits, *Transactions of The society of instrument and control engineers*, Vol. 54, No. 1, 146–152 (2018) (in Japanese)
- [10] L. Zaccarian and A. R. Teel: Nonlinear scheduled anti-windup design for linear systems, *IEEE Transaction on Automatic Control*, Vol. 49, No. 11, pp. 2055–2061 (2004)
- [11] S. S. Kia and F. Jabbari: Multi-stage Anti-Windup Compensation for Open-loop Stable Plants, *IEEE Transaction on Automatic Control*, Vol. 56, No. 9, pp. 2166–2172 (2011)
- [12] M. V. Kothare, M. Morari: Multiplier theory for stability analysis of anti-windup control systems, *IFAC Automatica*, Vol. 35, No. 5, pp. 917–928 (1999)
- [13] N. Wada, M. Saeki: Anti-windup Control of Systems with Input Saturation, *Journal of Systems, control and information*, Vol. 46, No. 2, pp. 84–90 (2002) (in Japanese)
- [14] M. Kosaka, H. Uda, E. Bamba: Anti-Windup for 2 Degree of Freedom Model Matching Control System Using Switch, *Journal of The Japan Society of Mechanical Engineers C*, Vol. 71, No. 703, pp. 995–1002 (2005) (in Japanese)

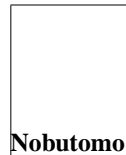
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